

# Gravity without Apology: The Science of Elasticities, Distance, and Trade\*

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## Abstract

Gravity as both fact and theory is one of the great success stories of recent research on international trade, and has featured prominently in the policy debate over Brexit. We first review the facts, noting the overwhelming evidence that trade tends to fall with distance. We then introduce some expository tools for understanding CES theories of gravity as a simple general-equilibrium system. Next, we point out some anomalies with the theory: mounting evidence against constant trade elasticities, and implausible predictions for bilateral trade balances. Finally, we sketch an approach based on subconvex gravity as a promising direction to resolving them.

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“Today, we stand on the verge of an unprecedented ability to liberate global trade for the benefit of our whole planet with technological advances dissolving away the barriers of time and distance. It is potentially the beginning of what I might call ‘post geography trading world’ where we are much less restricted in having to find partners who are physically close to us.”

– Liam Fox (2016)

# 1 Introduction: Gravity and International Trade

Recognition of the importance of gravity in international trade is one of the great successes of modern economics. To adapt a comment made about evolution by the late Harvard paleontologist Stephen Jay Gould (Gould (1981)), gravity in trade is both *fact* and *theory*. Countless empirical studies have shown a consistently and significantly negative effect of distance on trade volumes; and much theoretical work has shown that this pattern is consistent with almost all the major approaches to the theory of international trade, in the process opening the door to quantitative studies of the effects of trade barriers on trade flows and welfare. However, these relatively recent developments are not widely appreciated by economists who are not trade specialists. As for the general public, there is some awareness of the role of gravity in trade.<sup>1</sup> But “anti-gravity” continues to have popular appeal: witness the success of books such as *The Death of Distance* by Frances Cairncross (Cairncross (1997)) and *The World is Flat* by Thomas Friedman (Friedman (2005)), both embodying sentiments eloquently summarized in the opening quotation from Liam Fox, M.P., some months after he became United Kingdom (UK) Minister for International Trade.

In this paper, we seek to introduce this literature for the benefit of non-specialists, and to suggest some directions it might profitably take for the benefit of insiders. We first review the evidence for gravity, illustrating in a novel way how it shapes the spatial pattern of UK exports. We then present the archetypal model of “structural gravity”, and introduce some new ways of understanding how it works as a simple general-equilibrium system. Next we

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<sup>1</sup>A gravity equation featured on the front page of the *Financial Times* on April 19, 2016 in the context of discussions preceding the Brexit referendum, on which more below.

note some counter-factual implications of the assumption of constant elasticity of substitution (CES) preferences that underlies almost all general-equilibrium gravity models. Finally, we sketch some alternative specifications of gravity models. Throughout, we note the relevance of gravity and trade to ongoing debates in the UK on the likely effects of “Brexit”: the process of the UK exiting the European Union (EU).<sup>2</sup>

Of course, Brexit is about much more than economics. Just how much more is suggested by an anonymous quote from a senior member of the UK’s ruling Conservative Party:

“I don’t think we’ll be poorer out [of the EU], but if you told me my family would have to eat grass I’d still have voted to leave.”

– anon.; quoted by Robert Shrimmsley, *Financial Times*, Dec. 14, 2018

It is easy to mock this position. It is not clear if the family were informed. And it is very clear that the electorate were not: the 2016 UK referendum campaign featured an iconic red Brexit Bus sporting the slogan “We send the EU £350 million a week; let’s fund our NHS (National Health Service) instead”; there are no reports of a bus proclaiming “The grass is greener outside.” But perhaps there is too much mockery around these days, on both sides of the highly-polarized Brexit debate. Perhaps it is kinder to take the second half of the quote as merely a rhetorical device, a passionate endorsement of sincere, strongly-held views on the desirability of cutting links between the UK and the EU in order to restore Britain’s sovereignty, mirroring the sincere, strongly-held views of those who favour remaining in the EU on liberal internationalist grounds. By contrast, the first half of the quote makes a modest claim about the economic effects of Brexit. As we will show, the scientific evidence suggests overwhelmingly that this claim is false, though only modestly so.

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<sup>2</sup>The UK, officially the “United Kingdom of Great Britain and Northern Ireland” is often referred to as just “Britain.” It joined the European Economic Community (EEC), the predecessor of the EU, on 1 January 1973. Under the 1993 Maastricht Treaty, the EEC was renamed the European Community, one of the three pillars of the European Union (EU). The European Community in turn was abolished by the 2009 Treaty of Lisbon, and its institutions incorporated into the EU’s wider framework. In a referendum held on June 23, 2016 the UK voted to leave the EU by 51.89% to 48.11%. On March 29, 2017, the UK invoked Article 50 of the Treaty on European Union which began a two-year process of withdrawal, which in turn will be followed by a period of indefinite duration during which a new trading arrangement with the EU will be negotiated. At the time of writing (November 5, 2019), the Article 50 deadline of March 29, 2019 has been extended three times, and there is no certainty about what the outcome of the Brexit process will be.

In the rest of the paper, we focus on the economics of Brexit, and in particular its implications for international trade. There have been many studies of the trade effects of Brexit, most of them using the gravity model. Examples include Dhingra et al. (2017), Sampson (2017), Brakman, Garretsen, and Kohl (2018), and Mayer, Vicard, and Zignago (2019). There are also many other aspects of Brexit on which economists have already written, and no doubt there will be many more. A short list would include Davies and Studnicka (2018) on the stock-market response to the post-referendum depreciation of sterling; McGrattan and Waddle (2018) on the impact of Brexit on foreign investment in a neoclassical growth model; Alabrese, Becker, Fetzer, and Novy (2019) on the determinants of voting patterns in the Brexit referendum; and O’Rourke (2019) on the historical context. All these aspects are important, but the predicted effects on trade and real incomes have been the focus of most popular discussion of the economics of Brexit. So it seems appropriate to concentrate on them in order to explain why academic economists are almost unanimous in warning of the economic costs of Brexit, and to document the role that gravity has played in moulding that professional consensus.<sup>3</sup> The purpose of this paper is not to add another calibration of these costs, but rather to explore why the existing ones give the results they do.

The overwhelming conclusions of the gravity studies cited above might be called the “Three Iron Laws of the Economics of Brexit.” To be clear, these conclusions refer to trade in goods only: services trade also follows gravity, but the available data are not as comprehensive as for merchandise trade. Moreover, these conclusions follow from studies using static micro-founded general-equilibrium models, so they ignore transitional problems; for example, they have little to say about the hard-to-forecast costs of a “No-Deal” Brexit. They also ignore macroeconomic policy responses: in what follows any change in real income is equal to the change in real wages; the models are silent on whether these would be effected

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<sup>3</sup>Readers of some UK newspapers may be under the impression that the economics profession is deeply split on the issue. (It is tempting to draw parallels with other debates, such as on climate change, evolution, or vaccination, where an overwhelming scientific consensus is sometimes depicted as only one view among many equally valid ones.) However, the number of academic economists who support Brexit on economic grounds is tiny. See Neary (2019) for further discussion.

through a deflationary fall in nominal wages, or through an accommodative monetary policy coupled with a depreciation of sterling, as happened in the wake of the 2016 referendum.

What then are the “Three Iron Laws”? First, *the only good Brexit is a dead Brexit*: all realistic Brexit scenarios imply lower UK GDP than remaining in the EU. Second, *the harder the Brexit the higher the economic costs*: for example, a “hard” Brexit in which the UK completely withdraws from the EU Single Market and Customs Union will have higher costs than a “soft” one that entails some continued participation in these deep trade agreements. Third, *even a hard Brexit will not have “very” large costs*: the orders of magnitude from all the studies suggest a permanent but once-off loss of the order of 2% of GDP for a soft Brexit, and 6% or more of GDP for a hard one. These are significant economic costs, unprecedented for a deliberate policy choice by a peacetime government; to put them in context, the UK spent 7.26% of its GDP on the NHS in 2016-17.<sup>4</sup> But this is not Armageddon, or a wartime scenario. Passionate leavers who value sovereignty above all else should be prepared for a major reduction in UK GDP relative to what it would otherwise have been, but, conditional on an orderly exit, need have no fears of a grass-only menu.

The plan of the paper follows the outline given above. Section 2 sketches the facts of gravity from the perspective of UK exports. Section 3 explains the structural gravity model and shows how it can be interpreted as a simple general-equilibrium system. Section 4 considers some anomalous implications of CES demands and CES gravity. Section 5 outlines an approach that may help overcome them. Finally, Section 6 summarizes the paper, while the appendices gives details on data sources and technical derivations.

## 2 Gravity as Fact

This section uses some simple charts to illustrate the robustness of the gravity effect, both for geographic distance and for other distance variables such as membership of a common trade agreement and former colonial ties. The data are for UK merchandise exports to

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<sup>4</sup>See: <https://www.nuffieldtrust.org.uk/chart/nhs-spending-as-a-percentage-of-gdp-1950-2020>.

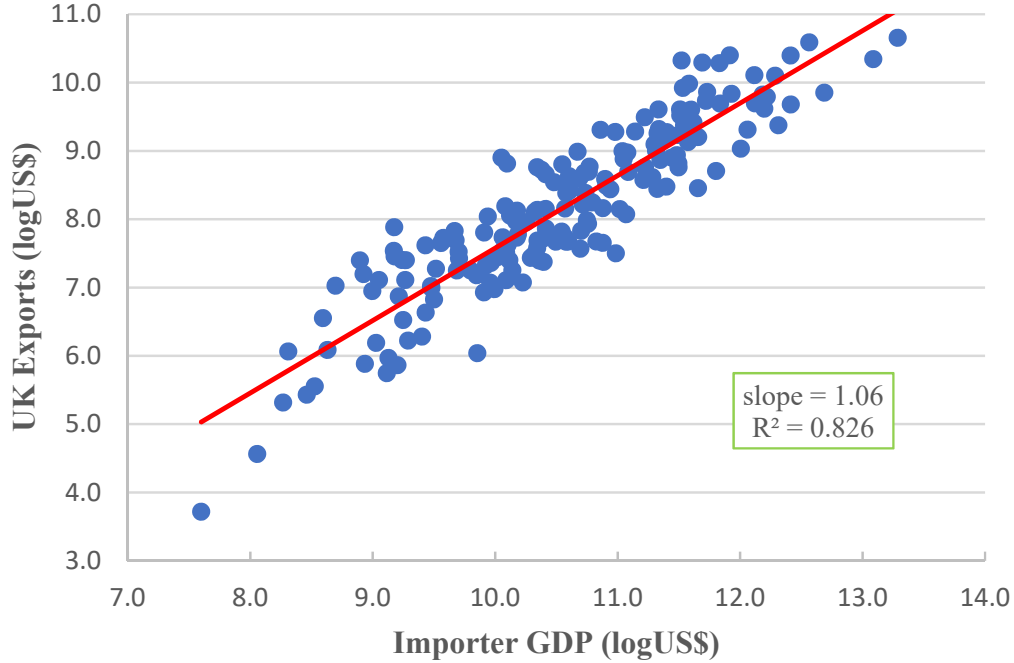


Figure 1: UK Exports and Importer GDP, 2017

181 countries in 2017 (the latest full year for which comparable data are available).<sup>5</sup> It goes without saying that this is not intended as a serious econometric exercise, though the tendencies we will point out are in line with the findings of almost all large-scale studies.

Figure 1 plots UK exports against importer GDP, both in logs. The positive relationship between the two is apparent, and confirmed by the simple regression line. The estimated slope coefficient is 1.061 with a standard error of 0.036: significantly different from zero but not from one. As we will see in the next section, most theoretical foundations of the gravity equation assume that this coefficient equals one. Hence, to allow a visual exploration of the effect of distance, it makes sense to impose a value of one, which allows us to focus on the ratio of UK exports to importer GDP.

Figure 2 plots this ratio against bilateral distance, both once again in logs.<sup>6</sup> This time

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<sup>5</sup>Figures 1 and 2 follow Head and Mayer (2014) who illustrate similar patterns for French exports. See Appendix A for data sources. Data on countries with zero UK exports recorded, and for which GDP data are not available, are omitted.

<sup>6</sup>Distance is measured as a population-weighted average of distances between major cities. Hence Ireland is closer to the UK than either Belgium or the Netherlands, and all three are closer than France. See Mayer and Zignago (2011) for discussion.

