

An update of the NECMOD - a 2009 reestimation

Bureau of Macroeconomic Forecasts

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Outline

- 1 Introduction
- 2 Data
- 3 The structure of the model
- 4 Impulse response analysis
- 5 Risk analysis
- 6 Concluding remarks

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Main changes

- Reestimation on the data sample from 1995 to 2008
- Improvement of seasonal adjustment algorithms
- Changes in the model's structure including:
 - the block of external sector with exchange rate determination derived from the taste-for-variety theory,
 - more precise definition of household wealth,
 - higher level of fiscal sector sophistication,
 - refurbished specification of dynamics of inventories.



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Data preparation

Official sources

- Main source: national accounts
- Labour market: LFS data (adjusted for data bias)
- General government: in line with ESA methodology (consistent with national accounts)
- Prices of consumer goods: in line with CPI basket
- Financial data: NBP

Own estimates

- E.g. residential investment, capital stock

Seasonal adjustment

Seasonally adjusted series may be required to satisfy a few constraints simultaneously:

- Additivity (e.g. GDP and its components),
- Non-linear constraints (e.g. deflators and nominal plus real values),
- End-of-year constraint (yearly sum of raw data equals sum of adjusted series).

Solution:

- Direct adjustment (each series is individually seasonally adjusted),
- Discrepancies are redistributed using balancing algorithm,
- Share of discrepancy, falling to particular variable dependent on its variation and quality of seasonal pattern.

A new balancing algorithm

- Approach generalizes the method proposed by van der Ploeg (1982) to non-linear constraints that appear in National Account data,
- The system of weights (shares of discrepancy falling to a particular variable) was based on
 - covariance-variance matrix (as proposed in Ploeg (1982)),
 - quality of seasonal adjustment of individual series as measured by several statistics,
- Standardisation of weights so that they are homogenous for all variables
- Weights are automatically updated

A new balancing algorithm - details

Aggregated seasonally adjusted series:

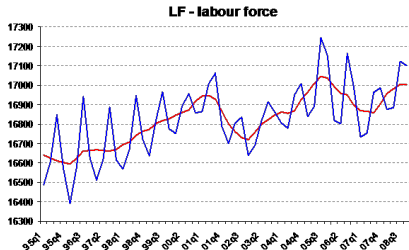
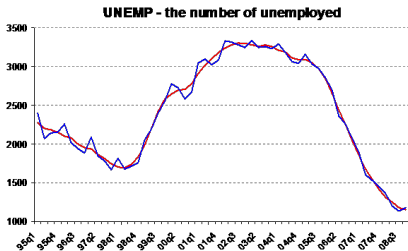
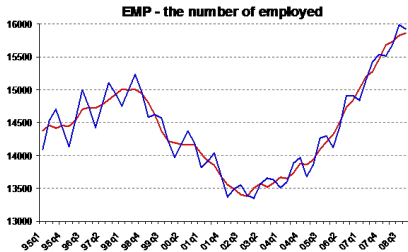
- GDP and its components
 - 20 variables and 8 equations
 - non-linear relations (deflators)
- Labour market
 - 12 variables and 4 equations
- Fiscal block
 - 49 variables and 13 equations
 - end-of-year constraint

Two stages:

- 1 Seasonal adjustment of individual series (all with Tramo/Seats)
- 2 Redistribution of discrepancies to ensure postulated relations between variables

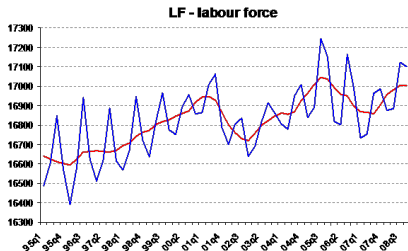
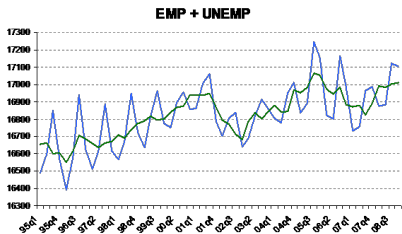


A new balancing algorithm: stage 1 - example



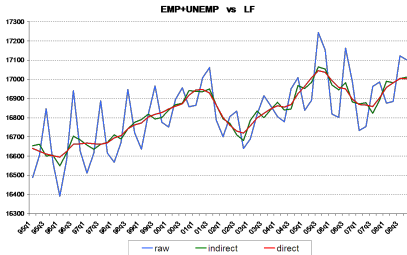


A new balancing algorithm: stage 1 - example, continued





A new balancing algorithm: stage 1 - example, continued



A new balancing algorithm: stage 2 - redistribution of discrepancies (method)

Ploeg (1982) approach with a modified variance-covariance matrix:

$$y_t^{bal} = y_t - S \cdot A \cdot (A' \cdot S \cdot A)^{-1} \cdot (A' \cdot y_t - a')$$

y_t^{bal} - vector of balanced seasonally adjusted variables in period t ,
 y_t - vector of unbalanced seasonally adjusted variables in period t ,
 S - modified variance covariance matrix (weights on diagonal),
 A - matrix of linear constraints,
 a - vector of autonomous terms related to linear constraints .

A new balancing algorithm: stage 2 - redistribution of discrepancies (weights)

Weights (shares of discrepancy falling to a particular variable):

(standard deviation of raw data) \times (seasonal adjustment quality index)

$$std(|y_t - y_{t-4}|) \times \frac{10}{8} \sum_j \left| \frac{statistic_j - optimalvalue_j}{limitvalue_j - optimalvalue_j} \right|$$

$statistic_j$ - is one from the list: Ljung-Box on residuals, Box-Pierce on residuals, Ljung-Box on squared residuals, Box-Pierce on squared residuals, normality, skewness, kurtosis, percentage of outliers, $limitvalue_j$ - the limits of the confidence intervals for the concerned test statistics (with highest confidence levels, e.g. 0.1%), $optimalvalue_j$ - the optimal values for the concerned test statistics: 3 for kurtosis and 0 for other statistics.

A new balancing algorithm: stage 2 - redistribution of discrepancies (non-linear constraints)

Problem of non-linear constraints, e.g.: $pgdp_t = gdp_n_t / gdp_t$, solved with iterative algorithm:

$$\min(y_t^{bal} - y_t) \cdot S^{-1} \cdot (y_t^{bal} - y_t)$$

subject to:

- linear constraints : $A \cdot y_t^{bal} = 0$
- non-linear constraints : nominal value = real value · deflator

$pgdp_t$ - GDP deflator, gdp_n_t - nominal GDP, gdp_t - Gross Domestic Product.



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Production function

Production function:

- combines factor inputs with a Cobb-Douglas technology

$$GDP_t = TFP_t \cdot EMP_t^{0.67} \cdot KN_t^{1-0.67}$$

GDP - gross domestic product, *TFP* - total factor productivity, *EMP* - employment, *KN* - net productive capital,

Productive capital:

- consists of corporate and public capital under assumption of their imperfect substitutability

$$KN_t = \left(0.70^3 \cdot KN_P_t^{-2} + (1 - 0.70)^3 \cdot KN_G_t^{-2} \right)^{-\frac{1}{2}}$$

KN_P - net corporate capital, *KN_G* - net public capital,

Labour:

- long run demand for labour is residually derived from production function

Capital accumulation

Capital accumulation (in line with perpetual inventory method):

- corporate capital

$$\begin{aligned}
 K_P_t &= (1 - LIK_P_t) \cdot K_P_{t-1} + 0.30 \cdot GFCF_P_t \\
 &+ 0.20 \cdot GFCF_P_{t-1} + 0.22 \cdot GFCF_P_{t-2} \\
 &+ 0.09 \cdot GFCF_P_{t-3} + 0.08 \cdot GFCF_P_{t-4} \\
 &+ 0.06 \cdot GFCF_P_{t-5} + 0.05 \cdot GFCF_P_{t-6}
 \end{aligned}$$

- public capital

$$\begin{aligned}
 K_G_t &= (1 - LIK_G_t) \cdot K_G_{t-1} + 0.14 \cdot GFCF_G_t \\
 &+ 0.15 \cdot GFCF_G_{t-1} + 0.18 \cdot GFCF_G_{t-2} \\
 &+ 0.18 \cdot GFCF_G_{t-3} + 0.15 \cdot GFCF_G_{t-4} \\
 &+ 0.12 \cdot GFCF_G_{t-5} + 0.09 \cdot GFCF_G_{t-6}
 \end{aligned}$$

K_P - gross corporate capital, K_G - gross public capital,

LIK_P - liquidation rate of corporate capital, LIK_G - liquidation rate of public capital,

$GFCF_P$ - gross fixed corporate capital formation, $GFCF_G$ - gross fixed public capital formation.

