Optimal Monetary Policy with $r^* < 0$

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Motivation

- Widespread consensus on a substantial decline in the natural rate of interest (r^*)
- With a ZLB constraint: increased incidence of binding ZLB episodes, greater macro instability
- Existing literature: optimal monetary policy under a ZLB constraint and a positive mean natural rate $(r^* > 0)$. Normal times: $r_t^n >> 0 \Rightarrow i_t > 0$, successful stabilization of inflation and the output gap. Occasional episodes with $r_t^n < 0 \Rightarrow i_t = 0$, macro instability. Key role for forward guidance.
- *This paper*: optimal monetary policy under a ZLB constraint with $r^* < 0$. "New normal": $r_t^n < 0$. Occasional episodes with $r_t^n > 0$. Summers' "secular stagnation" speech.

What does the optimal monetary policy look like in that environment?

What are its implications for macro outcomes?

Outline

- The optimal monetary policy problem
- Equilibrium under the optimal policy: The case of a constant natural rate
- Fluctuations in response to natural rate shocks.
- Implementation
- Concluding remarks

Related Literature

- Optimal monetary policy under the ZLB: Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2005), Adam and Billi (2006), Nakov (2008), who analyze the problem of optimal policy under commitment in the basic New Keynesian model with a ZLB constraint.
- Optimal choice of an inflation target, conditional on a given interest rate rule: Coibion et al. (2012), Bernanke et al. (2019), and Andrade et al. (2020, 2021).
- Equilibrium determinacy in regime-switching models: Davig and Leeper (2007), Farmer et al. (2009) and Barthélemy and Marx (2017, 2019).

The Optimal Monetary Policy Problem

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta y_t^2\right)$$

subject to

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t \tag{1}$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n) \tag{2}$$

$$i_t \geq 0 \tag{3}$$

$$r_t^n = r^* + z_t \tag{4}$$

all for $t = 0, 1, 2, \ldots$ where $z_t \sim AR(1)$ and

$$r^* < 0$$

A Brief Detour: A Microfounded NK Model with r*<0

- Based on the NK-OLG model in Galí (AEJM, 2021)
- Consumers: constant "life" and "activity" survival rates (γ, υ). Objective function for consumer born in period s:

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\gamma)^{t-s} Z_t \log C_{t|s}$$

where $\beta \equiv \exp\{-\rho\}$ and $z_t \equiv \log Z_t \sim AR(1)$

- Firms: attached to founder, hence survival rate $\gamma\upsilon$. Calvo pricing.
- Steady state:

$$r^* = \rho + \log \upsilon$$

- Condition for $r^* < 0$

$$\upsilon < \beta$$

- Linearized equilibrium conditions:

$$\pi_t = \beta\gamma\mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n)$$

with $r_t^n = r^* + (1-\rho_z)z_t$

The Optimal Monetary Policy Problem

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta y_t^2\right)$$

subject to

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t \tag{5}$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n) \tag{6}$$

$$i_t \geq 0 \tag{7}$$

$$r_t^n = r^* + z_t \tag{8}$$

all for $t = 0, 1, 2, \ldots$ where $z_t \sim AR(1)$ and

$$r^* < 0$$

The Optimal Monetary Policy Problem

- Optimality conditions:

$$\pi_t = \xi_{1,t} - \xi_{1,t-1} + \beta^{-1}\xi_{2,t-1} \tag{9}$$

$$\vartheta y_t = -\kappa\xi_{1,t} - \sigma\xi_{2,t} + \sigma\beta^{-1}\xi_{2,t-1} \tag{10}$$

$$\xi_{2,t} \geq 0 \tag{11}$$

$$\xi_{2,t}\ \left[r_t^n + \mathbb{E}_t\{\pi_{t+1}\} + \sigma(\mathbb{E}_t\{y_{t+1}\} - y_t)\right] = 0 \tag{12}$$

with initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

Optimal Policy: The Case of a Constant Natural Rate

- Assumption

$$r_t^n = r^* < 0$$

- Steady State

$$\pi = \beta^{-1}\xi_2 \geq 0$$

$$\vartheta y = -\kappa\xi_1 + \sigma(\beta^{-1} - 1)\xi_2$$

$$\xi_2 \geq 0 \quad ; \quad r^* + \pi \geq 0$$

$$\xi_2(r^* + \pi) = 0$$

$$\Rightarrow \quad \pi \geq -r^* > 0$$

$$\Rightarrow \quad \xi_2 > 0 \quad \Rightarrow \quad i = 0 \quad \Rightarrow \quad \pi = -r^*$$

