Playing with Benford's Law

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ABSTRACT: This paper presents a classroom experiment, the simulations, and a research which familiarize the students with the Benford's Law. This law is widely used in a tax fraud detecting procedures. This paper shows that: i) the Benford's Law can be useful in extending the simple perception of the probability which is presented at the lectures concerning the risk, ii) can be an excellent example of using data processing for the classroom tasks, iii) by the experience of the fraud detecting technique the students might change their attitude to cheating. The experiment and the prepared R codes can be used in the numerous courses, such as accounting, applied microeconomics, and quantitative methods.

Keywords: Benford's Law, probability perception, tax fraud, scientific fraud, R-CRAN

JEL Code: A2, C15, D80, M42

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Introduction

The Benford's Law (hereafter – BL) is a collection of the empirical evidence about the frequency distribution of the digits in the numerical data sets. The best-known version of the law states that in the data sets, which represent a collection of a "natural" data, the probability of seeing a digit on the first position is inversely related to its rank. For instance, number 1 appears as a first digit in about 30% of all cases, while number 9 appears in less than 5% of cases. The law is non-intuitive because one would rather expect that the digits are uniformly distributed, and the chances of seeing each digit on the first position are equal (with a chance of 1/9=11.1%). The other versions of the BL define the frequency distribution of the second digits, third digits, and their combinations. The history of the discovery of this law makes it even more mysterious. Originally designed by Newcomb (Newcomb & Nuw, 1881), the law was forgotten and rediscovered by Benford (1938), who used the law to explain the behaviour of the numerous data sets from the different domains of science. Based on those empirical observations, Benford proposed a new functional form of the distribution, which defined the law of first digit distribution more precisely. The BL can be treated as one of the most interesting representatives of the power laws, which are used in natural sciences and empirical research in economics (Gabaix, 2016).

Despite the lack of the sufficient theoretical explanations (Berger & Hill, 2011), the fact that Benford's distribution commonly appears in real-life data allows for using it as the indicator of whether given sample is real-life indeed. The real-life numbers are assumed to be defined by the range of the random events, thus following BL. In contrast, if the numbers are the product of human design, the distribution of the first digits should differ from the Benford's distribution. A special area of application of BL is the fight against the tax frauds (Drake & Nigrini, 2000).

It seems that the topics which can gain from introducing the BL to the curriculum are accounting and finance, and in general economic sciences. The popularity of BL, especially in the accounting textbooks, rises as the big data analysis enters everyday accounting practice (Janvrin & Watson, 2017). However, it can be seen wider, as proposed here, that: i) the law can be useful in extending the simple perception of the probability, which is presented at the microeconomic lectures concerning the risk; ii) the law can be a good example of using the sophisticated data processing for the applied tasks.

The paper is organized as follows: the first part is focused on presenting of the BL. The second part presents the potential benefits of using it in accounting, microeconomics, and quantitative methods courses. The last part focuses on the design of the experiment with technical notes on how to conduct and create further extension of using them for the in-class teaching activities.

1. History, formulation, and testing of Benford's Law

The history of the BL begins in XIX century, when the astronomer Newcomb (1881) noticed in the library that first pages of the logarithm books were the most thumbed, while the last pages left relatively undamaged. The scientist explained this by the fact the frequency of distribution of first digits is not uniform and is inversely dependent on the digit's rank. Initially, this distribution pattern was defined as

$$P_D = log_{10}\left(1 + \frac{1}{D}\right) \tag{1}$$

and implied that probability P_D of occurring the first digit D, which takes the value from the range $D = 1,2, \dots ,9$, can be defined as the logarithm to base 10 of $\left(1 + \frac{1}{D}\right)$. If the distribution of the first digit was uniform, the probability of occurring as a first number would be identical for each digit and approximately equal 11.1 %. However, according to the Newcomb's formula for a probability distribution (1), for numbers 1, 2, and 9, the probability of appearing as the first digit is close to 30%, 17%, and 4%, respectively. Despite of lack of theoretical justification, the pattern is general and irrespective to the units of measurement or data source, thus being referred as "universal property of real world measurements" (Sambridge & Tkalcic, 2010), or, more poetically, the "harmonic" nature of the world (Furlan, 1948).