Marginal product and cost of capital

Marginal product and user cost of capital:

- enterprises equalize the marginal product of capital with real user cost of capital

$$MPK_t = 0.70^3 \cdot (1 - 0.67) \cdot \frac{GDP_POT_t}{KN_t} \cdot \left(\frac{KN_t}{KN_P_t} \right)^3$$

MPK - marginal product of productive capital, *GDP_POT* - domestic potential output,

Real user cost of capital:

- sum of real interest rate, risk premium and depreciation rate, accounting for tax shield (net effective corporate tax burden) and inflow of structural funds

$$RUCC_t = \frac{R_RATE_t/4 + DISC_P_t}{(1 - G_CORP_TR_t)(1 + G_CORP_EU_t)}$$

RUCC - real user cost of corporate capital, *R_RATE* - real interest rate, *DISC_P* - discard rate of private capital, *G_CORP_TR* - effective tax burden levied on enterprises, *G_CORP_EU* - ratio of EU structural funds directed to firms and farmers to corporate investments,

Components of real user cost of capital

- Effective tax burden levied on enterprises

$$G_CORP_TR_t = \left(1 - \frac{EMP_A_t}{EMP_t}\right) \cdot \left(GR_CIT_TR_t - \frac{GE_SUB_NOFARM_N_t + GE_CAP_TRANS_N_t}{OPSURP_N_t} \right) + \frac{EMP_A_t}{EMP_t} \cdot GR_FARM_TR_t$$

EMP_A - employment in agriculture, *GR_CIT_TR* - effective rate of taxes on the income or profits of enterprises including holding gains, *GE_SUB_NOFARM_N* - general government (GG) subsidies excluding subsidies to farmers, *GE_CAP_TRANS_N* - GG capital transfers, *OPSURP_N* - net operating surplus of enterprises, *GR_FARM_TR* - effective rate of social security contribution paid by farmers,

- Ratio of EU structural funds directed to firms and farmers to corporate investments

$$G_CORP_EU_t = \frac{TRANS_GFCF_P_N_t + TRANS_GFCF_G_N_t}{GFCF_P_t \cdot PVA_t}$$

TRANS_GFCF_P_N - other EU transfers mainly for enterprises in PLN, *TRANS_GFCF_G_N* - EU structural funds for public capital development in PLN, *PVA* - value-added deflator,

Net operating surplus of enterprises

- Net operating surplus

$$\begin{aligned} OPSURP_N_t &= GDP_N_t - GR_PROD_TAX_N_t - YD_WF_N_t \\ &\quad - GR_CORP_N_t - GR_OTAX_CORP_N_t \\ &\quad - KN_P_t \cdot PGDP_t \cdot DISC_P_t \end{aligned}$$

$GR_PROD_TAX_N$ - GG revenues from taxes on production and imports, YD_WF_N - nominal wage bill, GR_CORP_N - social security contributions paid by employers, $GR_OTAX_CORP_N$ - GG revenues from other taxes on production and products,

Financial position of enterprises

Financial position of enterprises:

- serves as proxy for financial accelerator effect,

$$\begin{aligned}
 FINACC_t = & (OPSURP_N_t + 0.51 \cdot KN_P_t \cdot PGDP_t \cdot DISC_P_t \\
 & - GR_FARM_N_t - GR_CIT_N_t \\
 & + GE_SUB_NOFARM_N_t \\
 & + GE_CAP_TRANS_N_t - YD_NOS_N_t \\
 & - YD_PRO_NOGINT_t \\
 & + G_REF_t \cdot OFE_N_t + GE_INT_NRES_N_t \\
 & + CAB_INC_EUR_NOREM_t \cdot S_EUR_PLN_t) / GDP_N_t
 \end{aligned}$$

FINACC - corporate disposable income share in GDP, *PGDP* - GDP deflator, *GR_FRAM_N* - GG revenues from social security contributions paid by farmers, *GR_CIT_N* - taxes on income or profits of firms including holding gains, *YD_NOS_N* - nominal income of households from the operating surplus, *YD_PRO_NOGINT* - nominal income of households from property excluding interests on GG debt, *G_REF* - share of pensions paid from the GG budget, *OFE_N* - GG transfers to Open Pension Funds, *GE_INT_NRES_N* - interest on GG debt held by non-residents, *CAB_INC_EUR_NOREM* - current account income balance excluding remittances, *S_EUR_PLN* - EUR/PLN exchange rate, *GDP_N* - nominal gross domestic product,

Other investments

Public investments:

- sum of government expenditures and transfers from the structural funds aimed at development of infrastructure

$$GFCF_G_t = GE_GFCF_t + \frac{TRANS_GFCF_G_N_t}{PGFCF_G_t}$$

$$\Delta ge_gfcf_t = \Delta gdp_t$$

GE_GFCF - real public investments, $PGFCF_G$ - deflator of gross fixed public capital formation,

Inventories:

- tied to the potential output and the real interest rate

$$STOCK_t^* = (0.78 - 0.55 \cdot I_3MR_PVA_t) \cdot GDP_POT_t$$

$$INV_t = STOCK_t - STOCK_{t-1}$$

$STOCK$ - level of inventories, I_3MR_PVA - real 3-month interest rate deflated with value-added deflator, INV - change in inventories

Investment demand - short run solution

$$\begin{aligned}
 \Delta gfcf_p_t &= \underset{(0.51)}{0.32} \cdot (MPK_{t-1} - RUCC_{t-1}) + \underset{(0.05)}{0.39} \cdot gfcf_p_{t-1} \\
 &+ 0.39^{3/2} \cdot \Delta gfcf_p_{t-2} \\
 &+ (1 - 0.39 - 0.39^{3/2}) \Delta gdp_t \\
 &- \underset{(0.02)}{0.02} (\Delta_4(p_ener_t + s_usd_pln_t - pva_t)) \\
 &+ \underset{(0.2)}{0.1} \cdot \Delta FINACC_t - \underset{(0.01)}{0.05} (I01Q1 + I01Q2 - I02Q3 - I06Q2)
 \end{aligned}$$

Adjusted $R^2 = 0.62$

S.E. of regression = 0.022

LM test (p-value) = 0.14

Estimation sample: 1996q1 - 2008q4

P_ENER - index of global energy prices, S_USD_PLN - USD/PLN exchange rate,
 $IxxQy$ - one period dummy variable, where xx denotes year and y stands for quarter,

Labour demand - short run solution

$$\begin{aligned}
 \Delta emp_t = & \underset{(0.10)}{0.37} \cdot \Delta emp_{t-1} \\
 & + \underset{(0.10)}{(1 - 0.37)} \cdot \Delta \log(LF_t(1 - NAWRU_t)) \\
 & - \underset{(0.03)}{0.07} \cdot (-(gdp_{t-1} - 0.67 \cdot emp_{t-1} \\
 & + (1 - 0.67) \cdot kn_{t-1} - tfp_trend_{t-1})/0.67) \\
 & + \underset{(0.09)}{0.10} \cdot (\Delta gdp - \Delta tfp_trend_t/0.67) \\
 & + \underset{(0.64)}{1.91} \Delta \sum_{i=0}^4 \frac{ALMP_N_{t-i-1}}{PGDP_{t-i-1} \cdot GDP_POT_{t-i-1}} \\
 & - \underset{(0.05)}{0.02} (\Delta (wage_n_t + gr_corp_tr_t - pva_t) - \Delta tfp_trend_t/0.67)
 \end{aligned}$$

Adjusted $R^2 = 0.74$

S.E. of regression = 0.003

LM test (p-value) = 0.001

Estimation sample: 1997q1 - 2008q4

LF - labour force, NAWRU - non-accelerating wage inflation rate of unemployment, ALMP_N - expenditures on the active labour market policy

, WAGE_N - nominal gross average wage, GR_CORP_TR - effective rate of social security contributions paid by employers

Inventories - short run solution

$$\Delta stock_t = \underset{(0.07)}{0.83} \cdot \Delta stock_{t-1} + (1 - \underset{(0.07)}{0.83}) \cdot sales_t - \underset{(0.02)}{0.05} \cdot (STOCK_{t-1} - STOCK_{t-1}^*) / GDP_POT_{t-1}$$

Adjusted $R^2 = 0.69$

S.E. of equation = 0.006

LM test (p-value) = 0.13

Estimation period: 1996q1 - 2008q4

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Consumption - long run solution

Consumption:

- modelled in line with the permanent income hypothesis,
- the permanent income proxied by the weighted average of wealth and disposable income of households,
- set of explanatory variables extended to include households' net wealth ratio (difference between their assets and liabilities normalized by nominal wealth) to capture the effects of an increase in households' debt at the end of the sample period,

$$\begin{aligned} \mathit{comp}_t^* &= 0.86 \cdot \mathit{yd}_t + (1 - 0.86) \cdot \mathit{wealth}_t - 0.35 - 0.16 \cdot \mathit{I_3MR_CPI}_t \\ &\quad + 0.24 \cdot \mathit{HH_NET_WEALTH_RATIO}_t \end{aligned}$$

CONP - individual consumption, *YD* - real disposable income, *WEALTH* - wealth, *I_3MR_CPI* - real 3-month interest rate (deflated with CPI), *HH_NET_WEALTH_RATIO* - ratio of a difference between financial assets and liabilities of households to the nominal value of wealth,

Residential investment - long run solution

Residential investments:

- supply proportional to the potential output and affected by relative house prices

$$gfcf_h_t^* = -3.50 + gdp_pot_t + 0.27 \cdot (pgfcf_h_t - 0.32 \cdot GR_VAT_TR_t - pva_t)$$

GFCF_H - gross fixed residential capital formation, *GDP_POT* - domestic potential output,

Gross residential capital:

- demand proportional to the level of consumption and influenced by demographic structure of population

$$k_h_t^* = 0.22 + conp_t - 0.07 \cdot (rucc_h_t + pgfcf_h_t - cpi_t) + 2.03(POP_M_t/POP_t)$$

K_H - gross residential capital of households, *RUCC_H* - real user cost of residential capital, *POP_M* - middle-aged total population (25-44 years), *POP* - total population (including active and inactive on the labour market), ▶

Real user cost and stock of residential capital

Real user cost of residential capital:

$$RUCC_H_t = (I_H_t - INF_TARGET_t + PREMIUM_t)/4 + DISC_H_t$$

I_H - average interest on mortgage loans (weighted average of 5-year domestic and euro zone rates), INF_TARGET - inflation target, $PREMIUM$ - wedge on the mortgage credit tied to credit risk and shallowness of the market, $DISC_H$ - discard rate of housing capital,