Optimal Policy: The Case of a Constant Natural Rate

- Transitional dynamics

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \kappa \widehat{y}_t$$

$$\widehat{\pi}_t = \widehat{\xi}_{1,t} - \widehat{\xi}_{1,t-1} + \beta^{-1} \widehat{\xi}_{2,t-1}$$

$$\vartheta \widehat{y}_t = -\kappa \widehat{\xi}_{1,t} - \sigma \widehat{\xi}_{2,t} + \sigma \beta^{-1} \widehat{\xi}_{2,t-1}$$

$$\widehat{\xi}_{2,t} + \xi_2 \geq 0$$

$$\widehat{\pi}_{t+1} + \sigma(\widehat{y}_{t+1} - \widehat{y}_t) \geq 0$$

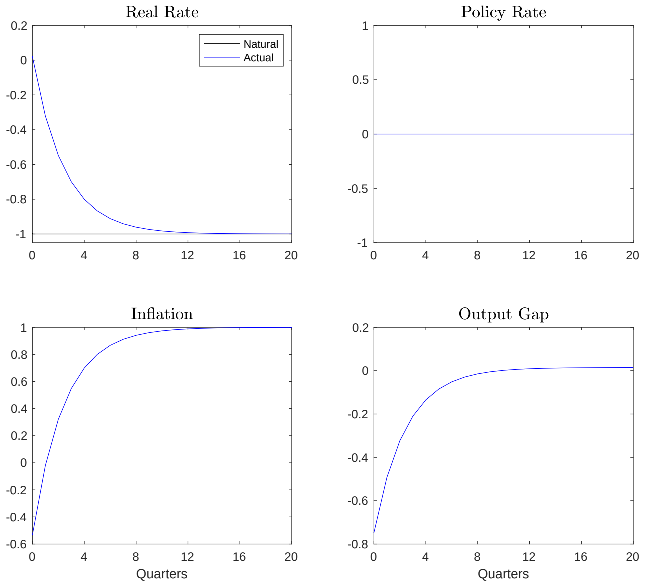
$$(\widehat{\xi}_{2,t} + \xi_2) \ [\widehat{\pi}_{t+1} + \sigma(\widehat{y}_{t+1} - \widehat{y}_t)] = 0$$

for $t = 0, 1, 2, ...$ with initial conditions $\widehat{\xi}_{1,-1} = -\xi_1$ and $\widehat{\xi}_{2,-1} = -\xi_2$ and such that $\lim_{t \to \infty} \widehat{x}_t = 0$ for $\widehat{x}_t \in \{\widehat{\pi}_t, \widehat{y}_t, \widehat{\xi}_{1,t}, \widehat{\xi}_{2,t}\}$

- Simulations for a calibrated economy

$\sigma = 1$, $\beta = 0.99$, $\kappa = 0.1717$, $\vartheta = 0.0191$ (Galí (2015))
$r = -0.0025$

Transitional dynamics under the optimal monetary policy



Fluctuations in Response to Natural Rate Shocks

- Stochastic equilibrium

$$\widehat{\pi}_t = \beta \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{\pi}_t = \widehat{\xi}_{1,t} - \widehat{\xi}_{1,t-1} + \beta^{-1}\widehat{\xi}_{2,t-1}$$

$$\vartheta \widehat{y}_t = -\kappa \widehat{\xi}_{1,t} - \widehat{\xi}_{2,t} + \beta^{-1}\widehat{\xi}_{2,t-1}$$

$$\widehat{\xi}_{2,t} + \xi_2 \geq 0$$

$$\sigma(\mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{y}_t) + \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + z_t \geq 0$$

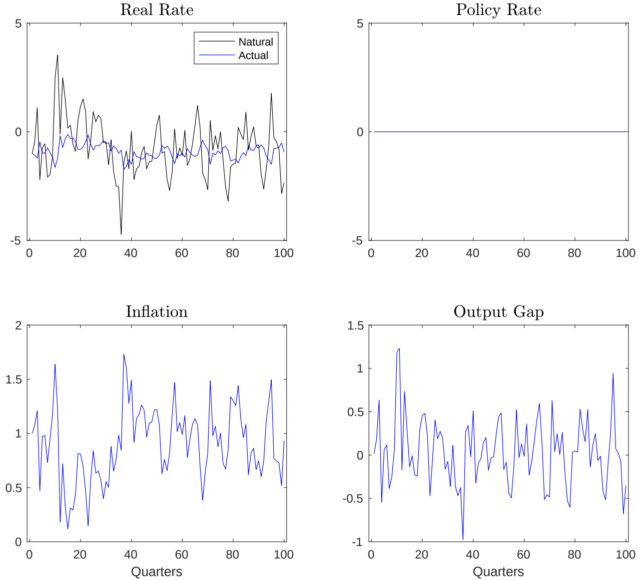
$$[\widehat{\xi}_{2,t} + \xi_2][\sigma(\mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{y}_t) + \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + z_t] = 0$$

for $t = 0, 1, 2, \ldots$ with initial conditions given by $\widehat{\xi}_{1,-1} = 0$ and $\widehat{\xi}_{2,-1} = 0$.