The law was rediscovered by Benford in 1938, who extended the rule to the form:

$$P_D = log_B\left(1+\frac{1}{D}\right) \tag{2}$$

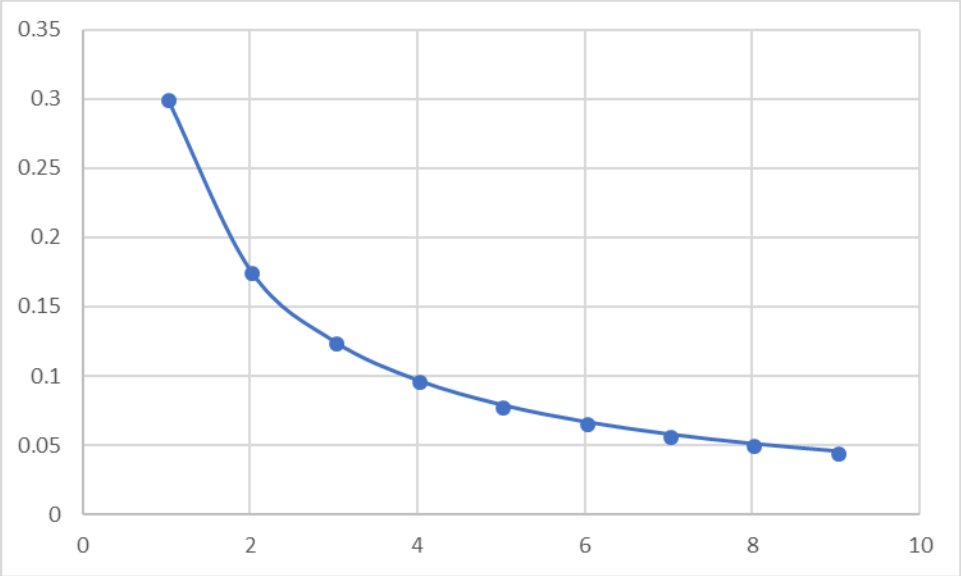
through introducing base B and variable D denoting the rank of the digit. Thus, it is possible to define the distribution pattern not solely for the digit D, but also for a combination of n digits. Tab.1 presents the precise expected frequency of the first four digits in accordance with the Benford's distribution; Fig.1 shows the probability distribution function for the first digit, which is the most commonly applied.

Table 1. The probabilities of occurring on the position of first, second, third, and fourth digit in the numbers coming from the natural dataset

Digit	Probability of occurring at the first position	Probability of occurring at the second position	Probability of occurring at the third position	Probability of occurring at the fourth position
0	N/A	0.11968	0.10178	0.10018
1	0.30103	0.11389	0.10138	0.10014
2	0.17609	0.10882	0.10097	0.10010
3	0.12494	0.10433	0.10057	0.10006
4	0.09691	0.10031	0.10018	0.10002
5	0.07918	0.09668	0.09979	0.09998
6	0.06695	0.09337	0.09940	0.09994
7	0.05799	0.09035	0.09902	0.09990
8	0.05115	0.08757	0.09864	0.09986
9	0.04576	0.08500	0.09827	0.09982

Source: Nigrini, 1999, p. 2.

Figure 1: The distribution of the leading digit: a visualization



Source: Own elaboration based on Nigrini, 1999, p. 2.

Benford (1938) analyzed 20’229 numbers related to the cities’ population, financial data, etc. coming from 20 different statistical sources and proved that the distribution of the first digits followed the pattern he defined.

One explicit and unified theoretical justification of the BL has not been developed so far (Sambridge et al., 2011; Berger & Hill, 2011). Nevertheless, the attempts to design it have resulted in the theorem on the random samples from the random distributions. This theorem suggests that even if some particular individual distributions of the real data do not follow the BL, the random samples from such distributions still do (Hill, 1995). As it is discussed in the next section, numerous studies explored the opportunity to exploit BL for the sake of the tax fraud revealing. From this perspective, the researchers need the statistical tools which would provide the reliable results concerning the deviations of the presumably “natural” data samples from the distributions predicted by the BL.

One of the most common methods of the statistical testing of the theoretical and empirical distributions’ consistence is a goodness-of-fit test. Its null hypothesis (H0) states that the sample of the data does not differ from the sample predicted by the BL. The test statistics is given with the typical Chi squared distribution (Cho & Gaines, 2007, p. 220):

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{3}$$

where O_i and E_i denote the observable and expected frequency, respectively. It follows the Chi^2 distributions with k degrees of freedom. One of the major disadvantages of this test is that it is extremely sensitive to the size of a sample, which makes it too rigid for the testing purposes (Ley, 1996; Giles, 2007).