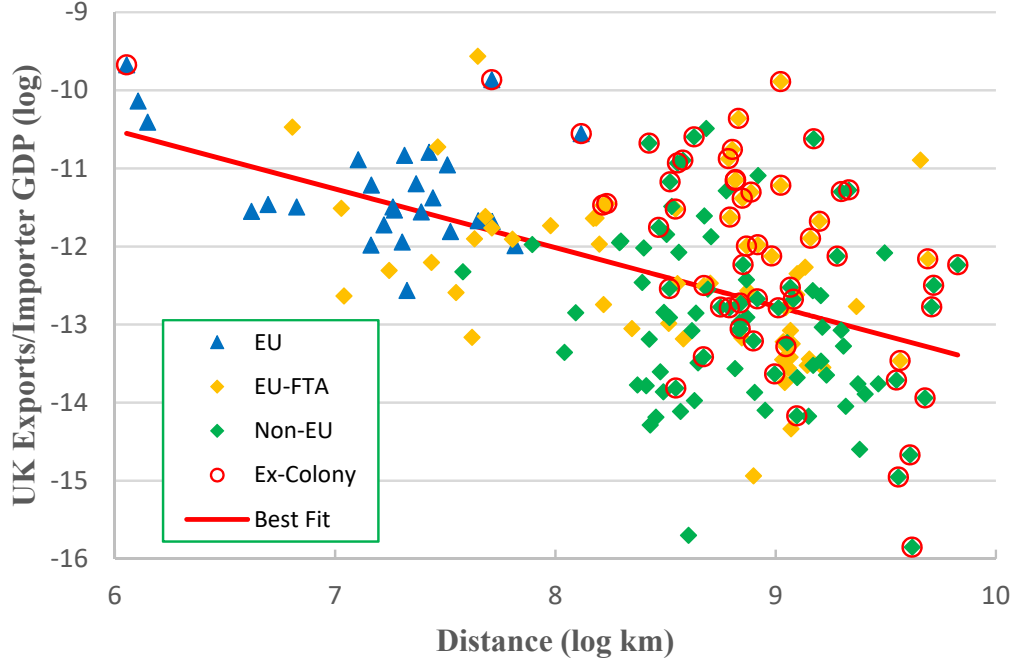


Figure 2: UK Exports/Importer GDP and Distance, 2017

the simple regression line is downward-sloping. Its estimated slope coefficient is  $-0.752$  with a standard error of  $0.098$ : significantly different from zero. For each export market, the symbols indicate if it is a member of the EU; if it has a free trade agreement (FTA) with the EU as of March 2019;<sup>7</sup> and if it is a former UK colony. Figure 3 illustrates the same data as Figure 2 but this time as a “bubble chart,” with the size of each bubble proportional to the level of UK exports to that country.

Figures 2 and 3 confirm that UK trade falls off with distance, when we control for the size of the importing country. Figure 3 also shows that the tendency to cluster around the best-fit line is even more pronounced for larger trading partners. Many of the extreme outliers in Figure 2 are barely visible when we scale by the value of exports as in Figure 3; while most of the largest export markets lie on or close to the best-fit line. (The best-fit line in Figure

<sup>7</sup>The EU currently has trade agreements with 84 countries, of which 66 are in the data used in the figures. The most recently concluded of these agreements, that with Japan, came into force on 1 February 2019. The UK benefits from these trade agreements as long as it remains an EU member, and is engaged in negotiations to roll them over post-Brexit. However, so far only a small number of these have been concluded, and it is not clear if they will yield the full benefits of the current agreements.

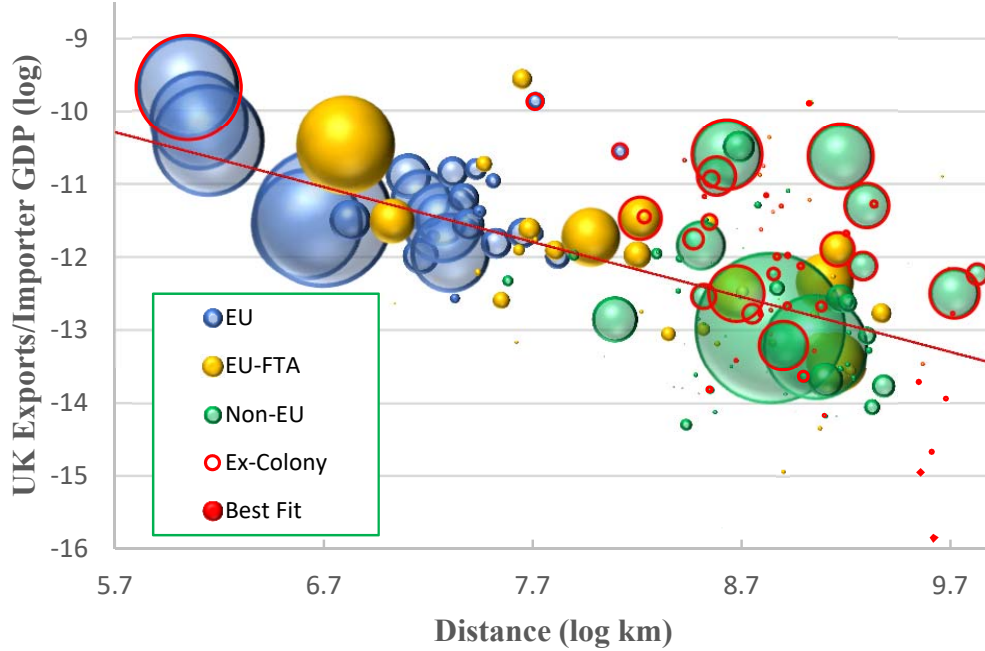


Figure 3: UK Exports/Importer GDP and Distance, Scaled by Exports, 2017

3 is the same as that in Figure 2.)

Is the relationship shown in these figures unique to the UK? The answer is definitely not. From the same data set (182 countries in 2017) we can plot similar charts for each of the other 181 exporting countries. It turns out that every single one of them has a negative gravity coefficient, all but one of them significantly different from zero.<sup>8</sup> For the 2017 data as a whole, the estimate of the distance coefficient is  $-0.977$ , with a standard error of  $0.021$ .<sup>9</sup> Moreover this is only one data set: the same pattern can be seen in almost all empirical gravity studies. A comprehensive survey by Head and Mayer (2014) reviewed 159 papers that had estimated gravity equations. They found an average estimate of the

<sup>8</sup>The exception is Andorra, whose gravity coefficient from the same simple regression as in Figure 2 is  $-0.214$  with a standard error of  $0.327$ . Given Andorra's tiny size (its population was 80,209 in 2017) and unique location, there are no doubt special factors at work.

<sup>9</sup>This comes from estimating a gravity equation for the complete trade matrix, with a full set of control variables, using Poisson Pseudo Maximum Likelihood to take account of heteroscedasticity and zero observations. (The complete matrix has 206 countries, and so  $n = 42,230$ .) By comparison, an OLS regression with a full set of controls but omitting zero observations, so  $n = 23,251$ , yields an estimated distance coefficient of  $-1.452$  with a standard error of  $0.021$ .

A less satisfactory way of controlling for zeros is to use OLS on the full sample with  $\log(1 + V_{jk})$  as dependent variable; this yields an estimated distance coefficient of  $-0.735$  with a standard error of  $0.034$ .

distance elasticity of  $-0.89$  over the 159 papers; using a preferred estimation methods, the average was higher at  $-1.10$ , with a standard deviation of  $0.41$  and a median of  $-1.14$ .

Moreover, the distance coefficient for goods trade has not fallen over time, contrary to suggestions in popular debates as discussed in the introduction. This has been called “the mystery of the missing globalization,” or “the puzzling persistence of the distance effect” (Disdier and Head (2008)). However, it is not really a mystery, when we reflect that in estimated gravity equations distance is relative. Studies that use data on both domestic sales and exports to estimate a border dummy variable typically find that it has fallen over time. Thus international trade *per se* has become easier, but the relative attractiveness of nearby versus foreign markets has not changed much. (See, for example, Anderson and Yotov (2010) and Yotov (2012).) Improvements in transport and communication technology have made it easier for UK firms to export to New Zealand, but also easier to export to Ireland.

We have focused so far on trade in goods only. Because standardized data on merchandise trade are much more widely available, the majority of gravity studies look only at this component of trade. However, it has been shown in many studies that distance also matters (though less so on average) for a whole range of international transactions. In rough order of distance coefficients that decrease in absolute value, significant effects of distance have been found for: services trade (Kimura and Lee (2006)), foreign direct investment (Kleinert and Toubal (2010), Keller and Yeaple (2013)), trade in equities (Portes and Rey (2005)), eBay transactions (Lendle, Olarreaga, Schropp, and Vézina (2016)), and Google hits (Cowgill and Dorobantu (2012)).

It is not only geographical distance that matters in gravity equations. Distance in other senses also affects trade, with variables such as common language, common legal system, common colonial origins, membership of the same FTA, and so on, invariably showing up as significant. Returning to Figures 2 and 3, some of these effects can be seen clearly for UK trade. In particular, recalling the tendency for the larger trading partners to cluster more closely around the best-fit line, it is noteworthy that most of the exceptions are former

colonies.<sup>10</sup> Both figures show that the UK tends to export more to its former colonies than to other countries, relative to what geographical distance alone predicts. This is in line with an extensive literature which finds that former colonial ties tend to increase trade. (See for example Head, Mayer, and Ries (2010).) It is also relevant to the Brexit debate. A recurring theme in the arguments in favour of Brexit has been characterized, perhaps ironically, as “Empire 2.0”: the hope that new trade agreements with the “Anglosphere”, former UK colonies including the US, would more than make up for the loss of preferential access to EU markets. But as Figure 3 shows, “Empire 1.0” casts a long shadow: controlling for distance, the UK trades much more with these countries (other than the US) than it does on average. Almost all of the countries with which there is a significant value of exports and which lie above the best-fit line are former colonies, from Australia and New Zealand at the far end of the globe, to the UAE, Hong Kong, Singapore and Malaysia in the Middle and Far East, to, perhaps most remarkably, all three former UK colonies that are EU members: Ireland, Malta and Cyprus.

What do these figures imply for Brexit? Gravity is not destiny. Yet it is hard not to look at Figure 3 without reflecting that the UK currently enjoys free and frictionless trade with the blue bubbles, and preferential access to the yellow ones; and without wondering at the wisdom of abandoning the first and risking the second in the hope of negotiating new trade agreements with the far-away green bubbles. Of course, this argument is far from rigorous: there is no explicit counter-factual. For that we need a theory that is consistent with the data and that yields predictions of how changes in trade policy would affect trade patterns. We turn to this in the next section.

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<sup>10</sup>The only prominent exception with no colonial ties to the UK is Switzerland, though anecdotal evidence suggests that exports to it in 2017 were boosted above trend by flows of gold bullion, reflecting balance-of-payment adjustments rather than merchandise trade. See: “Gold fingered for distorting Brexit Britain’s trade balance,” *Financial Times*, February 24, 2017; and “How gold takes the shine off Britain’s trade balance,” *Sky News*, April 18, 2018.

### 3 Gravity as Theory

“[I] have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power.”

– Isaac Newton (1713)

“The intent of this paper is to provide a theoretical explanation for the gravity equation applied to commodities.”

– Jim Anderson (1979)

In later editions of his *Principia*, Isaac Newton conceded to his critics that his mathematical theory of gravity did not give a primitive explanation of the forces between bodies. Yet in the 1979 *American Economic Review*, Jim Anderson provided a micro-founded theoretical explanation for the gravity equation of trade flows. The contrast between the two goes deeper than that. Newton gave an analytical solution for the force of gravity in the two-body problem only. Even today, while physicists can simulate the movements of planets and particles with extraordinary accuracy, there is no explicit expression for physical gravity in higher dimensions: the three- or  $n$ -body problem cannot be solved in closed form. Yet Jim Anderson in 1979 and other economists since then have been able to provide closed-form gravity expressions for trade between any number of countries. Why has it proved easier to derive results of this kind in economics than in physics? The reason is simple: planets do not have CES preferences! Almost all the many theoretical rationales that have been provided for the gravity equation in international trade assume that consumers have CES preferences.