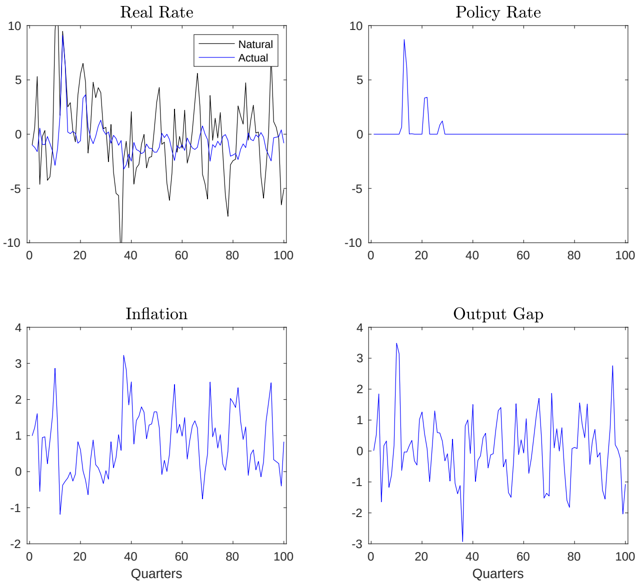
- Simulations

$(\rho_z, \sigma_z) = (0.5, 0.0025)$

Aggregate fluctuations under the optimal monetary policy



Aggregate fluctuations with higher shock volatility



Optimal Policy: Implementation

- Equilibrium outcomes under optimal policy: (i_t^*, y_t^*, π_t^*)
- Candidate rule:

$$i_t = i_t^*$$

for all t. Combined with non-policy block $\Rightarrow$ multiplicity of solutions in addition to (i_t^*, y_t^*, π_t^*)

$\Rightarrow$ *wanted*: policy rule that guarantees (i_t^*, y_t^*, π_t^*) is the only equilibrium.

Optimal Policy: Implementation

- Proposed rule:

$$i_t = \begin{cases} i_t^* + \phi_\pi^{(1)} \widetilde{\pi}_t + \phi_y^{(1)} \widetilde{y}_t & \text{if } \widetilde{\pi}_t \geq 0 \text{ and } \widetilde{y}_t \geq 0 \ \ \textit{(regime 1)} \\ i_t^* - \phi_\pi^{(2)} \widetilde{\pi}_t - \phi_y^{(2)} \widetilde{y}_t & \text{if } \widetilde{\pi}_t < 0 \text{ and } \widetilde{y}_t < 0 \ \ \textit{(regime 2)} \\ i_t^* + \phi_\pi^{(3)} \widetilde{\pi}_t - \phi_y^{(3)} \widetilde{y}_t & \text{if } \widetilde{\pi}_t \geq 0 \text{ and } \widetilde{y}_t < 0 \ \ \textit{(regime 3)} \\ i_t^* - \phi_\pi^{(4)} \widetilde{\pi}_t + \phi_y^{(4)} \widetilde{y}_t & \text{if } \widetilde{\pi}_t < 0 \text{ and } \widetilde{y}_t \geq 0 \ \ \textit{(regime 4)} \end{cases} \tag{13}$$

where $\widetilde{x}_t \equiv x_t - x_t^*$, $\phi_x^{(i)} \geq 0$ for $x \in \{\pi, y\}$ and $i \in \{1, 2, 3, 4\}$. More compactly:

$$i_t = i_t^* + \phi_{\pi,t} \, |\widetilde{\pi}_t| + \phi_{y,t} \, |\widetilde{y}_t| \tag{14}$$

where $\phi_{x,t} = \phi_x^{(i)}$ for $x \in \{\pi, y\}$ and $i \in \{1, 2, 3, 4\}$