One of the possible alternatives may be the Kolmogorov-Smirnov test for comparing two empirical distributions through defining the largest absolute difference between the cumulative distribution functions as the measure of the discrepancy:

$$\chi^2(k) = \sup|F_{n1}(x) - F_{n2}(x)| \cdot \sqrt{{n_1 \cdot n_2}/{(n_1 + n_{2)}}} \qquad (4)$$

where k is the total number of observations, $F_{n1}(x)$ and $F_{n2}(x)$ are empirical cumulative distributions of two samples, and n_1 and n_2 are the sizes of these samples. According to Stephens (1970), the Kolmogorov – Smirnov test is not biased by the sample size and gives more reliable conclusions. However, as noted by Noether (1963), the tests based on the null hypothesis of a continuous distribution may not be appropriate for testing the discrete distributions. As the result, the test values may be quite conservative in rejecting the null hypothesis, giving false-negative results.

2. *Application of the Benford's Law in teaching*

2.1.Accounting and finance – the tax frauds

The first proposition to apply the BL for checking the validity of the natural number's sample was made by Varian (1972). The BL had not been applied by the accountants and tax officers up to the late 80's, when Carslaw (1988) found out that the distribution of the digits in the numbers reflecting income streams from New Zealand was not in line with the BL. Nigrini's works (1996; 1999) on the digital analysis were actually helpful in finding the cases of the tax evasion in U.S. At the same time, some researchers (see, for instance, Ettredge & Srivastava, 1999) stated that the fact that the distribution of the first digits in the presumably real numbers differs from the BL does not necessarily imply the fraud. The phenomenon may also result from the flaws and operating inefficiencies in the operating data systems. Durtschi et al. (2004) defined the cases when the BL should or should not be applied in order to recognize the fraud. They claim that the BL can be used as the reliable detector of the fraud i) when the numbers are the combinations (products) of the other numbers (e.g. sales); ii) when the dataset is large enough; iii) when the mean is greater than the median, and the skewness is positive. In contrast, the BL will not give a reliable conclusions about the case of fraud under the condition that: i) the numbers are assigned (e.g. invoice numbers); ii) the numbers are the product of human mind design by themselves (e.g. prices, which tend to end with 99); iii) the numbers come from the accounts with a large proportion of firm-specific numbers; and iv) when there are minimum and maximum numbers built in.

In the last decade, there is a lot of financial and accounting literature which shows how to use the BL in tracking frauds and its applicability for forensic accounting (Nigrini, 2012). The presented there *know-how* focus on the operation side of the problems (Simkin, 2010). The focus on the applied aspects of the BL in accounting and financial literature caused spreading and extending of these methods (Nigrini, 2012). A new promising area of applications in finance and accounting can be the anti-money-laundering (Yang & Wei, 2010) and the financial statement analysis (Henselmann et al., 2012). The paper by Drake and Nigrini (2000) is a good example how to efficiently introduce students to the Benford's Law and digital analysis of accounting data. This paper can be treated as a very comprehensive instruction for students and practitioners of accounting and auditing.

Microeconomics - training the probability perception

The important issue related to the modern way of teaching microeconomics is the excessive use of "chalk-and-talk" approach as well as the insufficient attention towards the students' skills of solving the applied problems (Watt, 2011). The BL experiment can be treated as a one of the series of activating tools used during the applied microeconomics courses. Its main aim is to show how the simplified perception of the probability can result in the wrong perception of real-life economic phenomena. Even setting up a simple standard model requires, among others, realistic understanding of the probabilities, which economic education usually leaves beyond the agenda (Gigerenzer 2015). Majority of the economic textbooks present the problems, where the probabilities are either explicitly defined or can be derived using simple rules of probabilities counting (see, for instance, McConnel et al. 2014, Varian 2014, Frank & Cartwright 2013). Students are not taught of probabilities and of evaluating them on their own, but are getting information as given from the textbook problems.

The BL law can be treated as the Power Laws (hereafter PLs), which are a good source of the empirical evidence necessary to develop the skills of the realistic probability perception, since they facilitate the process of gaining background knowledge about the numerous empirically observed patterns. Firstly, PLs are present in the various fields of science and life. One of the best known and commonly mentioned beyond the field of formal science PLs is a Pareto principle. The size of cities tend to follow the trend predicted by the Zipf's Law (1936), as well as the firms' sizes and the stock exchanges cumulative returns (Gabaix, 2016). Secondly, the unique thing about PLs is that they break the common perception that the variables' values are distributed around the mean (i.e. that data generally fit the uniform or normal distribution pattern). Human tendency to average is so strong that even so well-known Pareto distribution of income is flattered in perception, as Norton and Ariely (2011) showed in their experimental study.