This assumption about preferences is very special, as we explore further in Section 4. Yet is a natural starting point for quantitative studies of trade; CES preferences are a standard benchmark for modeling consumer behaviour, they are widely used in many fields of economics other than trade, they are analytically very tractable, and they connect easily with observable data. Moreover there is a compensating richness on the supply side. The gravity equation has been shown to be consistent with a wide range of canonical trade models, each with different assumptions about the structure of production: the Armington (1969)

model of pure exchange, by Anderson (1979) and Anderson and van Wincoop (2003); models of monopolistic competition such as Krugman (1980), by Bergstrand (1985) and Helpman (1987); the heterogeneous-firms model of Melitz (2003), by Chaney (2008); and the multi-country Ricardian model of Eaton and Kortum (2002), by the authors. As highlighted in the synthesis of Arkolakis, Costinot, and Rodríguez-Clare (2012), all these frameworks yield the same “structural gravity” model, and the same parsimonious expression for the gains from trade.

To fix ideas, we focus here on the simplest Armington-based version of structural gravity. In Section 3.1 we introduce notation and show how CES demands combined with market-clearing yield the structural gravity equations; this section can safely be skimmed by trade specialists. In Section 3.2 we present a new pedagogic approach to understanding the structural model as a simple general-equilibrium system, while in Section 3.3 we show its usefulness with an application to Brexit.

### 3.1 From CES Demands to Structural Gravity

Consider a world of  $n$  countries, each populated by a representative consumer who is endowed with a fixed supply of a unique good. We assume that all countries have the same CES preferences, which implies that each country consumes all goods provided trade costs are less than infinite. The demand for country  $j$ ’s good in country  $k$  can be written as follows:

$$V_{jk} = p_{jk}x_{jk} = \eta_j \left( \frac{p_{jk}}{P_k} \right)^{1-\sigma} E_k \quad (1)$$

Here  $V_{jk}$  is the value of exports from  $j$  to  $k$ : we write the subscripts for exporting and importing countries in the same order as the direction of trade throughout.  $V_{jk}$  in turn equals the price  $p_{jk}$  times the quantity  $x_{jk}$  of exports. Crucially,  $p_{jk}$  is the delivered price of  $j$ ’s export in  $k$ , which equals the origin or “factory-gate” price  $p_j$  times an “iceberg” trade cost,  $t_{jk} \geq 1$ :  $p_{jk} = p_j t_{jk}$ . Iceberg costs imply that  $t_{jk}$  units of country  $j$ ’s good must be

shipped from  $j$  for one unit to arrive in country  $k$ ;  $t_{jk} - 1$  units “melt” in transit.

Turning to the right-hand side of (1), the determinants of demand are standard for a CES function. Sales are proportional to total expenditure in the importing country,  $E_k$ , reflecting the fact that CES preferences are homothetic. Conditional on expenditure, demand for good  $j$  depends on a preference parameter  $\eta_j$  and on its price  $p_{jk}$  relative to the cost of living in the importing country,  $P_k$ . The preference parameter depends only on the origin of the good  $j$ , since all importing countries have the same tastes. As for  $P_k$ , it takes the standard form of a CES price index, implicitly defined by:

$$(P_k)^{1-\sigma} = \sum_h \eta_h (p_{hk})^{1-\sigma} \quad (2)$$

Finally, the impact of relative prices on demand depends on the elasticity of substitution  $\sigma$ , which is also the elasticity of demand.

To go from CES demands to structural gravity, we add the conditions for goods-market equilibrium. These imply that, for every country  $j$ , its total sales to all countries (both exports,  $V_{jk}$ ,  $j \neq k$ , and sales to the home consumer,  $V_{jj}$ ), must equal the value of its GDP,  $Y_j$ :

$$\sum_k V_{jk} = Y_j \quad (3)$$

Combining this with the demand function (1), we see that a term in the taste parameter and the exporting country’s factory-gate price factors out:

$$Y_j = \sum_k V_{jk} = (\eta_j p_j)^{1-\sigma} \sum_k \left( \frac{t_{jk}}{P_k} \right)^{1-\sigma} E_k \quad (4)$$

Using this to eliminate the term  $(\eta_j p_j)^{1-\sigma}$  from  $V_{jk}$  in (1) and  $P_k$  in (2) yields the structural gravity equation:

$$V_{jk} = \underbrace{\left( \frac{t_{jk}}{\Pi_j P_k} \right)^{1-\sigma}}_{(1)} \underbrace{\frac{Y_j E_k}{Y_W}}_{(2)} \quad (5)$$

where:

$$(\Pi_j)^{1-\sigma} = \sum_h \left( \frac{t_{jh}}{P_h} \right)^{1-\sigma} \frac{E_h}{Y_W} \quad (P_k)^{1-\sigma} = \sum_h \left( \frac{t_{hk}}{\Pi_h} \right)^{1-\sigma} \frac{Y_h}{Y_W} \quad (6)$$

To understand the implications of this, consider the two composite terms in (5) in reverse order. The second term represents the level of free and frictionless trade predicted by the model: if there are no trade costs (so all the  $t_{jk}$  equal one), then the value of exports from  $j$  to  $k$  equals the product of exporter GDP  $Y_j$  and importer expenditure  $E_k$  deflated by world income  $Y_W$ .<sup>11</sup> Putting this differently, when prices are the same everywhere, each country  $k$  spends a proportion of its total expenditure on imports from every other country  $j$  that is equal to the exporter country's share in world GDP:  $V_{jk}/E_k = Y_j/Y_W$ . The first term in (6) shows how trade costs modify this: exports from  $j$  to  $k$  are lower the greater is the elasticity of import demand,  $\sigma - 1$ , and the higher is the bilateral trade cost  $t_{jk}$  relative to the product of two indices of the average trade costs faced by the exporter and the importer respectively,  $\Pi_j$  and  $P_k$ . Anderson and van Wincoop (2003) called these outward and inward “multilateral resistance” respectively, and the fact that they are dual to one another underlines the elegance of the structural gravity system.

Where do we go from (5) and (6)? The first step is estimation. As Anderson and van Wincoop (2003) showed, this can be done structurally, using non-linear methods. However, this approach is seldom used since it requires that we take a stand on the supply side of the model summarized by the  $Y_j$  terms in (5) and (6). Hence the irony that the structural gravity model is rarely estimated structurally. In practice a different approach is taken. Irrespective of which model of the production side of the economy is assumed, we can take

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<sup>11</sup>Total output and expenditure must be equal for the world as a whole, so  $Y_W = \sum_j Y_j = \sum_k E_k$ . Note that, with zero trade costs, the terms  $\Pi_j$  and  $P_k$  need not reduce to one: after all,  $P_k$  continues to represent the true cost of living. Whether they do or not depends on the choice of numéraire. However, their product must equal one. As is easy to check from (6), when all  $t_{jk}$  equal one,  $\Pi_j$  and  $P_k$  are independent of  $j$  and  $k$  (as they must be, since the producer and consumer prices of each good are the same in all countries); and one is the reciprocal of the other, so  $\Pi_j P_k = \Pi P = 1$ . See Section 3.3 below for further discussion of the choice of numéraire.

logs of (5) and write the result as:

$$\log V_{jk} = F_j + F_k + \mu \log t_{jk} + u_{jk} \quad (7)$$

Thus the value of exports from  $j$  to  $k$  takes a simple log-linear form, depending on importer and exporter fixed effects,  $F_j$  and  $F_k$ , and on a term specific to the “dyad”  $\{j, k\}$ . In practical applications, the latter term can be decomposed in a log-linear way, writing  $t_{jk} = \delta_{jk} \exp(\gamma' D_{jk})$ , where  $\delta_{jk}$  is geographic distance, and the vector  $D_{jk}$  includes a range of other bilateral “distance” measures such as contiguity, common language, colonial ties, membership of an FTA, etc. The coefficient of this term,  $\mu$ , is (minus) the elasticity of trade, and its relationship to underlying structural parameters depends on the assumptions made about the supply side of the model.

Given estimates of (7), the next step usually taken is simulation. In particular, by considering changes to the generalized distance term  $t_{jk}$ , it is possible to simulate the effects of detailed changes in trade policy. This is the approach taken in the studies of the effects of Brexit mentioned in the introduction, for example. Applications of this kind are becoming increasingly common.<sup>12</sup> Given our current state of knowledge, they provide the best available quantitative answer to questions such as “How will Brexit affect UK trade and GDP?” However, given the complexity of the multilateral trade linkages considered, they run the risk of seeming like “black boxes.” In the remainder of this section, we take a different, complementary approach. We ask what can be said about the qualitative properties of the model. Such a theoretical analysis is not possible in levels. However, it can be done in terms of local changes, following the standard approach of comparative statics.

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<sup>12</sup>For an overview of the issues that arise in implementing them, see Yotov, Piermartini, Monteiro, and Larch (2016).

### 3.2 The Structure of Simple Structural Gravity Models

Comparative statics for structural gravity have been explored by a number of authors, including Alvarez and Lucas (2007), Dekle, Eaton, and Kortum (2008) and Allen, Arkolakis, and Takahashi (2019). The approach adopted here also has similarities to the framework for aggregating from micro to macro in a multi-sectoral economy developed by Baqaee and Farhi (2017). It is most closely related to the classic exposition of the two-sector Heckscher-Ohlin model in Jones (1965), and its multi-sectoral extension in Jones and Scheinkman (1977).<sup>13</sup>

Following Jones (1965) and Jones and Scheinkman (1977), define the shares of trade in exporter GDP and importer expenditure as follows:

$$\lambda_{jk} = \frac{V_{jk}}{Y_j} \quad \theta_{jk} = \frac{V_{jk}}{E_k} \quad (8)$$

With balanced trade ( $E_j = Y_j, \forall j$ ), these shares are related to each other in exactly the same way as the analogous shares are in Jones (1965):  $\lambda_{jk}\theta_j = \theta_{jk}\theta_k$ , where  $\theta_j \equiv Y_j/Y_W$  is country  $j$ 's share in world GDP. An important special case is where country  $j$  is relatively “small”, so its share in world GDP is close to zero,  $\theta_j \approx 0$ , and all other countries are “large”.

In this case:

$$\lambda_{kj} = \frac{\theta_{kj}}{\theta_k}\theta_j \approx 0 \quad \text{and} \quad \theta_{jk} = \frac{\lambda_{jk}}{\theta_k}\theta_j \approx 0, \quad \forall k \neq j \quad (9)$$

So, every other country exports only an infinitesimal proportion of its output to  $j$ , and devotes only an infinitesimal proportion of its expenditure to imports from  $j$ .

Now, express changes in terms of these shares, using “hats” to denote local proportional changes,  $\hat{x} \equiv d \log x$ . It turns out to be most insightful to do this using the primitive equations of the model for consumer equilibrium and market-clearing, (2) and (3) respectively, rather than the structural gravity equations themselves, (5) and (6). First, we can express the requirement that the market for each good must clear as applying at the margin. Totally differentiating equation (3), the change in each country's GDP must equal a  $\lambda$ -weighted

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<sup>13</sup>Similar multi-sectoral results, using different notation, were obtained by Diewert and Woodland (1977).

average of the changes in its sales to each country (including home sales as well as exports):

$$Y_j = \sum_k V_{jk} \quad \Rightarrow \quad \hat{Y}_j = \sum_k \lambda_{jk} \hat{V}_{jk} \quad \Rightarrow \quad 0 = \sum_k \lambda_{jk} (\hat{t}_{jk} + \hat{x}_{jk}) \quad j = 1, \dots, n \quad (10)$$

The middle expression in (10) holds for all versions of the structural gravity model. The final one specializes to the Armington version, where GNP is just the home price times the exogenously given stock of output,  $Q_j$ :  $Y_j = p_j Q_j$ . In this form it says that a  $\lambda$ -weighted average of changes in production for each market must sum to zero, keeping in mind that production includes a trade cost component. Second, from (2), the change in the price index in each country must equal a  $\theta$ -weighted average of the changes in retail prices there, both of the home-produced good and of imports:

$$P_k = \left( \sum_h p_{hk}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad \Rightarrow \quad \hat{P}_k = \sum_j \theta_{jk} \hat{p}_{jk} \quad k = 1, \dots, n \quad (11)$$

(The right-hand side holds for any true cost-of living index, not just the CES.)