Gross residential capital of households:

$$K_H_t = (1 - LIK_H_t) \cdot K_H_{t-1} + \sum_{i=0}^8 GFCF_H_{t-i}$$

LIK_H - liquidation rate of housing capital,

Disposable income of households

$$\begin{aligned}
 YD_t = & (YD_WF_N_t + GE_UNEMP_N_t \cdot (1 - GR_CORP_TR_t) + \\
 & GE_PENSIONS_N_t + GE_FAMILY_N_t \cdot (1 - 0.16 \cdot GR_CORP_TR_t) \\
 & + GE_PRERETIRE_N_t + GE_SOCSECURITY_N_t + \\
 & GE_RELIEF_REST_N_t + GE_RELIEF_KIND_N_t \\
 & + TRANS_CAP_N_t + GE_SUB_FARM_N_t \\
 & - GR_EMP_N_t - GR_FARM_N_t - GR_HC_N_t \\
 & - GR_PIT_N_t - GR_PIT_CIT_N_t - GR_OTAX_HH_N_t + \\
 & REM_BALANCE_t \cdot S_EUR_PLN_t + YD_NOS_N_t + \\
 & YD_PRO_NOGIT_N_t + GE_INT_RES_N_t) / CPI_t
 \end{aligned}$$

YD_N - total nominal disposable income of households, YD_NOS_N - nominal income of households from the operating surplus, YD_PRO_N - nominal income of households from property, YD_WF_N - nominal wage bill, GE_UNEMP_N - general government (GG) expenditures on unemployment benefits, GR_CORP_TR - effective rate of social security contributions paid by employers, $GE_PENSIONS_N$ - GG expenditures on pensions, GE_FAMILY_N - GG expenditures on family benefits, $GE_PRERETIRE_N$ - GG expenditures on preretirement benefits, $GE_SOCSECURITY_N$ - GG expenditures on social benefits i.e. health, maternity benefits, $GE_RELIEF_REST_N$ - other GG expenditures on social assistance, $TRANS_CAP_N$ - Common Agricultural Policy transfers to farmers in PLN, $GE_RELIEF_KIND_N$ - GG social transfers in kind, $REM_BALANCE$ - remittances balance, S_EUR_PLN - EUR/PLN exchange rate,

$GE_SUB_FARM_N$ - GG subsidies to farmers, GR_FARM_N - social security contribution paid by farmers, GR_HC_N - GG revenues from compulsory health care contribution, GR_PIT_N - GG revenues from personal income tax, $GR_OTAX_HH_N$ - GG revenues from taxes on winnings from lottery or gambling and other current taxes, $GR_PIT_CIT_N$ - GG revenues from personal income tax levied on small enterprises, GR_EMP_N - social security contributions paid by employees,

Components of disposable income

Nominal income of households from property:

- function of nominal GDP, interest rate, wealth and households net wealth ratio,

$$yd_pro_nogint_n_t^* = gdp_n_t - 4.67 + 1.26 \cdot (WEALTH_t / GDP_t) \cdot I_3M_t + 2.11 \cdot HH_NET_WEALTH_RATIO_t$$

Nominal income of households from the operating surplus:

- expected to be proportional do GDP

$$yd_nos_n_t^* = -1.33 + gdp_n_t$$

Households' wealth

$$\begin{aligned}
 WEALTH_t = & (KN_P_t \cdot PVA_t \cdot (1 + GR_VAT_TR_t) \\
 & + KN_H_t \cdot PGFCF_H_t + G_DEBT_RES_N_t \\
 & + NFA_t - G_DEBT_NRES_N_t) / CPI_t
 \end{aligned}$$

KN_P - net corporate capital, *PVA* -foreign value added deflator, *GR_VAT_TR* - effective rate of VAT, *KN_H* - net residential capital, *PGFCF_H* - deflator of gross fixed residential capital formation, *G_DEBT_RES_N* - GG debt held by residents, *NFA* - net foreign assets in PLN, *G_DEBT_NRES_N* - GG debt held by non-residents, *CPI* - consumer price index,

Consumption - short run solution

$$\begin{aligned} \Delta \text{comp}_t = & \underset{(0.04)}{0.86} \cdot (\Delta \text{tfp_trend}_t / 0.67) + (1 - \underset{(0.04)}{0.86}) \cdot \Delta yd_t \\ & - \underset{(0.06)}{0.21} \cdot (\text{comp}_{t-1} - \text{comp}_{t-1}^*) \\ & - \underset{(0.07)}{0.06} \cdot \Delta(I_3MR_CPI_t) \end{aligned}$$

Adjusted R² = 0.08

S.E. of equation = 0.005

LM test (p-value) = 0.1082

Estimation period: 1997q2 - 2008q4

TFP_TREND - trend total factor productivity,

Residential investment - short run solution

$$\begin{aligned} \Delta gfcf_h_t &= \Delta gdp_pot_t - \underset{(0.09)}{0.24} \cdot (gfcf_h_{t-1} - gfcf_h_{t-1}^*) \\ &\quad - \underset{(0.04)}{0.18} \cdot (I02Q2_t - I02Q3_t) + \underset{(0.18)}{0.32} \cdot \Delta(pgfcf_h_{t-1}) \\ &\quad - 0.32 \cdot GR_VAT_TR_t - pva_{t-1}) \end{aligned}$$

Adjusted $R^2 = 0.43$

S.E. of equation = 0.054

LM test (p-value) = 0.29

Estimation period: 1996q1 - 2008q4

Housing prices - short run solution

$$\begin{aligned}
 \Delta pgf_{cf_h_t} = & \frac{0.34}{(0.04)} \cdot \Delta cpi_t + (1 - \frac{0.34}{(0.04)}) \cdot \Delta pgf_{cf_h_{t-1}} \\
 & - \frac{0.09}{(0.08)} \cdot (k_{h_{t-1}} - k_{h_{t-1}}^*) \\
 & - \frac{0.08}{(0.03)} \Delta(0.08 + I_{H_t} - \Delta_4 pgf_{cf_h_{t-1}}) + \frac{0.16}{(0.07)} \cdot GAP_t \\
 & + \frac{0.04}{(0.001)} \cdot (I03Q1_t - I03Q2_t) \\
 & + \frac{0.05}{(0.01)} \cdot (I06Q1_t + I06Q2_t + I06Q3_t + I06Q4_t)
 \end{aligned}$$

Adjusted $R^2 = 0.93$

S.E. of equation = 0.009

LM test (p-value) = 0.084

Estimation period: 1996q3 - 2008q4

Components of disposable income - short run solution

Nominal disposable income of households from property excluding interests on GG debt :

$$\begin{aligned} \Delta yd_pro_nogit_n_t &= -0.17 \cdot (yd_pro_nogit_n_{t-1} - yd_pro_nogit_n_{t-1}^*) \\ &\quad (0.06) \\ &\quad + 0.77 \cdot \Delta yd_pro_nogit_n_{t-1} \\ &\quad \quad (0.12) \\ &\quad + (1 - 0.76) \cdot \Delta gdp_n_{t-1} \\ &\quad \quad (0.11) \end{aligned}$$

Adjusted $R^2 = 0.50$

S.E. of equation = 0.04

LM test (p-value) = 0.20

Estimation period: 1997q2 - 2008q4

Nominal income of households from the operating surplus:

$$\begin{aligned} \Delta yd_nos_n_t &= \Delta gdp_n_t - 0.34 (yd_nos_n_{t-1} - yd_nos_n_{t-1}^*) \\ &\quad (0.09) \\ &\quad - 0.16 (I99Q2 - I99Q3) \\ &\quad (0.02) \end{aligned}$$

Adjusted $R^2 = 0.69$

S.E. of equation = 0.03

LM test (p-value) = 0.006

Estimation period: 1995q2 - 2008q4

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Participation rate

- refers to the discouragement and marginal worker effects,
- modelled separately for three different age groups (to reflect different factors impinging upon labour activity in each age group),
- participation rate fluctuates with changes in the demographic structure of population of Poland

$$\left(\frac{LF_t}{POP_t}\right)^* = \left(\frac{POP_Y_t}{POP_t}\right)\left(\frac{LF_Y}{POP_Y}\right)^* + \left(\frac{POP_M_t}{POP_t}\right)\left(\frac{LF_M_t}{POP_M_t}\right)^* + \left(\frac{POP_O_t}{POP_t}\right)\left(\frac{LF_O_t}{POP_O_t}\right)^*$$

LF - labour force, *POP* - total population, *LF_Y* - force 15-24 years, *POP_Y* - population 15-24 years, *LF_M* - labour force 25-44 years, *POP_M* - population 25-44 years, *LF_O* - labour force 45+ years, *POP_O* - population 45+ years,