2.2.Big data – new challenge

The development of the field of data science has provided the additional opportunities of applying the statistical analysis like testing of BL in many areas of management, accounting, business, and economics. It will be part of "big data" revolution, in which data are treated as a new production factor. The big data is referred as "*high-volume, high-velocity and/or high-variety information assets* […]" (Gartner, 2015, retrieved from Janvrin & Watson, 2017, p. 2). The field of data science as it is understood nowadays is relatively young, while it has already integrated to the wide range of related fields. Teachers should find a new way of incorporating this change to the existing didactic process.

Janvrin and Watson (2017) analyze the role of the "big data" for maintaining the accounting standards. Big data is even more relevant to the revealing tax fraud cases when applying the BL, because using automatic machine procedures is less costly than elaborating the human mind. Further development of data science and the standardization of reporting to tax and financial authorities may facilitate the development of the automated audits. For the accounting students, the new tools provide a possibility to analyze the structured and unstructured data using Business Intelligence (hereafter BI) technologies and avoiding time-consuming and laborious calculations in Excel. The BI are going to be a part of accounting students curriculum (Drake & Nigrini, 2000).

The "big data" revolution will also change the way of teaching economics. The new IT tools make the statistical analysis simpler and easier. Admittedly, students of economics have no access to typical BI tools due the relatively high costs, but the combination of open source R, R-studio and Markdown software makes it possible to conduct their own research without any costs. The availability of the ready-to-use procedures allow to carry out such data exploration with little programming skills and can be a good motivation to learn coding and statistics. The possibility of testing the BL seems to be wide open for student and teachers. The emerging areas with increasing number of applications make it more attractive (Berger & Hill, 2015). There are two main areas of using the BL in the classroom: i) testing the quality of economic data and investigation of scientific fraud and fake data; ii) testing the behaviour of the forecast in the econometric modelling.

Similarly to the tax and financial data, the economic data may contain the defects resulting from unreliable reporting or intentional manipulation. The exploitation of the data should be preceded by the procedures of testing the data quality. The literature showed that the credibility of the data used in an economic and social science research is moderate or even low (Ioannidis & Doucouliagos, 2013). The BL is one of the potential tools to investigate the problem of data falsification (Michalski & Stoltz, 2013). The application of these new IT tools make the procedure so simple that even the students can be investigators and discover the possible scientific frauds and non-naturality of the data. This can be their first step towards the critical thinking on the scientific evidence. The experience of investigations of this type can also reduce the own moral hazard of students through demonstrating the existence of the efficient tools for detecting the fraud.

The additional relatively new area of using the BL is an application in testing the robustness of the forecasts in the econometric modeling**.** If the input data fits the BL, then the output data should also fulfill this dependence. The new evidence shows that some outputs of the numerical algorithms, transformations of the random variables and the stochastic processes as well as the multidimensional systems fulfill the BL (Berger & Hill, 2015). A test based on the BL may be an interesting proposition to evaluate the models such as Dynamic Stochastic General Equilibrium Models, which have no statistical goodness-of-fit measures yet.