Equations (10) and (11) are the basic building blocks of the general-equilibrium system. Next, we add the demand functions, from (1), totally differentiated to give demands at the margin:

$$\hat{x}_{jk} = -\sigma \hat{p}_{jk} + (\sigma - 1) \hat{P}_k + \hat{E}_k \quad (12)$$

Using (11), we can write the own and cross-price derivatives of demand as follows:

$$\frac{\partial \log x_{jk}}{\partial \log p_{jk}} = -(\sigma(1 - \theta_{jk}) + \theta_{jk}) \quad \frac{\partial \log x_{jk}}{\partial \log p_{hk}} \Big|_{h \neq j} = (\sigma - 1) \theta_{hk} \quad (13)$$

A central implication of CES preferences is that these derivatives exhibit the gross substitutes property:  $-\frac{\partial \log x_{jk}}{\partial \log p_{jk}} > \frac{\partial \log x_{jk}}{\partial \log p_{hk}} > 0$ . As first noted by Alvarez and Lucas (2007), this property guarantees that the model is well-behaved with intuitive properties. Finally, to complete the general-equilibrium system, we add to (10), (11) and (12) three additional definitional

equations.<sup>14</sup> First is the link between prices and trade costs:

$$p_{jk} = p_j t_{jk} \quad \Rightarrow \quad \hat{p}_{jk} = \hat{p}_j + \hat{t}_{jk}. \quad (14)$$

Second is the link between national expenditure and GDP:

$$E_j = \kappa_j Y_j \quad \Rightarrow \quad \hat{E}_j = \hat{Y}_j. \quad (15)$$

The parameter  $\kappa_j$  equals one when aggregate trade is balanced, and is assumed to be exogenous: the model does not attempt to explain macroeconomic imbalances. Third is the specification of the supply side, which takes the simple Armington form:

$$Y_j = p_j Q_j \quad \Rightarrow \quad \hat{Y}_j = \hat{p}_j \quad (16)$$

Since the price of each country's output uniquely determines its income, we can also identify it with the wage:  $w_j = p_j$  implying that  $\hat{w}_j = \hat{p}_j$ . Naturally, the models with more sophisticated supply sides discussed earlier have richer mechanisms for wage determination.

### 3.3 An Application to Brexit

To illustrate in a stylized way how the general model can be used to understand the trade effects of Brexit, we specialize to three countries that we label  $A$ ,  $B$ , and  $E$ ; “ $B$ ” for “Britain”, “ $E$ ” for “Europe”, and “ $A$ ” for the rest of the world, or, for concreteness, “America”. This reduces the dimensionality of the general  $n$ -country model to three. Moreover, one market-clearing condition is redundant by Walras's Law, and one country's domestic price can be set equal to unity by choice of numéraire, so we can explore the model's properties in two dimensions. (This is a true numéraire or measuring rod: like the choice between Fahrenheit

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<sup>14</sup>With the further addition of the decomposition of changes in trade values into price and quantity components,  $\hat{V}_{jk} = \hat{p}_{jk} + \hat{x}_{jk}$ , this gives seven equations in all, which determine the seven endogenous variables,  $\hat{Y}_j$ ,  $\hat{V}_{jk}$ ,  $\hat{x}_{jk}$ ,  $\hat{p}_{jk}$ ,  $\hat{E}_k$ ,  $\hat{P}_k$ , and  $\hat{p}_j$ .

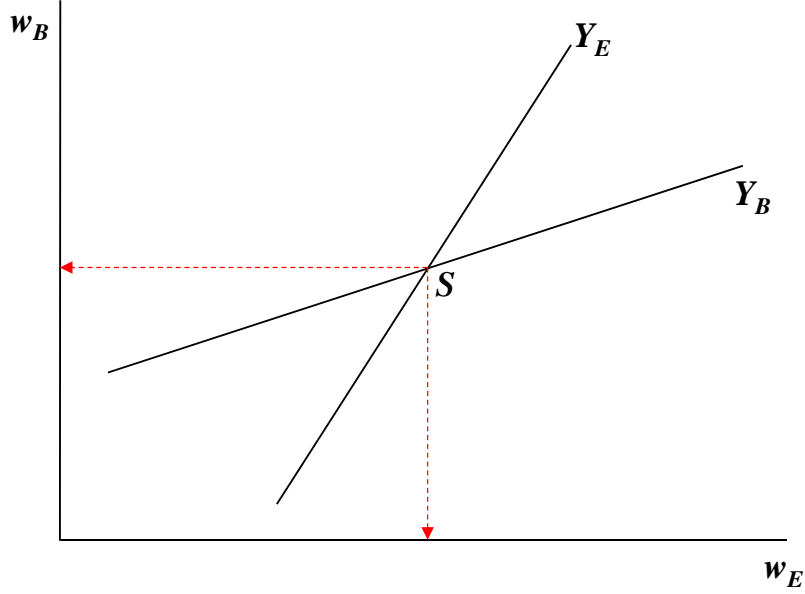


Figure 4: Determination of Equilibrium Wages in Countries  $B$  and  $E$

and Celsius, it does not affect the model's implications, though for framing reasons it may make us feel differently about them.) To fix ideas, we concentrate on countries  $B$  and  $E$ . Hence we omit the market-clearing condition for country  $A$ 's good, and select it as the numéraire, so all nominal variables are measured relative to prices in  $A$ . This allows us to understand the determination of equilibrium by illustrating in a simple diagram how the market-clearing conditions for outputs of  $B$  and  $E$  determine equilibrium wages, which are equivalent to prices:  $w_B = p_B$  and  $w_E = p_E$ .

Figure 4 illustrates in  $\{w_E, w_B\}$  space. The curve labelled  $Y_B$  indicates the combinations of wages in  $B$  and  $E$  consistent with market-clearing for country  $B$ 's output. Its properties can be deduced from (10) and (12).<sup>15</sup> Starting at any point along the  $Y_B$  locus, a higher wage in  $B$  leads to excess supply, a higher wage in  $E$  leads to excess demand, while a uniform increase in both wages leads, from gross substitutability, to excess supply. Hence the  $Y_B$  locus must be upward-sloping but with a slope less than  $45^\circ$  as shown. A symmetric argument

<sup>15</sup>For detailed derivations, see Appendix B.

shows that the market-clearing locus for country  $E$ 's output must also be upward-sloping, but with a slope greater than  $45^\circ$  as shown by the  $Y_E$  locus. The intersection of the two loci therefore determines the unique equilibrium wages  $w_B$  and  $w_E$ . Of course, the two loci need not have the same slope; for example, if  $B$  is a relatively small economy, then the  $Y_E$  locus is vertical.

Having illustrated the determination of equilibrium, we can now explore how it responds to shocks. This shows how easy it is to explore alternative trade policy scenarios in the gravity model.

As a first step, we decompose trade costs into those that are “natural”, denoted  $\delta_{jk}$ , and those that are policy-induced, denoted  $\tau_{jk}$ :<sup>16</sup>

$$t_{jk} = \delta_{jk}\tau_{jk} \tag{17}$$

The former includes distance of course, as well as historically given factors that encourage or discourage trade, such as colonial ties or common language. For simplicity, we assume that all trade costs are bilaterally symmetric, and that policy costs yield no revenue. The first of these assumptions is perhaps less innocent than it seems: the distance from Britain to Europe is the same in each direction, but the costs of transporting goods need not be, if for example the mix of goods shipped in the two directions is very different. By contrast, the second assumption is perhaps more innocent than it seems: the majority of policy-induced trade barriers in the modern world economy are not tariffs but rather standards and technical barriers, that are not primarily, if at all, revenue-raising. Finally, given our focus on Brexit, we assume that trade costs between America and Europe remain fixed throughout.

Next, we want to consider some alternative trade policy scenarios. For simplicity we consider only three, as summarized in Table 1. In all three, the natural trade costs are the same: Britain is always geographically closer to Europe than to America, so  $\delta_{BE} < \delta_{BA}$ . The differences relate to the policy-induced trade costs. First is the status quo, where

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<sup>16</sup>See Maggi, Mrázová, and Neary (2018).

Scenario	$\delta_{BE}$	$\tau_{BE}$	$\delta_{BA}$	$\tau_{BA}$
Status quo	low	low	high	high
“Cake and Eat”	low	low	high	low
“Global Britain”	low	high	high	low

Table 1: Alternative Trade Policy Scenarios

Notes: (1) All trade costs are assumed to be bilaterally symmetric.  
(2) Revenue from policy costs is ignored.

membership of the EU’s Customs Union and Single Market ensures that Britain faces lower artificial barriers with Europe than with America:  $\tau_{BE}$  is low while  $\tau_{BA}$  is high. Second is what we can call the “Cake and Eat” scenario: lower trade costs with America plus unchanged trade costs with Europe ensure the best of both worlds for Britain. Third is what we can call the “Global Britain” scenario: withdrawing from the Single Market and the EU Customs Union raises trade costs with Europe but leaves Britain free to negotiate an alternative trade agreement with America, so  $\tau_{BA}$  falls. We focus throughout on the direct economic consequences of each of these two scenarios relative to the status quo, ignoring political-economy considerations, such as the clear incentive of EU countries to maintain the integrity of the Single Market, or the difficulties for Britain of negotiating new trade agreements on favourable terms with non-EU countries.

Consider first the “Cake and Eat” scenario. The only change this implies relative to the status quo is a fall in the trade cost between Britain and America,  $\tau_{BA}$ . Because trade costs fall in both directions, a lower  $\tau_{BA}$  has an ambiguous effect on the demand for British output:

$$\hat{X}_B = -(\sigma - 1) \underbrace{\{\lambda_{BA}(1 - \theta_{BA})\}}_{(1): +} \underbrace{-\lambda_{BB}\theta_{AB}}_{(2): -} \hat{\tau}_{BA} \quad (18)$$

The ambiguity arises because bilateral trade liberalization has two opposing effects. The first effect reflects increased opportunities in the export market: a reduction in the cost of shipping good  $B$  to  $A$  raises exports. (This effect is dampened but cannot be reversed by the

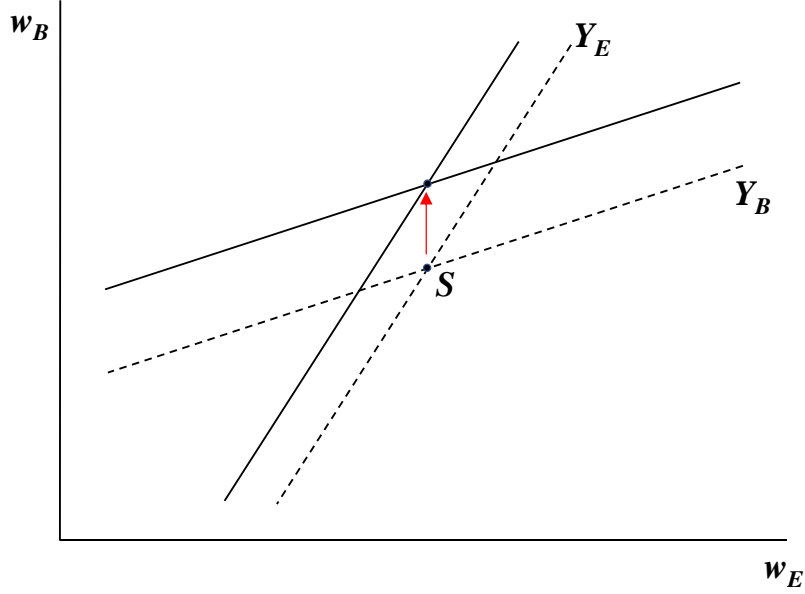


Figure 5: “Cake and Eat”: The Case where Nominal Wages Rise in Britain

induced rise in the price index in  $A$  represented by  $\theta_{BA}$ , which in any case can be ignored when  $B$  is small.) The second effect reflects increased competition in the home market: imports into  $B$  are cheaper, which diverts home demand away from home goods.<sup>17</sup> For the moment, assume that the first effect dominates: this case is easier to analyse diagrammatically, and as we will see the answer it gives applies more generally. Hence the market-clearing locus for good  $B$  is shifted upwards as shown in Figure 5, tending to raise the equilibrium wage  $w_B$ .