- Non-Policy block, in deviations from optimal path:

$$\widetilde{\pi}_t = \beta \mathbb{E}_t\{\widetilde{\pi}_{t+1}\} + \kappa \widetilde{y}_t \tag{15}$$

$$\widetilde{y}_t = \mathbb{E}_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(\widetilde{i}_t - \mathbb{E}_t\{\widetilde{\pi}_{t+1}\}) \tag{16}$$

where $\widetilde{i}_t \equiv i_t - i_t^*$

Optimal Policy: Implementation

- Regime switching model representation

$$\begin{bmatrix} \widetilde{y}_t \\ \widetilde{\pi}_t \end{bmatrix} = \mathbf{A}_t \begin{bmatrix} \mathbb{E}_t\{\widetilde{y}_{t+1}\} \\ \mathbb{E}_t\{\widetilde{\pi}_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}^{(1)} \equiv \frac{1}{\sigma + \phi_y^{(1)} + \kappa\phi_\pi^{(1)}} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi^{(1)} \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y^{(1)}) \end{bmatrix}$$

$$\mathbf{A}^{(2)} \equiv \frac{1}{\sigma - \phi_y^{(2)} - \kappa\phi_\pi^{(2)}} \begin{bmatrix} \sigma & 1 + \beta\phi_\pi^{(2)} \\ \sigma\kappa & \kappa + \beta(\sigma - \phi_y^{(2)}) \end{bmatrix}$$

$$\mathbf{A}^{(3)} \equiv \frac{1}{\sigma - \phi_y^{(3)} + \kappa\phi_\pi^{(3)}} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi^{(3)} \\ \sigma\kappa & \kappa + \beta(\sigma - \phi_y^{(3)}) \end{bmatrix}$$

$$\mathbf{A}^{(4)} \equiv \frac{1}{\sigma + \phi_y^{(4)} - \kappa\phi_\pi^{(4)}} \begin{bmatrix} \sigma & 1 + \beta\phi_\pi^{(4)} \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y^{(4)}) \end{bmatrix}$$

Equilibrium Determinacy in (Possibly Endogenous) Regime Switching Models

- A benchmark regime switching model

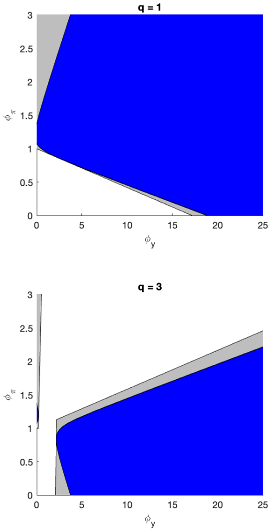
$$\mathbf{x}_t = \mathbf{A}_t \mathbb{E}_t\{\mathbf{x}_{t+1}\} \tag{17}$$

where $\mathbf{x}_t$ is an $(n \times 1)$ vector of non-predetermined variables and A_t is an $(n \times n)$ non-singular matrix. Assume $\mathbf{A}_t \in \mathcal{A} \equiv \{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, ..., \mathbf{A}^{(Q)}\}$.

- Goal: establish *sufficient* conditions on $\mathcal{A}$ that guarantee that $\mathbf{x}_t = 0$ all t is the only bounded solution to (17), i.e. $\lim_{T\to+\infty} \mathbb{E}_t\{\|\mathbf{x}_{t+T}\|\} > M\|\mathbf{x}_t\|$ for any $M > 0$ and $\mathbf{x}_t \neq 0$, and where $\|\cdot\|$ is the usual L^2 norm.
- Define $\left\|\mathbf{A}^{(q)}\right\| \equiv \max_{\mathbf{x}} \left\|\mathbf{A}^{(q)}\mathbf{x}\right\|$ subject to $\|\mathbf{x}\| = 1$. In addition, $\alpha \equiv \max\{\left\|\mathbf{A}^{(1)}\right\|, \left\|\mathbf{A}^{(2)}\right\|, ..., \left\|\mathbf{A}^{(Q)}\right\|\} > 0$.

Theorem *[sufficient condition for determinacy]*: If $\alpha < 1$, then $\mathbf{x}_t = 0$ for all t is the only bounded solution to (17)

Determinacy Regions



Optimal Policy: Implementation

- Determinacy regions
- Constant coefficient case
- Comparison with determinacy conditions in single-regime models
- Discussion: time inconsistency, credibility

Concluding remarks

- Optimal monetary policy with a ZLB constraint and $r^* < 0$.
- The optimal policy aims to approach *gradually* a steady state with positive average inflation and a binding ZLB.
- Around that steady state, inflation and the output gap display (second-best) fluctuations in response to shocks. Those fluctuations coexist with a nominal rate that remains at its ZLB most (or all) of the time.
- The central bank can implement the optimal policy as a (locally) unique equilibrium by means of an appropriate state-contingent rule.
- In order to establish that result, we derive a sufficient condition for local determinacy in a general model with endogenous regime switches, a finding that may be of interest beyond the problem studied in the present paper.