Equilibrium participation rate 15-24

$$\left(\frac{LF_Y_t}{POP_Y_t}\right)^* = 1 - 0.36 \cdot \overline{UNRATE}_t$$

$$- 0.33 \cdot (1 - D99Q1_t) + 0.01 \cdot (1 - D97Q1_t)$$

$$- 0.3 \cdot (GR_EMP_TR_t + GR_PIT_TR_t$$

$$+ GR_HC_TR_t) - 0.3 \cdot (GR_VAT_TR_t$$

$$+ W_CORE_t \cdot GR_GAM_TR_t + (W_ENER_t$$

$$+ W_CORE_t) \cdot GR_EXT_TR_t) - 0.009 \cdot \min w_t$$

$$- 1.54 \cdot STUDENT_t - 0.009 \cdot RR_REM_t))$$

UNRATE - unemployment rate, *DxxQy* - shift dummy variable where xx means year and y denotes quarter when the shift takes place, *GR_EMP_TR* - effective rate of social security contributions paid by employees, *GR_PIT_TR* - effective rate of personal income tax, *GR_HC_TR* - effective rate of compulsory health care contribution, *GR_VAT_TR* - effective rate of VAT, *W_CORE* - weight of core inflation in CPI basket, *GR_GAM_TR* - effective rate of gambling taxes, *W_ENER* - weight of energy prices in CPI basket, *GR_EXT_TR* - effective rate of excise duties, *MINW* - relation of minimum wage to the nominal average gross wage in the economy, *STUDENT* - ratio of non-extramural students to population 15-24, *RR_REM* - replacement rate for remittances,

Equilibrium participation rate 25-44

$$\begin{aligned}
 \left(\frac{LF_M_t}{POP_M_t}\right)^* &= 0.89 + 0.02 \cdot \overline{UNRATE}_{t-1} \\
 &\quad - 0.01 \cdot rr_nfl_m_{t-1} \\
 &\quad - 0.1 \cdot (GR_EMP_TR_{t-1} + GR_PIT_TR_t \\
 &\quad + GR_HC_TR_{t-1}) - 0.1 \cdot (GR_VAT_TR_t \\
 &\quad + W_CORE_{t-1} \cdot GR_GAM_TR_{t-1} + (W_ENER_{t-1} \\
 &\quad + W_CORE_{t-1}) \cdot GR_EXT_TR_{t-1}) \\
 &\quad - 0.01 \cdot (1 - D99Q1_t) - 0.003 \cdot (1 - D97Q1_t)
 \end{aligned}$$

RR_NFL_M - replacement rate for non-participants 25-44 (including disability and retirement benefits and social reliefs),

Equilibrium participation rate 45+

$$\begin{aligned}
 \left(\frac{LF_{-}O_t}{POP_{-}O_t}\right)^* &= -0.40 - 0.28 \cdot \overline{UNRATE}_{t-1} \\
 &\quad - 0.18 \cdot rr_nfl_o_t - 0.3 \cdot (GR_EMP_TR_t \\
 &\quad + GR_PIT_TR_t + GR_HC_TR_t) \\
 &\quad - 0.3 \cdot (GR_VAT_TR_t \\
 &\quad + W_CORE_t \cdot GR_GAM_TR_t + (W_ENER_{t-1} \\
 &\quad + W_CORE_t) \cdot GR_EXT_TR_t) \\
 &\quad + WORKAGE_t - 0.09 \cdot rr_relief_kind_t \\
 &\quad + 0.04 \cdot (1 - D97Q1_t)
 \end{aligned}$$

RR_NFL_O - replacement rate for non-participants 45+ (including disability, retirement benefits and social relief), $WORKAGE$ - share of working age population in the population 45+, RR_RELIEF_KIND - replacement rate for social transfers in kind,

Labour force 15-24 - short run

$$\begin{aligned} \Delta lf_y_t = & \underset{(0.15)}{-0.30} \cdot \left(\frac{LF_Y_{t-1}}{POP_Y_{t-1}} - \left(\frac{LF_Y_{t-1}}{POP_Y_{t-1}} \right)^* \right) \\ & + \Delta pop_y_t + \underset{(0.18)}{0.42} \cdot (\Delta(wage_n_{t-1} - cpi_{t-1})) \\ & - \Delta tfp_trend_{t-1} / 0.67) + \underset{(0.03)}{0.05} \cdot \Delta minw_t \end{aligned}$$

Adjusted $R^2 = 0.35$

S.E. of equation = 0.01

LM test (p-value) = 0.11

Estimation period: 1996q1 - 2008q4

WAGE_N - nominal gross average wage, CPI - consumer price index, TFP_TREND - trend total factor productivity,

Labour force 25-44 - short run

$$\begin{aligned}
 \Delta lf_m_t = & -\frac{0.40}{(0.13)} \cdot \left(\frac{LF_M_{t-1}}{POP_M_{t-1}} - \left(\frac{LF_M_{t-1}}{POP_M_{t-1}} \right)^* \right) \\
 & + \frac{0.15}{(0.09)} \cdot \Delta lf_m_{t-1} + \frac{(0.14)^2}{(0.10)} \cdot \Delta lf_m_{t-2} \\
 & + \Delta pop_m_t - \frac{0.01}{(0.01)} \cdot \Delta rr_nfl_m_t
 \end{aligned}$$

Adjusted $R^2 = 0.69$

S.E. of equation = 0.002

LM test (p-value) = 0.013

Estimation period: 1996q1 - 2008q4

Labour force 45+ - short run

$$\begin{aligned}
 \Delta lf_o_t = & -0.14 \cdot \left(\frac{LF_O_{t-1}}{POP_O_{t-1}} - \left(\frac{LF_O_{t-1}}{POP_O_{t-1}} \right)^* \right) + \Delta pop_o_t \\
 & -0.08 \cdot \Delta rr_nfl_o_t - 0.08 \cdot 0.5 \cdot \Delta rr_relief_kind_t \\
 & + \Delta WORKAGE_t + 0.08(\Delta wage_n_{t-1} \\
 & - \Delta cpi_{t-1} - \Delta tfp_trend_{t-1}/0.67)
 \end{aligned}$$

Adjusted $R^2 = 0.32$

S.E. of equation = 0.007

LM test (p-value) = 0.014

Estimation period: 1996q1 - 2008q4

- 3 The structure of the model
 - Production sector
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Wages - long run solution

- set in the bargaining process between firms and workers,
- determined by long-run labour demand and supply curves together with short-term dynamics of labour productivity and unemployment rate,

$$\begin{aligned}
 wage_n_t^* = & 5.51 + (1/0.67) \cdot tfp_trend_t + cpi_t - 0.5 \cdot (GR_VAT_TR_t \\
 & + (W_ENER_t + W_CORE_t) \cdot GR_EXT_TR_t \\
 & + W_CORE_t \cdot GR_GAM_TR_t) \\
 & - 0.88 \cdot UNRATE_t + 0.06 \cdot rr_unemp_t + 0.5 \cdot (GR_PIT_TR_t \\
 & + GR_HC_TR_t + GR_EMP_TR_t) - 0.5 \cdot GR_CORP_TR_t \\
 & - 0.11 \cdot rr_rem_t + 5.51 \cdot \Delta pop_y_t \\
 & + 0.47 \cdot \frac{POP_Y_t}{POP_t} \cdot minw_t
 \end{aligned}$$

WAGE_N - gross nominal average wage in the economy, *TFP_TREND* - trend total factor productivity, *CPI* - consumer price index, *GR_VAT_TR* - effective rate of VAT, *W_ENER* - weight of energy prices in CPI basket, *W_CORE* - weight of core inflation in CPI basket, *GR_EXT_TR* - effective rate of excise duties, *GR_GAM_TR* - effective rate of gambling taxes, *UNRATE* - unemployment rate, *RR_UNRATE* - replacement rate for unemployed (including unemployment benefits), *GR_PIT_TR* - effective rate of personal income tax, *GR_HC_TR* - effective rate of compulsory health care contribution, *GR_EMP_TR* - effective rate of social security contributions paid by employees, *GR_CORP_TR* - effective rate of social security contributions paid by employers, *RR_REM* - replacement rate for remittances, *POP_Y* - population 15-24, *POP* - total population, *MINW* - relation of minimum wage to the average gross nominal wage in the economy,

NAWRU

- defined uniquely at equilibrium level of labour demand and supply,
- the responsiveness of NAWRU to economic developments over the cycle is mainly determined by labour share in GDP and semi-elasticity of wages with respect to unemployment,

$$\begin{aligned}
 NAWRU_t = & (1 - 0.67 + 0.88)^{-1} \cdot ((cpi_t - pva_t + (0.08/0.68) \cdot d04q3 \\
 & - 0.5 \cdot (GR_VAT_TR_t + (W_ENER_t + W_CORE_t) \cdot GR_EXT_TR_t \\
 & + W_CORE_t \cdot GR_GAM_TR_t)) + 0.5 \cdot (GR_PIT_TR_t \\
 & + GR_HC_TR_t + GR_EMP_TR_t + GR_CORP_TR_t) \\
 & + 0.06 \cdot rr_unemp_t + 0.47 \cdot \left(\frac{POP_Y_t}{POP_t}\right) \cdot minw_t \\
 & + 0.47 \cdot (pimp_t + GR_TAR_TR_t - pva_t \\
 & - \left(\frac{EMP_A_t}{LF_t}\right) + (1 - 0.67) \cdot [lf_t - k_n_t] \\
 & + ((1/0.67) - 1) \cdot tfp_trend_t + 0.5 - 0.11 \cdot rr_rem_t)
 \end{aligned}$$

NAWRU - non-accelerating wage inflation rate of unemployment, PVA - value added deflator, DxxQy - shift dummy variable where xx means year and y denotes quarter when the shift takes place, PIMP^C - imports prices corrected for the equilibrium exchange rate fluctuations, GR_TAR_TR - effective rate of import duties, EMP_A - employment in agriculture, LF - labour force, KN - net productive capital, TFP_TREND - trend total factor productivity,