The market-clearing locus for good  $E$  is also shifted:

$$\hat{X}_E = (\sigma - 1)(\lambda_{EA} \underbrace{\theta_{BA}}_{\text{red bracket}} + \lambda_{EB} \underbrace{\theta_{AB}}_{\text{red bracket}}) \hat{\tau}_{BA} \quad (19)$$

This effect is unambiguous, as European exports face tougher competition in both the British and American markets. Hence, the  $Y_E$  locus shifts to the left, as shown in Figure 5. However,

<sup>17</sup>The contrast between these two effects is reminiscent of the distinction between trade creation and trade diversion in the theory of customs unions. The analogy is useful though not perfect, as these effects would arise even if there were only two countries.

these effects are not very strong if Britain is small, when the terms underlined in red tend to zero. Americans devote a relatively small proportion of their expenditure to British output, so  $\theta_{BA}$  is small; and Europe exports a relatively small proportion of its GDP to Britain, so  $\lambda_{EB}$  is small. The net effect is therefore a rise in  $w_B$  and an ambiguous change in  $w_E$ , as illustrated in Figure 5. What is unambiguous is that  $w_B$  rises relative to  $w_E$ ; and since an absolute increase in  $w_B$  implies that it rises relative to the numéraire  $w_A$ , it follows that real wages in Britain must increase.

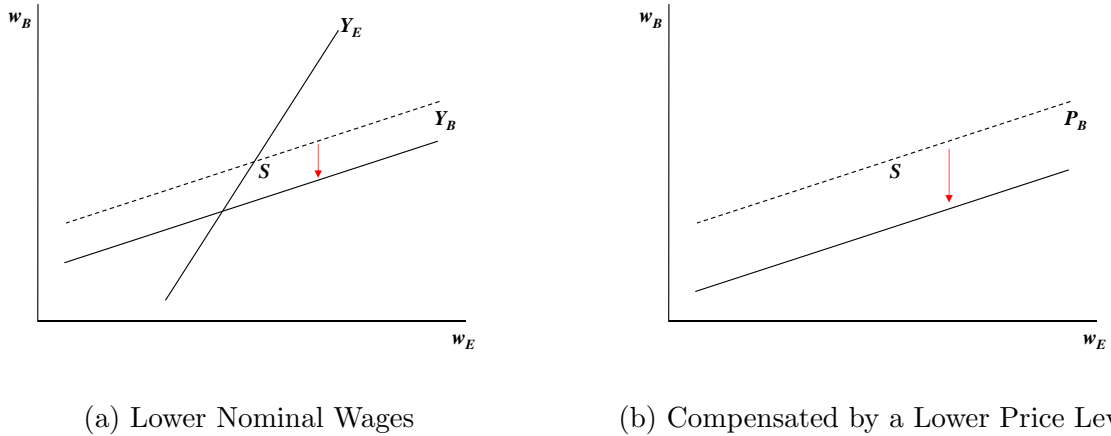


Figure 6: “Cake and Eat”: The Case where Nominal Wages Fall in Britain

For completeness, recalling the ambiguity in (18), we must also consider the case where a lower  $\tau_{BA}$  reduces demand for  $Y_B$ , as illustrated in panel (a) of Figure 6. This leads to a lower nominal wage in Britain, but nonetheless the real wage there is likely to rise. This is because the same condition which implies a lower nominal wage, namely where  $\theta_{AB}$  is large enough that home demand for  $B$  falls, also implies that the price level in Britain falls a lot: the British consumer spends a lot on imported American goods, so benefits from the fall in the trade cost. As we show in Appendix B, when Britain is small these two effects exactly cancel, so the effect of higher exports dominates: the real wage in  $B$  definitely rises. (It can be shown that this result holds for any number of countries.) Hence the overall conclusion is an unsurprising one: having your cake and eating it too is good for you; in this stylized

gravity model, Britain unambiguously gains from a bilateral reduction in trade costs with America coupled with unchanged trade costs with Europe.

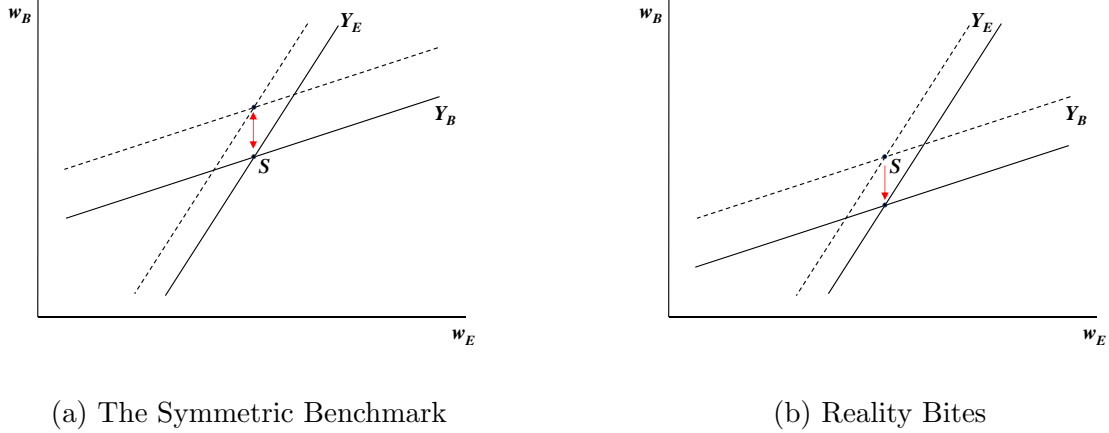


Figure 7: “Global Britain”

What then of the second scenario, “Global Britain”, where trade barriers with America come down but those with Europe go up? To understand this case it is helpful to begin with a hypothetical symmetric benchmark in which America and Europe are identical from the perspective of Britain: equally distant, equally rich (so they have the same GDP), and equally restrictive (so their initial policy trade barriers are the same). Moreover, we assume that the proportionate reduction in  $\tau_{BA}$  is small, and is exactly equal, except opposite in sign, to the proportionate increase in  $\tau_{BE}$ . A moment’s reflection should make it clear that in this case of complete symmetry between  $A$  and  $E$  the Global Britain scenario implies no net effect relative to the status quo. As Panel (a) of Figure 7 illustrates, the effect of a small increase in  $\tau_{BE}$  exactly offsets that of a small reduction in  $\tau_{BA}$ .

The point of this symmetric benchmark is of course to highlight the fact that, even at this level of abstraction, there are many reasons why the Global Britain scenario is likely to be asymmetric. The first and probably the most important reason for a departure from symmetry concerns the depth of economic integration with the two partner countries. The Single Market combined with the EU Customs Union is a very deep trade agreement: rather

than merely abolishing tariffs, it also eliminates substantial non-tariff barriers to trade in both goods and services; moreover it imposes regulatory alignment, underpinned by a framework of commercial law subject to the rulings of the European Court of Justice. It is highly unlikely that any future trade agreement between Britain and America could attain this degree of integration. And, stepping outside the three-country model for a moment, even if the UK and US were to enter into such a deep trade agreement, it would not be matched by agreements with Britain's other non-EU partners. Thus, Britain can never attain with the rest of the world the same degree of economic integration that it currently enjoys with the EU. In terms of the model's parameters, all this implies that  $\tau_{BE}|_S < \tau_{BA}|_{GB}$ : the policy barriers to trade between Britain and Europe in the Single Market status quo are unambiguously lower than those between Britain and America in any plausible Global Britain scenario.

There are other reasons why the symmetric benchmark is misleading. A second dimension of asymmetry is size. Defenders of the economic case for Brexit often claim that the UK will gain by switching its trade away from the EU towards faster-growing countries. However, what matters is not absolute size, but size mediated by distance. As we have seen in Section 2, the EU27 accounts for 40% of 2017 UK trade, and countries that have trade agreements with the EU add another 15%. This dominance of EU countries in UK trade is partly a direct result of EU membership but mainly a consequence of geographic proximity; to the extent that the latter matters it is likely to persist in any post-Brexit scenario. A third source of asymmetries arises from the difference between increases in low policy costs and decreases in high ones. This is moot for infinitesimal changes in trade costs as in our comparative statics exercises, but it matters for discrete changes. Because  $\tau_{BE}$  is initially much lower than  $\tau_{BA}$ , the loss from a 10%-point increase in  $\tau_{BE}$  is greater in absolute value than the gain from a 10%-point decrease in  $\tau_{BA}$ . Finally, to the extent that distance imposes fixed costs on trade, the effects of changes in  $\tau_{BE}$  and  $\tau_{BA}$  will differ. To take a simple example, if the iceberg trade cost  $t_{jk}$  decomposes in an additive rather than a multiplicative way as in (17), then a

reduction in policy costs with country  $j$  has a smaller effect the further away it is:

$$t_{jk} = \delta_{jk} + \tau_{jk} \quad \Rightarrow \quad \hat{t}_{jk} = (1 - \omega_{jk})\hat{\tau}_{jk}, \quad \omega_{jk} \equiv \frac{\delta_{jk}}{t_{jk}} \quad (20)$$

For this reason too, the costs of raising trade barriers with nearby EU countries are likely to be higher than the benefits of lowering them against far-away trading partners.

All these reasons combined show that the benchmark case of symmetry between the two potential foreign partners is a poor reflection of the actual options facing the UK in choosing between alternative trade agreements. Moreover, all four departures from symmetry work in the same direction: against the neutral outcome of the symmetric benchmark, and in favour of an outcome such as that in Panel (b) of Figure 7, where Global Britain is poorer than in the status quo.

## 4 Gravity Anomalies

So far, we have presented an exposition of gravity theory and empirics in an uncritical way. In this section, we turn to consider some anomalous features of the structural gravity model. In Section 4.1 we review the growing evidence against the model's key underlying assumption of CES preferences, while in Section 4.2, we show that, under plausible conditions, CES-based structural gravity imposes very strong counter-factual restrictions on bilateral trade balances. Note that there is no contradiction between the two parts of the paper: Sections 2 and 3 have presented the current consensus on the facts about gravity and the theories underlining them, while Sections 4 and 5 will present some more speculative thoughts on how the models might be improved. Science is provisional, so there is always room for improvement on current models and techniques, despite which, they provide the best answer we can currently give to applied questions.<sup>18</sup>

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<sup>18</sup>The analogy with evolution is helpful here too. Gould (1981) notes that creationists often point, wrongly, to disagreements between evolutionary scientists over the detailed mechanisms of evolution as evidence of fundamental disagreement over the validity of evolution as fact.

## 4.1 Gravity Assumptions: CES Preferences

It has been known for some time that CES preferences have very strong implications when embedded in models of monopolistic competition. In particular, they imply that mark-ups should be the same across all firms, and that the pass-through from costs to prices should be always 100%. There is a substantial body of evidence from industrial organization and international macroeconomics which estimates less than 100% rates of cost or exchange-rate pass-through. See, for example, Weyl and Fabinger (2013) and Gopinath and Itskhoki (2010) respectively. However, it is only relatively recently that credible micro evidence has become available that allows for joint tests of these two predictions. In particular, De Loecker, Goldberg, Khandelwal, and Pavcnik (2016), building on De Loecker and Warzynski (2012), study a large sample of Indian firms, and find both that markups are heterogeneous and that pass-through is less than 100%.<sup>19</sup> In the remainder of this section, we draw on Mrázová and Neary (2017) to illustrate just how strongly this evidence is inconsistent with the CES assumption.

The starting point of the approach in Mrázová and Neary (2017) is that, for many purposes, it is more insightful to consider demand functions not in quantity-price space as is standard, but in the space of the elasticity,  $\varepsilon \equiv -\frac{p}{xp'}$ , and convexity,  $\rho \equiv -\frac{xp''}{p'}$ , of demand, where  $p(x)$  is the inverse demand function. Figure 8 illustrates. In this space, the first- and second-order conditions for profit-maximization require that a monopolistically competitive firm can only be in equilibrium at a point that lies in an “admissible region,” defined by the conditions  $\varepsilon > 1$  and  $\rho < 2$ : the demand function must be elastic and “not too” convex. This corresponds to the area above and to the left of the solid dark lines in Figure 8.<sup>20</sup>

The next step is to establish which points in  $\{\rho, \varepsilon\}$  space correspond to a given demand function in  $\{x, p\}$  space. One special case is the CES: each CES demand function corresponds

<sup>19</sup>The findings of De Loecker et al. are particularly persuasive because the methods they use to estimate markups place no restrictions on technology or market structure.

<sup>20</sup>The first-order condition is that marginal revenue  $p + xp'$  equal marginal cost  $c$ , assumed to be exogenous for each firm; the second-order condition is that marginal revenue is decreasing in output:  $2p' + xp'' < 0$ . It is easy to check that these imply the boundaries of the admissible region as shown.

to a single value of the demand elasticity,  $\varepsilon = \sigma$ , as well as a single value of  $\rho$ , and so is represented by a single point in  $\{\rho, \varepsilon\}$  space. The curve labelled “CES” in Figure 8 is the locus of all such points.<sup>21</sup> This curve is also an important benchmark for comparative statics properties. As already noted, CES demands imply full proportional pass-through from marginal costs to price:  $\frac{d \log p}{d \log c} = 1$ . It is easy to check that points on a demand curve implying a greater degree of pass-through must correspond to points in this space that lie to the right of the CES locus: here demand is more convex than in the CES case, so Mrázová and Neary (2019b) call this the “superconvex” region. Conversely, points in the “subconvex” region to the left of the CES locus correspond to less than proportional pass-through. Subconvexity is also equivalent to “Marshall’s Second Law of Demand,” the hypothesis that the elasticity of demand falls with sales.

As for non-CES demand functions, Mrázová and Neary (2017) show that, subject only to relatively mild technical restrictions, any such demand function can be represented by a smooth curve in  $\{\rho, \varepsilon\}$  space. They call such a curve the “demand manifold” corresponding to the original demand function. Figure 8 illustrates the demand manifolds for some widely-used demand functions: linear, “CARA” (constant absolute risk-aversion), translog, and LES (linear expenditure system, also known as Stone-Geary).<sup>22</sup> All of these lie in the subconvex portion of the admissible region. Moreover, they exhibit a property that Mrázová and Neary (2017) call “manifold invariance”: while the demand function  $p(x; \phi)$  depends on a vector of parameters  $\phi$ , many demand manifolds are invariant with respect to some or all elements of  $\phi$ .<sup>23</sup> This makes it much easier to understand the implications of different assumptions about the form of demand.

The final step is to use Figure 8 to illustrate the results of De Loecker et al. (2016). We begin with the fact that their results give estimates, with confidence intervals, of the average

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<sup>21</sup>The CES demand function implies values for the elasticity and convexity of  $\varepsilon = \sigma$  and  $\rho = \frac{\sigma+1}{\sigma}$  respectively. Eliminating  $\sigma$  yields the expression for the locus of CES point-manifolds:  $\rho = \frac{\varepsilon+1}{\varepsilon}$ .

<sup>22</sup>The manifolds for these demand functions are given by  $\rho = 0$ ,  $\rho = \frac{1}{\varepsilon}$ ,  $\rho = \frac{2}{\varepsilon}$ , and  $\rho = \frac{3\varepsilon-1}{\varepsilon^2}$ , respectively. From the perspective of a monopolistically competitive firm, the translog demand function is observationally equivalent to the almost ideal demand system of Deaton and Muellbauer (1980).

<sup>23</sup>Necessary and sufficient conditions for manifold invariance are given in Mrázová and Neary (2017).

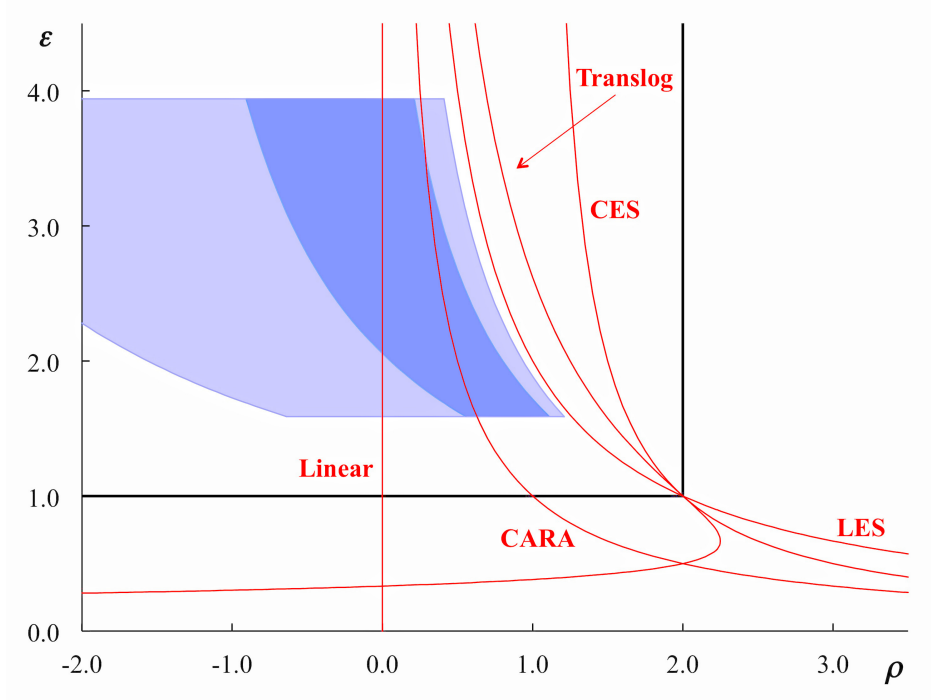


Figure 8: Evidence against CES Demands

Source: Mrázová and Neary (2017), based on data from De Loecker et al. (2016)

markup,  $m$ , and pass-through coefficient,  $\kappa$ , for their sample of firms. Assuming that the market is monopolistically competitive, these expressions can be written as functions of the elasticity and convexity of demand:

$$(i) \quad m \equiv \frac{p - c}{c} = \frac{1}{\varepsilon - 1} \quad (ii) \quad \kappa \equiv \frac{d \log p}{d \log c} = \frac{\varepsilon - 1}{\varepsilon} \frac{1}{2 - \rho} \quad (21)$$

These equations can then be solved to back out the values of the elasticity and convexity implied by the markup and pass-through estimates:

$$(i) \quad \varepsilon = \frac{m + 1}{m} \quad (ii) \quad \rho = 2 - \frac{1}{\kappa} \frac{1}{m + 1} \quad (22)$$

The shaded regions in Figure 8, taken from Mrázová and Neary (2017), show the results of doing this, using the data from De Loecker et al. (2016). They give estimates of the pass-through coefficient using both ordinary least squares (OLS) and instrumental variables

(IV): the dark-blue region in the figure represents the confidence region implied by the OLS estimate, while the light-blue region represents the confidence region implied by the IV estimate.<sup>24</sup>

It is clear from Figure 8 that all CES demands lie outside the implied confidence regions. (The data also reject LES and Translog demands, though less strongly.) More tests of this kind are needed of course, but taken in conjunction with the evidence on pass-through mentioned earlier, it seems reasonable to conclude that there is substantial micro-economic evidence against the hypothesis of CES demands. All of this suggests that the assumption of CES preferences which underlies the structural gravity model is open to question.

## 4.2 Gravity Predictions: Bilateral Trade Balances

The previous sub-section noted that the key assumption of CES preferences has implications that are inconsistent with a growing body of microeconomic evidence. In this sub-section we turn to a prediction of the structural gravity model itself that is not confirmed by the data.

As we have seen, the structural gravity model predicts bilateral trade flows  $V_{jk}$ . Hence it also predicts their *ratios*, which equal the bilateral trade balances  $V_{jk}/V_{kj}$  in ratio form between each pair of countries.<sup>25</sup> However, under reasonable assumptions about bilateral trade costs, the model’s predictions for bilateral trade balances are very stark and are not borne out by the data. This was first pointed out in the frictionless trade case by Davis and Weinstein (2002), who called the anomalous prediction “the mystery of the excess trade balances”.<sup>26</sup> The result is also derived for a very general structural gravity model by Allen, Arkolakis, and Takahashi (2019), though they do not emphasise the implications for bilateral trade balances. Here we give a self-contained presentation of the result and its implications.

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<sup>24</sup>For details on the estimates and the calculations, see Mrázová and Neary (2017), Online Appendix B17.

<sup>25</sup>There is a better-known precedent for considering ratios of bilateral trade flows: the *products* of bilateral trade flows predicted by gravity models are widely used to infer trade costs and the elasticity of trade. See, for example, Head and Ries (2001), Jacks, Meissner, and Novy (2008), and Caliendo and Parro (2015).

<sup>26</sup>Other empirical work on the result includes Badinger and Fichet de Clairfontaine (2018), Cuñat and Zymek (2018), and Felbermayr and Yotov (2019).

We begin with the simplest case. Assume that overall trade is balanced, so national income and expenditure are equal for all countries:  $Y_j = E_j, \forall j$ . As for trade costs, we assume that they are bilaterally symmetric:  $\tau_{jk} = \tau_{kj}, \forall j, k$ . Now recall the structural gravity equation from (5), and divide exports from  $j$  to  $k$  by exports from  $k$  to  $j$ :

$$\frac{V_{jk}}{V_{kj}} = \left( \frac{\Pi_j}{P_j} \right)^{\sigma-1} \bigg/ \left( \frac{\Pi_k}{P_k} \right)^{\sigma-1} \quad (23)$$

Given the assumptions we have made, the terms in income, expenditure, and bilateral trade costs vanish, leaving only the ratios of outbound to inbound multilateral resistance for each country. However, with bilaterally symmetric trade costs, these are proportional to each other:  $P_j = \lambda \Pi_j$ , as first pointed out by Anderson and van Wincoop (2003).<sup>27</sup> Hence the model implies that all bilateral trade balances are zero! It hardly needs checking that this prediction is overwhelmingly rejected by the data: there is considerable variation in bilateral trade balances across countries, whence the “mystery of the excess trade balances.”

Of course, the assumptions made are strong, but they can be relaxed. Taking trade costs first, we can replace the assumption of bilateral symmetry with what Allen and Arkolakis (2016) call *quasi*-symmetric bilateral trade costs:<sup>28</sup>

$$t_{jk} = t_j^X \bar{t}_{jk} t_k^M, \quad \bar{t}_{jk} = \bar{t}_{kj} \quad (24)$$

Here the dyad-specific term  $\bar{t}_{jk}$  is symmetric as before, and in addition each country has two idiosyncratic trade cost terms, one that applies to all its exports and the other to all its imports. This allows among other things for home bias and for border effects; see, for example, Head and Ries (2001). However, as Allen, Arkolakis, and Takahashi (2019) show,

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<sup>27</sup>There is some potential confusion in the literature on this point. Anderson and van Wincoop (2003) go further than proportionality between  $P_j$  and  $\Pi_j$  and set  $\lambda = 1$ . They call this “an implicit normalization”; it would be more conventional to call it a choice of numéraire. As such, it is perfectly valid, though it is not advisable if another nominal variable is also chosen as numéraire: see Baldwin and Taglioni (2007) and Balistreri and Hillberry (2007).

<sup>28</sup>This assumption can be found in Eaton and Kortum (2002) and Anderson and van Wincoop (2003).

the ratio of outbound and inbound multilateral resistances for a given country, though no longer the same across all countries, is now equal to the ratio of that country's export and import trade cost parameters:  $\frac{\pi_j}{P_j} = \frac{t_j^X}{t_j^M}$ . As a result, quasi-symmetric bilateral trade costs do not affect the prediction that all bilateral trade balances are zero. Second, we can relax the assumption of overall trade balance, allowing for  $E_j \neq I_j$ . This does allow for non-zero bilateral trade balances, but only in a restricted way:

$$\frac{V_{jk}}{V_{kj}} = \frac{I_j}{E_j} \bigg/ \frac{I_k}{E_k} \quad (25)$$

Thus, the bilateral trade balance between countries  $j$  and  $k$  equals the ratio of their overall trade balances. The implications of this can be brought out by rewriting (25) in logs:

$$v_{jk} - v_{kj} = \rho_j - \rho_k \quad \text{where:} \quad \rho_j \equiv \log \frac{I_j}{E_j} \quad (26)$$

Thus, with quasi-symmetric trade costs, the  $\frac{1}{2}n(n-1)$  bilateral trade balances,  $v_{jk} - v_{kj}$ , are uniquely determined by  $n$  country-specific aggregate trade balances  $\rho_j$ . This reduces the dimensionality of the bilateral trade balance terms by a factor of  $\frac{2}{n-1}$ . Putting this differently, with quasi-symmetric trade costs, the vector of bilateral trade balances between any country  $j$  and all other countries is independent of  $j$ , except for a factor of proportionality.