Wages - short run solution

- short-run deviations of wages from the long-term path are tied to wage inertia, growth rate of labour productivity and unemployment rate,

$$\begin{aligned}
 \Delta wage_n_t &= 0.05 \cdot (wage_n_{t-1} - wage_n_{t-1}^*) \\
 &\quad (0.02) \\
 &\quad + 0.05(pva_{t-1} - pva_{t-1}^*) \\
 &\quad (0.03) \\
 &\quad + 0.73 \cdot \Delta wage_n_{t-1} + (1 - 0.73) \cdot \Delta cpi_{t-1} \\
 &\quad (0.12) \qquad (0.12) \\
 &\quad + (1 - 0.73) \cdot (\Delta gdp_{t-1} - \Delta emp_na_{t-1}) - 0.28 \Delta UNRATE_{t-1} \\
 &\quad (0.11) \qquad (0.21)
 \end{aligned}$$

Adjusted $R^2 = 0.74$

S.E. of equation = 0.007

LM test (p-value) = 0.001

Estimation period: 1996q1 - 2008q4

ULCNA - unit labour costs in non-agriculture, GDP - gross domestic product, EMP_NA - non-agriculture employment,

- 3 The structure of the model
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General characteristic of the external sector

- process of trade integration interpreted jointly with the growth convergence,
- the equilibrium exchange rate assures stabilization of the net foreign assets to GDP ratio at its equilibrium level,
- degree of pass-through of the real exchange rate into trade prices and price elasticities of exports and imports jointly determine the optimal response of the exchange rate to the divergence of growth rates between Poland and its trading partners as well as to the evolution of the net foreign assets position,
- the net foreign assets accumulation equation relates the category to the sum of the past and present current account balances and accounts for the valuation effects

Exports and imports - long run solution

- export activity is proportional to domestic potential product,

$$\begin{aligned} gdp_exp &= gdp_ext_t - 19.78 + 1.51 \cdot gdp_pot_t \\ &\quad - 0.78 \cdot pexp_t - (pva_ext_t + s_neer_t) \\ &\quad + 0.22 \cdot D04Q03 \end{aligned}$$

- import intensity is proportional to potential product of the country's trade partners,

$$\begin{aligned} gdp_imp &= gdp_t - 12.2 + 1.51 \cdot gdp_ext_pot_t \\ &\quad - 1.52 \cdot (pimp_t + GR_TAR_TR_t - pva_t) + 0.04 * D04Q3 \end{aligned}$$

GDP_EXP - exports, GDP_EXT - foreign GDP (weighted average of GDP in the euro area, the UK, and the USA), GDP - gross domestic product,

Export and import prices - long run solution

● Export prices

$$pexp_t^* = pva_ext_t + s_neer_t - 3.02 - 0.52 \cdot s_reer_t + 0.07 \cdot D04Q3$$

PEXP - export deflator, *PVA_EXT* - foreign value added deflator, *S_NEER* - nominal effective exchange rate, *S_REER* - real effective exchange rate, *DxxQy* - shift dummy variable where xx means year and y denotes quarter when the shift takes place,

● Import prices

$$pimp_t^* = pva_t - GR_TAR_TR_t - 4.08 + 0.66 \cdot s_reer_t + 0.04 \cdot D04Q3$$

PIMP - deflator of imports, *PVA* - deflator of value-added, *GR_TAR_TR* - effective rate of import duties,

● Import prices excluding prices of oil and gas

$$pimp_core_t = (pimp_t - 0.07 \cdot (p_oil_t + s_usd_pln_t) - 0.03 \cdot (p_gas_t + s_usd_pln_t)) / 0.90$$

PIMP_CORE - deflator of imports excluding prices of oil and gas, *P_OIL* - price of oil BRENT,

S_USD_PLN - exchange rate USD/PLN, *P_GAS* - global gas prices (price of Russian gas per 1000 cubic meters),

Equilibrium exchange rate

$$\begin{aligned}
 \Delta s_{reer_eq_t} = & ((1 - 1.51)/(1 + 0.78 \cdot 0.52 \\
 & + 1.51 \cdot 0.66 - 0.52 - 0.66)) \cdot (\Delta gdp_pot_t \\
 & - \Delta gdp_ext_pot_t) + (1/(1 + 0.78 \cdot 0.52 + 1.52 \cdot 0.66 - 0.52 \\
 & - 0.66)) \cdot (\Delta TCAB_t - \Delta CAB_TRANS_INC_GDP_t)/OPEN_t \\
 & + (1/(1 + 0.78 \cdot 0.52) + 1.52 \cdot 0.66 - 0.52 \\
 & - 0.66)) \cdot \Delta GR_TAR_TR_t
 \end{aligned}$$

S_REER_EQ - the equilibrium exchange rate, GDP_POT - domestic potential output, GDP_EXT_POT - foreign potential output (weighted average of the potential output in euro area, the UK, and the USA), $TCAB$ - the equilibrium current account to GDP ratio, $CAB_TRANS_INC_GDP$ - ratio of current account income and transfer balances to GDP, $OPEN$ - measure of openness - ratio of imports and exports to GDP, GR_VAT_TR - effective rate of VAT,

NFA and balance of payments

Net foreign assets in PLN:

$$NFA_t = NFA_{t-1} + NFA_{t-1} \cdot \left(\frac{S_{NEER_t}}{S_{NEER_{t-1}}} - 1 \right) + CAB_{-t} \cdot S_{EUR_PLN_t}$$

$$NFA_{t_GDP_t} = NFA_t / GDP_{-N_t}$$

NFA - net foreign assets in PLN, *CAB* - current account balance (including the capital account) in EUR, *S_{EUR_PLN}* - EUR/PLN exchange rate, *NFA_GDP* - net foreign assets to GDP ratio,

Current account balance (including the capital account):

$$CAB_t = CAB_{NT_t} + CAB_{TRANS_EUR_NOREM_t} + CAB_{INC_EUR_NOREM_t} + REM_BALANCE_t$$

CAB_{NT} - trade balance of goods and services in EUR, *CAB_{TRANS_EUR_NOREM}* - current account transfers balance excluding remittances in EUR, *CAB_{INC_EUR_NOREM}* - current account income balance excluding remittances in EUR, *REM_BALANCE* - remittances balance,

Balance of payments - actual vs equilibrium

Trade balance of goods and services:

$$CAB_NT_t = (GDP_EXP_t \cdot PEXP_t - GDP_IMP_t \cdot PIMP_t) / S_EUR_PLN_t$$

Ratio of current account balance to GDP:

$$CAB_GDP_t = \frac{CAB_t \cdot S_EUR_PLN_t}{GDP_N_t}$$

The equilibrium current account to GDP ratio:

$$\begin{aligned} TCAB_t = & 0.025 \cdot (-2.6 - NFA_GDP_{t-1}) + (NFA_GDP_{t-1}) \\ & \cdot ((1 + INF_TARGET_t)^{0.25}) - 1 + \Delta gdp_pot_t \\ & - NFA_GDP_{t-1} \cdot ((1 + INF_TARGET_t)^{0.25} \\ & - 1 - 0.005) / ((1 + INF_TARGET_t)^{0.25} + \Delta gdp_pot_t) \end{aligned}$$

Exports and imports - short run solution

$$\begin{aligned} \Delta gdp_exp_t &= \Delta gdp_ext_t - \frac{0.12}{(0.07)} \cdot (gdp_exp_{t-1} - gdp_exp_{t-1}^*) \\ &+ \frac{0.36}{(0.15)} \cdot \Delta(gdp_exp_{t-1} - gdp_ext_{t-1}) \\ &- \frac{0.26}{(0.14)} \cdot \Delta(pexp_{t-1} - pva_{t-1}) + \frac{0.29}{(1.34)} \cdot \Delta(gdp_ext_{t-1} - \frac{tfp_ext_{t-1}}{0.67}) \\ &+ \frac{0.78}{(0.32)} \cdot \Delta gdp_pot_{t-1} \end{aligned}$$

Adjusted $R^2 = 0.02$

S.E. of equation = 0.027

Q test (p-value) = 0.008

Estimation period: 1996q2 - 2008q4

$$\begin{aligned} \Delta gdp_imp_t &= gdp_t - \frac{0.13}{(0.05)} \cdot (gdp_imp_{t-1} - gdp_imp_{t-1}^*) \\ &- \frac{0.55}{(0.09)} \cdot \Delta(pimp_t + GR_TAR_TR_t - pva_t) \\ &+ \frac{2.16}{(0.22)} \cdot \Delta \log((0.4 \cdot GFCF_t + 0.2 \cdot CONP_t + 0.5 \cdot CONGOV_t \\ &+ 0.4 \cdot GDP_EXP_t) / TFP_TREND_t^{0.67}) \end{aligned}$$

Adjusted $R^2 = 0.73$

S.E. of equation = 0.022

Q test (p-value) = 0.120

Estimation period: 1995q2 - 2008q4

TFP_EXT - foreign trend total factor productivity, $GFCF$ - gross fixed capital formation, $CONP$ - individual consumption, $CONGOV$ - government consumption,

Export and import prices - short run solution

$$\begin{aligned} \Delta pexp_t = & \underset{(0.09)}{-0.42} \cdot (pexp_{t-1} - pexp_{t-1}^*) \\ & + \underset{(0.18)}{0.38} \cdot \Delta pva_{t-1} + \underset{(0.10)}{0.26} \Delta s_reer_t \end{aligned}$$

Adjusted $R^2 = 0.24$

S.E. of equation = 0.030

Q test (p-value) = 0.041

Estimation period: 1995q3 - 2008q4

$$\begin{aligned} \Delta pimp_t = & \underset{(0.08)}{-0.33} \cdot (pimp_{t-1} - pimp_{t-1}^*) \\ & + \underset{(0.17)}{0.41} \cdot \Delta pva_{t-1} + \underset{(0.09)}{0.34} \Delta s_reer_t \\ & + \underset{(0.02)}{0.02} \Delta(0.7 \cdot (p_oil_{t-1} + s_usd_p \ln_{t-1})) \\ & + 0.3 \cdot (p_gas_{t-1} + s_usd_p \ln_{t-1}) \end{aligned}$$

Adjusted $R^2 = 0.27$

S.E. of equation = 0.028

Q test (p-value) = 0.041

Estimation period: 1995q3 - 2008q4

Exchange rate - short run solution

$$\begin{aligned}
 \Delta s_{reer}_t = & -\underset{(0.12)}{0.33} \cdot (s_{reer}_{t-1} - s_{reer}_{eq_{t-1}} - \underset{(0.07)}{1.78}) \\
 & -\underset{(0.32)}{0.69} \cdot (I_{3MR_PVA}_t - I_{3MR_EXT}_t) - \\
 & -\underset{(0.52)}{1.67} \cdot GAP_{t-1} - \underset{(2.12)}{2.27} \cdot \Delta G_BALANCE_GDP_t \\
 & -\underset{(0.59)}{1.18} \cdot \Delta CAB_GDP_t \\
 & -\underset{(0.78)}{0.67} \cdot (I_{5Y} - INF_TARGET_t - I_{5Y_EUR}_t - 0.02)
 \end{aligned}$$

Adjusted $R^2 = 0.31$

S.E. of equation = 0.041

LM test (p-value) = 0.083

Estimation period: 2001q1 - 2008q4

I_{3MR_PVA} - real 3-month interest rate deflated with the value added deflator, I_{3MR_EXT} - real 3-month foreign interest rate deflated with the foreign value added deflator, GAP - output gap, $G_BALANCE_GDP$ - general government balance as percentage of GDP, GDP_N - nominal gross domestic product, I_{5Y} - yield on 5-year government bonds, I_{5Y_EUR} - yield on 5-year Bunds,

Outline

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GDP identity

- Real gross domestic product

$$GDP_t = CONP_t + CONGOV_t + GFCF_t \\ + INV_t + GDP_EXP_t - GDP_IMP_t$$

GDP - gross domestic product, *CONP* - individual consumption, *CONGOV* - government consumption, *GFCF* - gross fixed capital formation, *INV* - change in inventories, *GDP_EXP* - exports, *GDP_IMP* - imports,

- Nominal gross domestic product

$$GDP_N_t = CONP_N_t + CONGOV_N_t + GFCF_N_t \\ + INV_N_t + GDP_EXP_N_t - GDP_IMP_N_t \\ = CPI_t \cdot CONP_t + PCONGOV_t \cdot CONGOV_t \\ + GFCF_N_t + PVA_t \cdot INV_t + PEXP_t \cdot GDP_EXP_t \\ - PIMP_t \cdot GDP_IMP_t$$

X_N - nominal value of variable *X*, *CPI* - consumer price index, *PCONGOV* - government consumption deflator, *PVA* - value added deflator, *PEXP* - export prices, *PIMP* - import prices,

Aggregate investment demand

$$GFCF_t = GFCF_P_t + GFCF_G_t + GFCF_H_t$$

$$\begin{aligned} GFCF_N_t &= PVA_t \cdot (1 + GR_VAR_TR_t) \cdot GFCF_P_t \\ &\quad + PGFCF_G_t \cdot GFCF_G_t \\ &\quad + PGFCF_H_t \cdot GFCF_H_t \end{aligned}$$

$GFCF_P$ - gross fixed corporate capital formation, $GFCF_G$ - gross fixed public capital formation, $GFCF_H$ - gross fixed residential capital formation, GR_VAR_TR - effective rate of VAT, $PGFCF_G$ - deflator of gross fixed public capital formation, $PGFCF_H$ - deflator of gross fixed residential capital formation,

Deflator, potential output, output gap

- GDP deflator

$$PGDP_t = \frac{GDP_N_t}{GDP_t}$$

- Potential output

$$GDP_POT_t = (LF_EQ_t \cdot POP_t \cdot (1 - NAWRU_t))^{0.67} \cdot (KN_t)^{(1-0.67)} \cdot TFP_TREND_t$$

- Output gap

$$GAP_t = \frac{GDP_t}{GDP_POT_t} - 1$$

PGDP - GDP deflator, *GDP_POT* - domestic potential output, *LF_EQ* - the equilibrium participation rate, *POP* - total population, *NAWRU* - non-accelerating wage inflation rate of unemployment, *KN* - net productive capital, *TFP_TREND* - trend total factor productivity, *GAP* - output gap,

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Prices - long run solution (1)

Value added deflator:

$$pva_t^* = -3.38 + 0.68 \cdot ulcna_t + (1 - 0.68) \cdot (pimp_t + GR_TAR_TR_t) + 0.09 \cdot D04Q3$$

Core CPI:

$$corecpi_t^* = 3.19 + 0.53 \cdot ulcna_t + (1 - 0.53) \cdot (pimp_core_t + GR_TAR_TR_t) + GR_VAT_TR_t + GR_GAM_TR_t + GR_EXT_REST_TR_t + 0.6 \cdot BS_TREND_t$$

PVA - value added deflator, *ULCNA* - unit labour costs in non-agriculture, *PIMP^C* - imports prices corrected for the equilibrium exchange rate fluctuations, *GR_TAR_TR* - effective rate of import duties, *DxxQy* - shift dummy variable where xx means year and y denotes quarter when the shift takes place, *CORECPI* - core CPI index (CPI excluding food and energy prices), *PIMP_CORE* - import prices excluding prices of oil and gas, *GR_TAR_TR* - effective rate of import duties, *GR_VAT_TR* - effective rate of VAT, *GR_GAM_TR* - effective rate of gambling taxes, *GR_EXT_REST_TR* - effective rate of excise duties on goods other than related to energy (fuels and gas), *BS_TREND* - trend in the core inflation equation, truncated from 2002Q3

Prices - long run solution (2)

Food prices:

$$\begin{aligned} foodcpi_t^* &= 1.32 + 0.41 \cdot (p_food_t + s_usd_pln_t) \\ &\quad + (1 - 0.41) \cdot corecpi_t + 0.41 \cdot GR_VAT_TR_t \end{aligned}$$

Energy prices:

$$\begin{aligned} energcpi_t^* &= 0.66 + 0.27 \cdot (p_ener_t + s_usd_pln_t \\ &\quad + GR_VAT_TR_t) + (1 - 0.27) \cdot (corecpi \\ &\quad - GR_EXT_REST_TR_t) + GR_EXT_ENER_TR_t \end{aligned}$$

FOODCPI - food prices CPI based index, *P_FOOD_BASE* - world price index of agricultural products, *S_USD_PLN* - USD/PLN exchange rate corrected for the equilibrium exchange rate fluctuations, *S_REER_EQ* - the equilibrium exchange rate, *ENERCPI* - energy prices CPI based index, *P_ENER* - index of global energy prices, *GR_EXT_ENER_TR* - effective rate of excise duties on goods related to energy (fuels and gas),

Value added deflator - short run solution

$$\begin{aligned}
 \Delta pva_t = & \left((1 + \overline{inf_target}_{t+2})^{0.25} - 1 \right) \cdot \left(1 - \frac{0.10}{(0.05)} - \frac{0.20}{(0.05)} - \frac{0.31}{(0.06)} \right) \cdot \Delta pva_{t-1} \\
 & + \frac{0.21}{(0.05)} \cdot pva_{t+1} + \frac{0.682}{(0.06)} \cdot \frac{0.31}{(0.06)} \cdot \Delta ulcna_{t-1} \\
 & + (1 - 0.682) \cdot \frac{0.31}{(0.06)} \cdot \Delta (pimp_{t-1} + GR_TAR_TR_{t-1}) \\
 & - \frac{0.05}{(0.009)} \cdot (pva_{t-1} - pva_{t-1}^*) \\
 & + \frac{0.04}{(0.004)} \cdot I04Q2_t
 \end{aligned}$$

Adjusted $R^2 = 0.45$

S.E. of equation = 0.007

J statistic(p-value) = 0.1

Estimation period: 1997q4 - 2008q4

$\overline{INF_TARGET}$ - 4-quarter moving average of inflation target, $IxxQy$ - one period dummy variable, where xx denotes year and y stands for quarter,

Core inflation - short run solution

$$\begin{aligned}
 \Delta corecpi_t = & \left((1 + \overline{inf_target}_{t+2})^{0.25} - 1 \right) \cdot \left(1 - \frac{3.35 \cdot 0.10}{(2.01) (0.05)} - \frac{0.21 \cdot 2.49}{(0.05) (0.65)} \right. \\
 & \left. - \frac{0.11 \cdot 0.31}{(0.04) (0.06)} \right) + \frac{3.35 \cdot 0.10}{(2.01) (0.06)} \cdot \Delta corecpi_{t-1} \\
 & + \frac{0.21 \cdot 2.49}{(0.05) (0.65)} \cdot corecpi_{t+1} + \frac{0.11 \cdot 0.53 \cdot 0.31}{(0.03) (0.06)} \cdot \Delta ulcna_{t-1} \\
 & + \frac{0.11 \cdot 0.31}{(0.03) (0.06)} \cdot (1 - 0.53) \cdot \Delta (pimp_core_{t-1} + GR_TAR_TR_{t-1}) \\
 & + \frac{0.36}{(0.14)} \cdot \frac{(-0.05)}{(0.01)} \cdot (corecpi_{t-1} - corecpi_{t-1}^*)
 \end{aligned}$$