Structural gravity models based on CES preferences thus make the very stark prediction that bilateral asymmetries in trade costs are the only source of bilateral asymmetries in trade balances. Is this plausible? Both symmetric and quasi-symmetric bilateral trade costs are a simplifying assumption, greatly reducing the number of independent trade costs. It is easy to think of cases where they might be expected to fail. Composition effects are an obvious example: a resource-exporting country might be expected to incur different transport costs on its exports than on its imports. Yet it is difficult to believe that trade cost asymmetries alone can save the models from their singular inability to allow for the observed diversity in bilateral trade balances. Empirical results find that trade balances are related to exactly

the same variables as one-way trade flows themselves: notably country size and distance. (See for example the empirical results in Davis and Weinstein (2002).) This makes no sense from a CES perspective, but suggests that relaxing the constant elasticity assumption may provide a route to a better explanation of bilateral trade balances. In the next section we turn to explore an approach to doing this.

## 5 Subconvex Gravity

As we saw in the last section, the constant elasticities of demand and of trade that are a central feature of CES-based structural gravity models have anomalous implications once we move away from aggregate trade flows. This suggests that it is worth exploring alternative approaches to modeling trade flows. At the same time, relaxing the assumption of CES preferences requires a sacrifice of theoretical tractability and ease of estimation. As a tractable compromise that relaxes the CES assumption but does not lead to an intractable specification, we explore in this section the case of demands that are generated by additively separable preferences.<sup>29</sup> These have the advantage that, as with CES, all cross-price effects are summarized in terms of a single parameter. At the same time they allow for subconvexity, and so are consistent with the empirical evidence in Section 4.1. Moreover, they nest not just CES itself, but also both sub- and superconvex cases, so we can test for subconvexity.

Additively separable preferences imply a simple first-order condition: the marginal utility of each good  $j$  in each consuming country  $k$  depends only on its own consumption, and equals its price times country  $k$ 's marginal utility of income. This can then be solved for demands that, as in the CES case, depend only on exporter and importer terms and on the trade cost between  $j$  and  $k$ :

$$u'(\eta_j^{-1}x_{jk}) = \lambda_k p_{jk} \quad \Rightarrow \quad x_{jk} = \eta_j f(\lambda_k p_j t_{jk}) \quad (27)$$

Here  $\eta_j$  is a taste shifter for country  $j$ 's good, written in a way that is consistent with the CES

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<sup>29</sup>This section draws on work in progress: Carrère, Mrázová, and Neary (2019).

specification in (1). Multiplying by price and taking a first-order approximation expresses changes in the volume of trade as a function of changes in the origin-country taste shifter and price, the destination-country marginal utility of income, and the bilateral trade cost:

$$\hat{V}_{jk} = \hat{\eta}_j - (\sigma_{jk} - 1)\hat{p}_j - \sigma_{jk}\hat{\lambda}_k - (\sigma_{jk} - 1)\hat{t}_{jk} \quad (28)$$

The only difference from the CES case is that the elasticity is variable. Of course, this is a major difference: the elasticity is not only variable but differs between each distinct pair of countries. The assumption of additive separability is helpful here, since it implies that the elasticity depends only on the volume of trade:  $\sigma_{jk} \equiv \sigma(x_{jk})$ .<sup>30</sup> In addition, subconvexity, for which there is substantial micro-econometric evidence as we saw in the last section, implies that the elasticity is decreasing in the volume of trade:  $\sigma_{jk}$  is decreasing in  $x_{jk}$ .

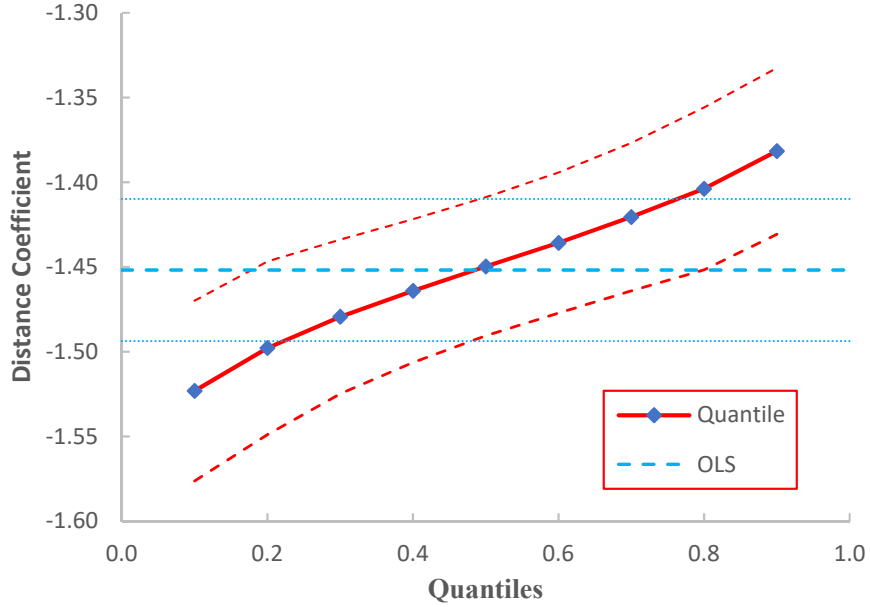


Figure 9: Quantile and OLS Estimates of the Distance Coefficient

Taking (28) to data poses a challenge, as the  $\sigma_{jk}$  coefficients on the right-hand side depend on  $x_{jk}$ , which is a component of the dependent variable on the left-hand side. Allowing the coefficients to vary continuously with export volume is not feasible. However, we can allow

<sup>30</sup>See Goldman and Uzawa (1964).

them to vary discretely by using quantile regression.<sup>31</sup>

Let quantile  $q \in (0, 1)$  denote the value of the dependent variable which splits the data into proportions  $q$  below and  $1 - q$  above. Quantile regression for quantile  $q$  selects the coefficient estimates to minimize a weighted sum of the absolute deviations from the regression equation,  $|e_i|$ , where the weights assign asymmetric penalties  $q|e_i|$  for underprediction and  $(1 - q)|e_i|$  for overprediction. Thus the quantile regression estimator for quantile  $q$  minimizes the loss function:

$$L(\beta_q) = \sum_{i: y_i \geq x'_i \beta_q} q |y_i - x'_i \beta_q| + \sum_{i: y_i < x'_i \beta_q} (1 - q) |y_i - x'_i \beta_q| \quad (29)$$

Note that each quantile regression is estimated over the whole sample but with different penalties depending on the quantile we are interested in. To implement this we first order the observations by  $V_{jk}$ , and divide them into quantiles. In practice, we use ten divisions, so we work with deciles. Note that the specification ideally requires that we order the observations by the volume of trade  $x_{jk}$ , not the value, but this is not possible with the data available. We then estimate for each quantile  $q$  the following regression:

$$\log V_{q,jk} = F_{q,j} + F_{q,k} + \mu_q \log t_{jk} + u_{q,jk} \quad (30)$$

where the coefficient vector is  $\beta_q = (F_{q,j}, F_{q,k}, \mu_q)'$ . This can be compared with the corresponding equation in the CES case, (7).

Figure 9 shows the estimated distance coefficients, with bootstrapped 95% confidence intervals, from the quantile regressions. We use the Method of Moments-Quantile Regression estimation procedure of Machado and Santos Silva (2019) to estimate the quantile coefficients; this makes it possible to estimate a quantile regression when there is a large number of fixed effects. For reference, Figure 9 also presents the OLS estimate.<sup>32</sup> Table 2 presents

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<sup>31</sup>For a previous application of quantile regression in a gravity context, see Baltagi and Egger (2016).

<sup>32</sup>As already noted in Section 2, footnote 9, this equals  $-1.452$  with a standard error of  $0.021$ .

the results of significance tests for differences between the quantile and OLS estimates of the distance coefficient.<sup>33</sup>

	$\beta_{OLS}$	$\beta_{10}$	$\beta_{20}$	$\beta_{30}$	$\beta_{40}$	$\beta_{50}$	$\beta_{60}$	$\beta_{70}$	$\beta_{80}$	$\beta_{90}$
$\beta_{10}$	<b>2.06</b>									
$\beta_{20}$	1.36	0.67								
$\beta_{30}$	0.87	1.23	0.53							
$\beta_{40}$	0.40	<b>1.70</b>	0.98	0.48						
$\beta_{50}$	0.07	<b>2.14</b>	1.44	0.95	0.48					
$\beta_{60}$	0.53	<b>2.54</b>	<b>1.85</b>	1.39	0.94	0.47				
$\beta_{70}$	1.02	<b>2.92</b>	<b>2.26</b>	<b>1.83</b>	1.41	0.96	0.50			
$\beta_{80}$	1.48	<b>3.27</b>	<b>2.63</b>	<b>2.24</b>	<b>1.85</b>	1.43	0.99	0.50		
$\beta_{90}$	<b>2.13</b>	<b>3.83</b>	<b>3.22</b>	<b>2.86</b>	<b>2.50</b>	<b>2.09</b>	<b>1.65</b>	1.16	0.63	

Table 2: *t*-Statistics Testing the Significance of Differences Between the Quantile and OLS Estimates of the Distance Coefficient\*

\*Values that are significantly different at the 5% level are indicated in red

Overall, Figure 9 and Table 2 present persuasive evidence for subconvexity. The quantile estimates of the distance coefficient are significantly decreasing (in absolute value) in the value of trade, and the one-size-fits-all CES-based constant coefficient hypothesis is rejected for both relatively high and relatively low trade flows. Of course, it would be very desirable to replicate this study with other data sets. Our findings are in line with those of Novy (2013) who uses a different technique to allow for variation in the distance coefficient.<sup>34</sup> For now it seems reasonable to conclude that subconvex gravity deserves serious consideration.

What would be the implications of moving from a CES-dominated view of gravity to one that allows for subconvexity? As far as the two anomalies discussed in Section 4 are concerned, it is clear that subconvex gravity points to a resolution of both. By construction, it is consistent with microeconomic evidence in favour of variable markups and less than

<sup>33</sup>Tests comparing an estimated quantile coefficient with the OLS estimate are two-sided, with a 5% threshold of 1.96. Tests comparing two estimated quantile coefficients are one-sided, with a 5% threshold of 1.64.

<sup>34</sup>Novy uses an OLS specification, with slope dummies on the distance coefficient for each quantile of the predicted value of trade. After this paper was completed, we came across a related paper by Chernozhukov, Fernandez-Val, and Weidner (2018). They develop a different estimator for this class of problem, and give an application to trade data from 1986, where they find evidence that the distance coefficient rises in absolute value with the value of trade, implying that import demand is superconvex.

100% pass-through. It also allows for the elasticity of import demand to vary systematically across destinations, for which there is some evidence. (See Novy (2013), for example.) As for the trade balances puzzle, with subconvexity bilateral balances now depend on distance, and by more the greater the imbalance. We have already noted that there is evidence confirming this in previous work; we hope to explore how strong this is in our future research.

More generally, it is not possible to solve for a closed-form structural gravity equation in the general subconvex case. However, two other routes to working with gravity models of this kind are open. First, the local comparative statics results presented in Section 3 continue to apply: inspecting the equations in Appendix B shows that all the own- and cross-price elasticities of import demand are indexed by the two countries involved. While it is true that the elasticity of substitution is the same in all cases, the elasticities also depend on the physical and value shares in world trade coefficients,  $\lambda_{jk}$  and  $\theta_{jk}$ . Hence the qualitative conclusions drawn in Section 3, based on the derivations in Appendix B, require no modification whatsoever when we replace the CES assumption with subconvex preferences. Second, computer modeling using parameterized forms of subconvex gravity poses no major problems in principle. In future work we hope to explore this approach.

Finally, we can ask what if any are the implications of subconvex gravity for predictions about the effects of trade policy changes such as Brexit? With subconvexity, elasticities of import demand are higher in smaller markets and lower in larger ones. In general that is likely to lead to more nuanced predictions about the effects of trade policy changes in multi-country settings. However, in the specific context of estimating the effects of a policy shock such as Brexit the differences are unlikely to be major. In the “Cake and Eat” scenario, the larger benefits of reducing trade costs in smaller non-EU markets are likely to be offset by the greater difficulty of raising exports to bigger markets. And in the more plausible “Global Britain” scenario, the same applies to the effects of increased access costs in EU markets: smaller reductions in exports to larger markets traded off against larger falls in smaller markets. The cumulative effects of these changes need to be explored in fully specified

empirical models. But *a priori* it seems likely that the implications of subconvex gravity for the estimated effects of Brexit will not be major: the results of previous studies are likely to be robust.