Adjusted $R^2 = 0.92$

S.E. of equation = 0.003

J statistic(p-value) = 0.1

Estimation period: 1997q4 - 2008q4

Food and energy prices - short run solution

$$\begin{aligned} \Delta foodcpi_t &= \underset{(0.12)}{0.58} \cdot \Delta foodcpi_{t-1} + \underset{(0.12)}{0.38} \cdot \Delta corecpi_{t-1} \\ &\quad - \underset{(0.04)}{0.13} \cdot (foodcpi_{t-1} - foodcpi_{t-1}^*) \\ &\quad + (1 - \underset{(0.12)}{0.58} - \underset{(0.12)}{0.38}) \\ &\quad \cdot \Delta(p_food_base + s_usd_p \ln - s_reer_eq) \end{aligned}$$

Adjusted $R^2 = 0.48$

S.E. of equation = 0.01

LM test (p-value) = 0.12

Estimation period: 1996q2 - 2008q4

$$\begin{aligned} \Delta enercpi_t &= \underset{(0.12)}{0.25} \cdot \Delta enercpi_{t-1} - \underset{(0.06)}{0.10} \cdot (enercpi_{t-1} - enercpi_{t-1}^*) \\ &\quad + \underset{(0.01)}{0.04} \cdot \Delta(p_ener_t + s_usd_p \ln_t - s_reer_eq) \\ &\quad + (1 - \underset{(0.12)}{0.25} - \underset{(0.02)}{0.04}) \cdot \Delta(corecpi_t - GR_EXT_REST_TR_t) \\ &\quad + (1 - \underset{(0.12)}{0.25}) \cdot \Delta GR_EXT_ENER_TR_t \end{aligned}$$

Adjusted $R^2 = 0.25$

S.E. of equation = 0.01

LM test (p-value) = 0.17

Estimation period: 1996q2 - 2008q4

3 The structure of the model

- Production sector
- Households' sector
- Labour supply
- Labour market
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- Aggregate demand and supply
- Cost and prices
- **Fiscal sector**
- The monetary authority and interest rates

General government revenues

$$\begin{aligned} GR_{N_t} = & GR_{PROD_TAX_N_t} + GR_{INC_TAX_N_t} \\ & + GR_{TCONTR_N_t} + GR_{PROP_INC_N_t} \\ & + GR_{OTHER_CURT_N_t} + GR_{OUTPUT_N_t} \\ & + GR_{CAP_TRANS_N_t} \end{aligned}$$

GR_N - total general government (GG) revenues, $GR_{PROD_TAX_N}$ - GG revenues from taxes on production and imports, $GR_{INC_TAX_N}$ - GG revenues from income and wealth taxes, GR_{TCONTR_N} - total social contributions, $GR_{PROP_INC_N}$ - GG's property income, $GR_{OTHER_CURT_N}$ - GG revenues from other current transfers, GR_{OUTPUT_N} - GG market output, output for own final use and payments for other non-market output, $GR_{CAP_TRANS_N}$ - GG revenues from capital transfers,

General government expenditures

$$\begin{aligned}
 GE_N_t = & GE_RELIEF_KIND_N_t + GE_SOC_CASH_N_t \\
 & + GE_SUB_FARM_N_t + GE_SUB_NOFARM_N_t \\
 & + GE_OTHER_TRANS_N_t + GE_EU_N_t \\
 & + GE_FIN_N_t + GE_WF_N_t + GE_CON_N_t \\
 & + GE_GFCF_N_t + GE_CAP_TRANS_N_t
 \end{aligned}$$

GE_N - total GG expenditures, $GE_RELIEF_KIND_N$ - GG expenditures on social transfers in kind, $GE_SOC_CASH_N$ - GG expenditures on social benefits other than social transfers in kind, $GE_SUB_FARM_N$ - GG subsidies to farmers, $GE_SUB_NOFARM_N$ - GG subsidies excluding subsidies to farmers, $GE_OTHER_TRANS_N$ - other GG transfers without EU budget contribution, GE_EU_N - GG contribution to the EU budget, GE_FIN_N - interest on the GG debt, GE_WF_N - GG compensations for employees, GE_CON_N - GG intermediate consumption, GE_GFCF_N - nominal GG investments, $GE_CAP_TRANS_N$ - GG capital transfers,

Social expenditures

$$\begin{aligned}
 GE_SOC_CASH_N &= GE_PENSIONS_N - (1 - G_REF) * OFE_N \\
 &+ GE_UNEMP_N + GE_FAMILY_N \\
 &+ GE_PRERETIREMENT_N \\
 &+ GE_RELIEF_REST_N \\
 &+ GE_SOCSECURITY_N
 \end{aligned}$$

$GE_SOC_CASH_N$ - GG expenditures on social benefits other than social transfers in kind, $GE_PENSIONS_N$ - GG expenditures on pensions, G_REF - share of pensions paid from the GG budget, OFE_N - GG transfers to Open Pension Funds, GE_UNEMP_N - GG expenditures on unemployment benefits, GE_FAMILY_N - GG expenditures on family benefits, $GE_PRERETIREMENT_N$ - GG expenditures on preretirement benefits, $GE_RELIEF_REST_N$ - GG expenditures on other social benefits, $GE_SOCSECURITY_N$ - GG expenditures on social insurance benefits together with grants,

General government balance and debt

General government balance:

$$G_BALANCE_N_t = GR_N_t - GE_N_t$$

General government debt:

$$G_DEBT_N = G_DEBT_DOM_RES_N + G_DEBT_DOM_NRES_N \\ + G_DEBT_FOR_RES_N + G_DEBT_FOR_NRES_N$$

$G_BALANCE_N$ - GG balance, G_DEBT_N - GG debt, $G_DEBT_DOM_RES_N$ - GG debt in domestic currency held by residents, $G_DEBT_DOM_NRES_N$ - GG debt in domestic currency held by non-residents, $G_DEBT_FOR_RES_N$ - GG debt in foreign currencies held by residents, $G_DEBT_FOR_NRES_N$ - GG debt in foreign currencies held by non-residents,

Cost of debt servicing

Total servicing cost of the GG debt:

$$GE_FIN_N_t = GE_INT_DOM_RES_N_t + GE_INT_DOM_NRES_N_t \\ + GE_INT_FOR_RES_N_t + GE_INT_FOR_NRES_N_t$$

GE_FIN_N - interests on the GG debt, $GE_INT_DOM_RES_N$ - interest on the GG debt in domestic currency held by residents, $GE_INT_DOM_NRES_N$ - interest on the GG debt in domestic currency held by non-residents, $GE_INT_FOR_RES_N$ - interest on the GG debt in foreign currencies held by residents, $GE_INT_FOR_NRES_N$ - interest on the GG debt in foreign currencies held by non-residents,

Interest on the GG debt in domestic currency:

$$\frac{GE_INT_DOM_N_t}{G_DEBT_DOM_N_t} \propto W_SHORT_DOM_t \cdot I_3M_t \\ + (1 - W_SHORT_DOM_t) \cdot \overline{I_5Y_t}$$

$GE_INT_DOM_N$ - interest on the GG debt in domestic currency, W_SHORT_DOM - variable controlling pass-through of short-term interest rate into servicing cost of public debt in domestic currency, $\overline{I_5Y_t}$ - 20-quarters average of yield on 5-year government bonds,

Cost of debt servicing (2)

Interests on the GG debt in foreign currencies:

$$\begin{aligned}
 GE_INT_FOR_N_t \propto & \sum_{i=1}^5 G_DEBT_FOR_N_{t-4i} \\
 & \cdot I_5Y_EUR_{t-4i} \cdot (GDEBT_EXCHANGE_{t-4i}^t \\
 & / GDEBT_EUR_{t-4i}^{t-4i})
 \end{aligned}$$

$GE_INT_FOR_N$ - interests on the GG debt in foreign currencies, I_5Y_EUR - yield on 5-year Bunds, $GDEBT_EUR$ - share of GG debt denominated in euro in the total GG debt denominated in foreign currencies, $GDEBT_EXCHANGE$ - effective exchange rate relevant for GG debt denominated in foreign currencies,

$$\begin{aligned}
 GDEBT_EXCHANGE_t^s = & GDEBT_EUR_s \cdot S_EUR_PLN_t \\
 & + (1 - GDEBT_EUR_s) \cdot S_USD_PLN_t
 \end{aligned}$$

S_EUR_PLN - EUR/PLN exchange rate, S_USD_PLN - USD/PLN exchange rate,

Fiscal policy

- Fiscal policy rule closes the fiscal balance,
- As long as fiscal policy rule remains passive, fiscal revenues and expenditures are generated so that their ratios to GDP are constant

Outline

-
-
- 3 The structure of the model**
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Monetary policy rule, long term interest rate - short run

$$\begin{aligned}
 I_{-3M}_t = & \frac{0.86}{(0.02)} \cdot I_{-3M}_{t-1} + (1 - \frac{0.86}{(0.02)} \cdot (I_{-3MR_EQ}_t + INF_{t+1})) \\
 & + \frac{1.04}{(0.36)} \cdot INF_{t+1} - \overline{INF_TARGET}_{t+3} + \frac{0.29}{(0.15)} \cdot GAP_t
 \end{aligned}$$

Adjusted $R^2 = 0.99$

S.E. of equation = 0.005

J statistic(p-value) = 0.830

Estimation period: 2000q3 - 2008q3

$$I_{-5Y}_t = \frac{1}{17} \cdot I_{-3M}_t + (1 - \frac{1}{17}) \cdot I_{-5Y}_{t+1} + \frac{0.001}{(0.002)}$$