## 6 Conclusion

In this paper we have provided an overview of the role of gravity in international trade, developed some pedagogic tools to illustrate it, and discussed some ways in which the standard models could be extended. We have emphasised that gravity in trade is both fact and theory. At the level of fact, there is overwhelming evidence that trade tends to fall with distance, as even a superficial examination of UK export patterns confirms. At the level of theory, the structural gravity model is consistent with a range of theoretical underpinnings, including Ricardian comparative advantage and monopolistic competition with heterogeneous firms. We have emphasised that it is no more and no less than a simple general-equilibrium system, and have presented new analytic tools for understanding it in small-scale applications. We have also noted some difficulties with the standard models, “gravity anomalies,” which arise from the underlying assumption of CES preferences, and which imply that models with a constant elasticity of trade cannot tell the whole story about trade patterns. Finally, we have sketched an approach based on subconvex gravity that provides a promising way forward. Relaxing the assumption of a constant elasticity of trade makes the model more consistent with microeconomic evidence on markups and pass-through, and also avoids the CES model’s stark predictions about bilateral trade balances. However, it is unlikely to change the “Three Iron Laws” of the economics of Brexit outlined in recent gravity-based studies.

# Appendices

## A Data Sources

The data used in charts and regressions on trade flows and gravity variables (such as distance, colonial ties, etc.) come mainly from the CEPII BACI database:

[http://www.cepii.fr/CEPII/en/bdd\\_modele/presentation.asp?id=8](http://www.cepii.fr/CEPII/en/bdd_modele/presentation.asp?id=8)

GDP data are from the World Bank’s World Development Indicators:

<http://datatopics.worldbank.org/world-development-indicators/>

All reported regression results use these data without amendment. To highlight features of UK trade, the data in Figures 2 and 3 amend these data slightly in two respects. First, the “ex-colony” variable in the database refers to whether a country was a colony in 1945. In the figures, we extend this list to include as former UK colonies a number of countries (Australia, Canada, Ireland, New Guinea, New Zealand, South Africa) that obtained independence or self-government before that date. Second, the FTA variable in the database refers to whether an FTA has been notified to the WTO as of 2016. In the figures we add trade agreements that were not in effect in 2016. Information on trade agreements other than the EU Customs Union and Single Market in which the UK participates at the time of writing (November 5, 2019) comes from the EU website, listing all EU trade agreements with third countries:

[http://ec.europa.eu/trade/policy/countries-and-regions/  
negotiations-and-agreements/](http://ec.europa.eu/trade/policy/countries-and-regions/negotiations-and-agreements/)

Additional information on UK trade policy can be found at the UK Trade Policy Observatory at the University of Sussex:

<http://blogs.sussex.ac.uk/uktpo/>

## B Solving the Three-Country Case

To confirm the properties of Figures 4 and 5, consider first the general market-clearing condition (10), specialized to the case of country  $B$  in the three-country model. Recall that  $w_A$  is constant by choice of numéraire; that trade costs between  $A$  and  $E$  are assumed to be constant:  $\hat{t}_{AE} = 0$ ; and that trade costs between  $A$  and  $B$  and between  $B$  and  $E$  are assumed to be symmetric:  $\hat{t}_{AB} = \hat{t}_{BA} = \hat{t}_A$  and  $\hat{t}_{BE} = \hat{t}_{EB} = \hat{t}_E$ . Substituting from the changes in the price indices in (11) into (10) gives:

$$\varepsilon_{BB}\hat{w}_B + \varepsilon_{BE}\hat{w}_E + \varepsilon_{Bt_A}\hat{t}_A + \varepsilon_{Bt_E}\hat{t}_E = 0 \quad (31)$$

where the elasticities of excess demand for country  $B$ 's output with respect to prices and trade costs can be written in full as follows:

$$\begin{bmatrix} \varepsilon_{BB} \\ \varepsilon_{BE} \\ \varepsilon_{Bt_A} \\ \varepsilon_{Bt_E} \end{bmatrix} = \begin{bmatrix} -(\sigma - 1)\lambda_{BB}(1 - \theta_{BB}) - \lambda_{BE}(\sigma(1 - \underbrace{\theta_{BE}}_{\text{red}}) + \underbrace{\theta_{BE}}_{\text{red}}) - \lambda_{BA}(\sigma(1 - \underbrace{\theta_{BA}}_{\text{red}}) + \underbrace{\theta_{BA}}_{\text{red}}) \\ (\sigma - 1)\lambda_{BB}\theta_{EB} + \lambda_{BE}((\sigma - 1)\theta_{EE} + 1) + (\sigma - 1)\lambda_{BA}\theta_{EA} \\ -(\sigma - 1)(\lambda_{BA}(1 - \underbrace{\theta_{BA}}_{\text{red}}) - \lambda_{BB}\theta_{AB}) \\ -(\sigma - 1)(\lambda_{BE}(1 - \underbrace{\theta_{BE}}_{\text{red}}) - \lambda_{BB}\theta_{EB}) \end{bmatrix} \quad (32)$$

Terms underlined in red are zero when country  $B$  is infinitesimally small. The expressions for the elasticities in (32) confirm the properties noted in the text:  $\varepsilon_{BB}$  and  $\varepsilon_{BE}$  are negative and positive respectively, while  $\varepsilon_{Bt_A}$  and  $\varepsilon_{Bt_E}$  are ambiguous in sign.

We can repeat the analogous substitutions for the excess demand for country  $E$ 's good:

$$\varepsilon_{EB}\hat{w}_B + \varepsilon_{EE}\hat{w}_E + \varepsilon_{Et_A}\hat{t}_A + \varepsilon_{Et_E}\hat{t}_E = 0 \quad (33)$$

where:

$$\begin{bmatrix} \underbrace{\varepsilon_{EB}} \\ \varepsilon_{EE} \\ \underbrace{\varepsilon_{Et_A}} \\ \underbrace{\varepsilon_{Et_E}} \end{bmatrix} = \begin{bmatrix} \underbrace{\lambda_{EB}}((\sigma-1)\theta_{BB}+1) + (\sigma-1)\lambda_{EE}\underbrace{\theta_{BE}} + (\sigma-1)\lambda_{EA}\underbrace{\theta_{BA}} \\ -\underbrace{\lambda_{EB}}(\sigma(1-\theta_{EB})+\theta_{EB}) - (\sigma-1)\lambda_{EE}(1-\theta_{EE}) - \lambda_{EA}(\sigma(1-\theta_{EA})+\theta_{EA}) \\ (\sigma-1)(\lambda_{EA}\underbrace{\theta_{BA}} + \underbrace{\lambda_{EB}}\theta_{AB}) \\ -(\sigma-1)(\underbrace{\lambda_{EB}}(1-\theta_{EB}) - \lambda_{EE}\underbrace{\theta_{BE}}) \end{bmatrix} \quad (34)$$

These expressions have similar properties to those in (32), with two exceptions. First, country  $E$  is not directly involved in an increase in trade costs between  $A$  and  $B$ , so demand for its good changes as a result only to the extent that the price indices in  $A$  and  $B$  rise; hence  $\varepsilon_{Et_A}$  is unambiguously positive. Second, when country  $B$  is infinitesimally small, neither its home price nor either of the trade costs it faces have any effect on the demand for country  $E$ 's output.

Combining (31) and (33):

$$\begin{bmatrix} -\varepsilon_{BB} & -\varepsilon_{BE} \\ -\varepsilon_{EB} & -\varepsilon_{EE} \end{bmatrix} \begin{bmatrix} \hat{w}_B \\ \hat{w}_E \end{bmatrix} = \begin{bmatrix} \varepsilon_{Bt_A} \\ \varepsilon_{Et_A} \end{bmatrix} \hat{t}_A + \begin{bmatrix} \varepsilon_{Bt_E} \\ \varepsilon_{At_E} \end{bmatrix} \hat{t}_E \quad (35)$$

Let  $\Delta \equiv \varepsilon_{BB}\varepsilon_{EE} - \varepsilon_{BE}\varepsilon_{EB}$  denote the determinant of the coefficient matrix on the left-hand side. Because demands exhibit gross substitutability, we know that  $-\varepsilon_{BB} > \varepsilon_{BE} > 0$  and  $-\varepsilon_{EE} > \varepsilon_{EB} \geq 0$ . It follows that  $\Delta$  is positive. Solving for the effect of a change in the bilateral trade cost with  $A$ ,  $t_A$ , on the wage in country  $B$  gives:

$$\hat{w}_B = \frac{1}{\Delta} \begin{vmatrix} \varepsilon_{Bt_A} & -\varepsilon_{BE} \\ \varepsilon_{Et_A} & -\varepsilon_{EE} \end{vmatrix} \hat{t}_A = \frac{1}{\Delta} (-\varepsilon_{Bt_A}\varepsilon_{EE} + \varepsilon_{BE}\varepsilon_{Et_A}) \hat{t}_A \quad (36)$$

Assume now that country  $B$  is small, so that  $\varepsilon_{EB} = \varepsilon_{Et_A} = 0$ . In Figures 4 to 7 this implies that the  $X_E$  locus is vertical and is unaffected by changes in  $t_A$ , so  $w_E$  is independent of  $t_A$ .

In algebraic terms, it implies that the change in  $w_B$  simplifies to:

$$\hat{w}_B = -\frac{\varepsilon_{Bt_A}\hat{t}_A}{\varepsilon_{BB}} \quad (37)$$

The denominator is negative, so the sign of the change in the wage rate depends on the numerator. Recalling (32), this is the same as the change in the excess demand for country  $B$ 's output, as shown in (18) in the text. As noted there, it is the sum of a positive “trade creation” effect and a negative “trade diversion” one.

Our main interest is in the change in the real wage in country  $B$ , which in general equals:

$$\hat{w}_B - \hat{P}_B = (1 - \theta_{BB})\hat{w}_B - \theta_{EB}\hat{w}_E - (\theta_{AB}\hat{t}_A + \theta_{EB}\hat{t}_E) \quad (38)$$

We consider the special case where  $\hat{w}_E = 0$  because  $B$  is small; and we focus on the effect of a fall in  $t_A$ :  $\hat{t}_A < 0$ . Substituting for  $\hat{w}_B$  from (37):

$$\hat{w}_B - \hat{P}_B = -(1 - \theta_{BB})\frac{\varepsilon_{Bt_A}\hat{t}_A}{\varepsilon_{BB}} - \theta_{AB}\hat{t}_A = -\frac{1}{\varepsilon_{BB}}((1 - \theta_{BB})\varepsilon_{Bt_A} + \theta_{AB}\varepsilon_{BB})\hat{t}_A \quad (39)$$

Next we substitute for  $\varepsilon_{Bt_A}$  and  $\varepsilon_{BB}$  from (32):

$$\hat{w}_B - \hat{P}_B = \frac{1}{\varepsilon_{BB}}((1 - \theta_{BB})(\sigma - 1)(\lambda_{BA} - \lambda_{BB}\theta_{AB}) + \theta_{AB}((\sigma - 1)\lambda_{BB}(1 - \theta_{BB}) + \sigma(\lambda_{BE} + \lambda_{BA})))\hat{t}_A \quad (40)$$

As noted in the text, the source of ambiguity is the trade diversion effect represented by the second expression in the first set of parentheses: lowering the trade cost between  $A$  and  $B$  reduces the price level in  $B$  (to an extent depending on  $\theta_{AB}$ ), which lowers domestic demand for the home good (to an extent depending on  $\lambda_{BB}$ ). However, though this tends to reduce the wage in  $B$ , the lower price level tends to increase the real wage (to an extent that also

depends on  $\theta_{AB}$ ). In fact the two terms exactly cancel:

$$\hat{w}_B - \hat{P}_B = \frac{1}{\varepsilon_{BB}} ((\sigma - 1)(1 - \theta_{BB})\lambda_{BA} + \sigma\theta_{AB}(1 - \lambda_{BB})) \hat{t}_A \quad (41)$$

Thus (bearing in mind that  $\varepsilon_{BB}$  is negative), a reduction in  $t_A$  unambiguously raises the real wage in  $B$ , as stated in the text. It can be shown that this result holds for any number of countries, provided  $B$  is small: the downward effect of the trade cost reduction on the home price index is always enough to counteract the negative trade diversion effect on nominal wages of making imports more attractive and hence the home good less attractive to home consumers.

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