Adjusted $R^2 = 0.94$

S.E. of equation = 0.008

J statistic(p-value) = 0.614

Estimation period: 1999q1 - 2008q4

I_{-3M} - WIBOR 3M, I_{-3MR_EQ} - the equilibrium interest rate, INF - CPI inflation,

$\overline{INF_TARGET}$ - 4-quarter moving average of inflation target, GAP - output gap, I_{-5Y} - yield on 5-year government bonds,

Money demand

$$m3_t^* = 1.25 \cdot (I_3M_{t-1} - I_5Y_{t-1}) - 1.47 \cdot gdp_n_{t-1} + 5.25$$

$$\begin{aligned} \Delta m3_t = & -0.34 \cdot (m3_{t-1} - m3_{t-1}^*) \\ & (0.09) \\ & + 0.44 \cdot \Delta m3_{t-1} + 0.75 \cdot \Delta gdp_n_t \\ & (0.13) \quad (0.22) \\ & + 0.15 \cdot \Delta s_neer_{t-1} \\ & (0.05) \end{aligned}$$

Adjusted $R^2 = 0.34$

S.E. of equation = 0.013

LM test (p-value) = 0.042

Estimation period: 2000q3 - 2008q4

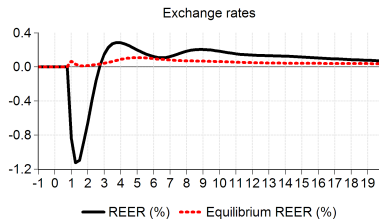
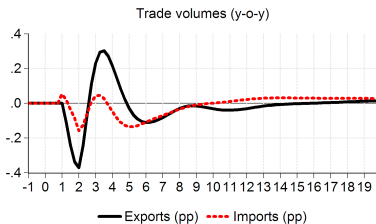
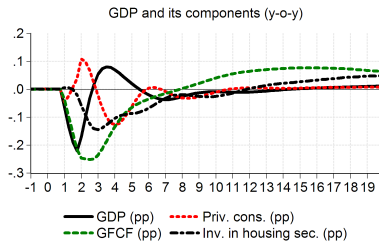
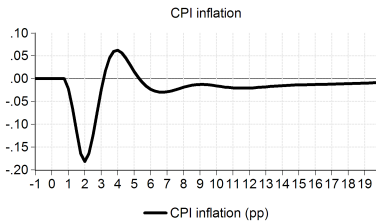
M3 - M3+ monetary aggregate, GDP_N - nominal gross domestic product, S_NEER - nominal effective exchange rate,



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- 2 Data
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- 4 Impulse response analysis**
- 5 Risk analysis
- 6 Concluding remarks

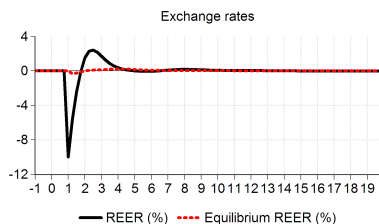
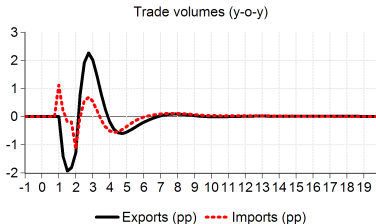
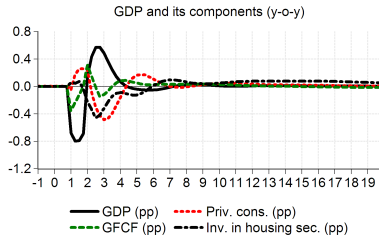
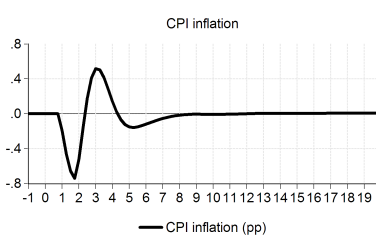
Monetary impulse*



*The monetary impulse is defined as an unexpected one quarter increase in the short-term interest rate by 100 basis points.



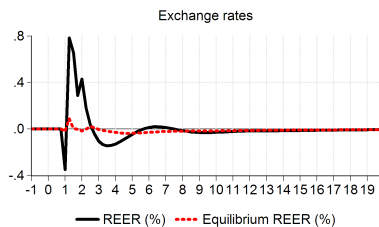
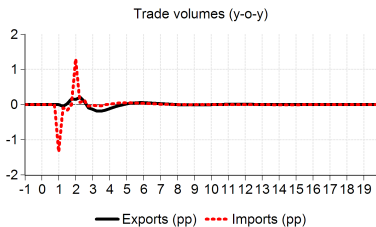
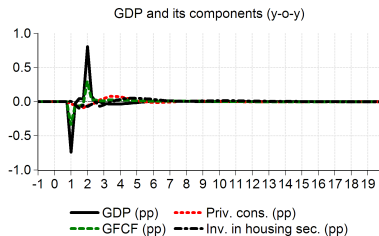
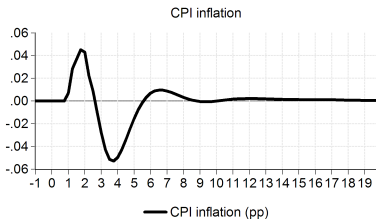
Exchange rate impulse*



*The exchange rate impulse is defined as an unexpected one quarter appreciation of the real (and nominal) effective exchange rate by 10%.



Fiscal impulse*



*The fiscal impulse is defined as a four-quarter unexpected cut in the purchases of goods and services by the general government sector, equivalent to 1% of GDP.



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Procedure of risk analysis

The final estimate of future uncertainty depends on three components:

- past forecast errors,
- the model's structure,
- the anticipated change in the uncertainty of selected exogenous variables.

When choosing series shocked in stochastic simulations the following criteria are taken into account:

- The selected variables should have prevailing impact on uncertainty of inflation and/or GDP growth (the property is assessed on the basis of model multipliers),
- Uncertainty of the variables should non-negligibly change between forecasting rounds,
- Uncertainty of the selected time series should be feasible to measure and forecast.

Procedure of risk analysis

Shocked time series:

- Crude oil prices on the world markets
- Natural gas prices on the world markets
- Coal prices on the world markets
- EUR/USD exchange rate
- 3M interest rates in the euro zone
- 3M interest rates in the USA
- Price of agricultural commodities on the world markets
- Weighted GDP of main trading partners of Poland
- Weighted potential GDP of main trading partners of Poland
- Weighted value added deflator of main trading partners of Poland

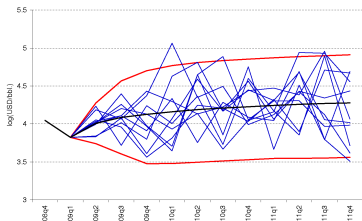
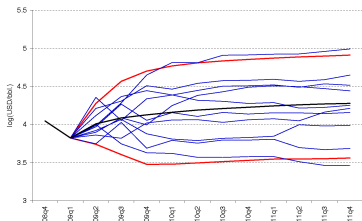
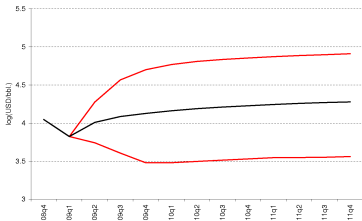
Simulation of paths of exogenous variables

Procedure of simulating exogenous variables needs to have the following properties:

- the expected value derived on the basis of simulated paths of variables conforms to central paths given by experts,
- the expected value of stochastic disturbances is zero,
- the autocorrelation of variables observed in the sample is retained,
- the cross correlation of shocks among particular variables is retained.



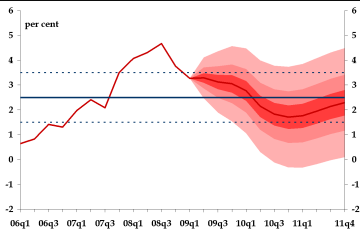
Simulation of paths of exogenous variables - exmple



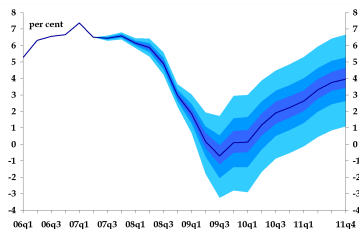
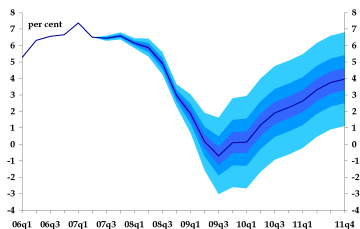
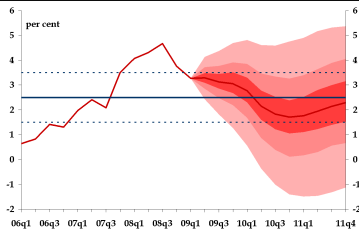


Illustrative results

Uncertainty of the past NBP's forecast errors



Final results





Outline

- 1 Introduction
- 2 Data
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Conclusion

- Major challenges tackled in the current version of the model originate in the gradual process of economic convergence and integration of Polish economy into European and global environment,
- Compared to the previous model version, closer look was given to coherency of specification of the trade block jointly with the equilibrium exchange rate,
- An advantage of the model is its explicit treatment of policies (i.e. inflow of structural funds) and phenomena (i.e. stronger inflow of remittances) connected with Polish participation in the European Union,
- Current version of the model has not only better simulation properties but also offers more promising framework for forecasting in conditions of strong fluctuations of global demand and high medium-run volatility of the exchange rate.