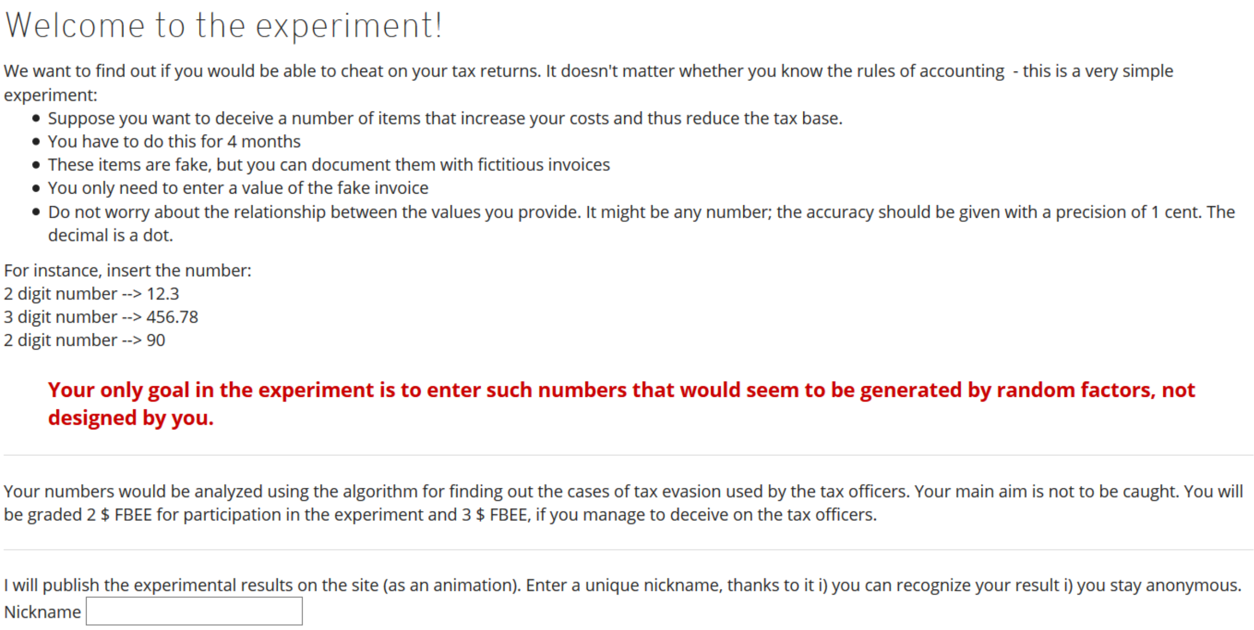
3. *The experiment, simulation and research – the BL in the classroom - case study*

The didactic tools presented below were used to introduce the BL during the Applied Microeconomics course, but they can also be used at the accounting / finance or data science classes. The first stage of the study is to conduct the experiment; the students have no prior knowledge when providing the answers, and this is expected to have a positive effect on their motivation to learn the subject through a creation of a kind of "information deprivation". The second stage is devoted to the analysis of the BL, including theorical formulation of the law, real-life examples, and an application of the BL in the tax fraud detection. The demonstration of the Monte-Carlo simulation is presented to prove the evidence collected by the other authors. Finally, the students' answers are presented. The third stage of the study incorporates the analysis of the IT tools necessary for conducting the experiment and the data analysis. The tax fraud problem was a main storytelling (or a narrative framework) used in order to make the problem interesting for the students. The main goal of using the tools was behind the story. It was attempted to show to the students that: i) the simple perception of the probability can result in the inefficiency in the analysis of the real-life problems; ii) the possibility of the fraud in the big data era is much smaller, and this is explained not only by the tax and financial issues, but also by the developments in the scientific research and public statistics.

The experiment

During the experiment, the students were asked to put 60 numbers to the fictitious invoices. Their goal was to generate such data, which would seem to be generated by the random factors, not by the human mind. Students received a small amount of additional grading points for a participation in the experiment, and they also gained extra points for avoiding being caught. The incentive may seem to be non-ethical, as the students received extra points for non-ethical behavior; however, the motivation incentive scheme is correct. The chance that they would receive the points for an efficient cheating is very small, as the antifraud procedures based on the BL work fine. Even after the lecture, student who knew and understood the BL, had the problems with a fictional cheating on the tax authorities because they could not control the frequencies of all the digits. In addition, the students should experience that their effort to cheat is useless. This experience (i.e. demonstrating how easily the fraud can be detected) is a crucial factor, which can influence their further behavior and motivate to avoid the frauds (Joyner, 2011), especially, in the light of the public presentation of the antifraud procedures results. To avoid the possibility of boycotting the experiments caused by transparency of the decision, students were asked to enter the anonymous nickname during the experiment (see the first screen of the experiment at Fig. 2). They could recognize their results while still remaining anonymous on the animation presenting the results of the antifraud algorithm (see Fig. 2).

Figure 2: Initial screen in Benford Law experiment



Welcome to the experiment!

We want to find out if you would be able to cheat on your tax returns. It doesn't matter whether you know the rules of accounting - this is a very simple experiment:

- Suppose you want to deceive a number of items that increase your costs and thus reduce the tax base.
- You have to do this for 4 months
- These items are fake, but you can document them with fictitious invoices
- You only need to enter a value of the fake invoice
- Do not worry about the relationship between the values you provide. It might be any number; the accuracy should be given with a precision of 1 cent. The decimal is a dot.

For instance, insert the number:
2 digit number --> 12.3
3 digit number --> 456.78
2 digit number --> 90

Your only goal in the experiment is to enter such numbers that would seem to be generated by random factors, not designed by you.

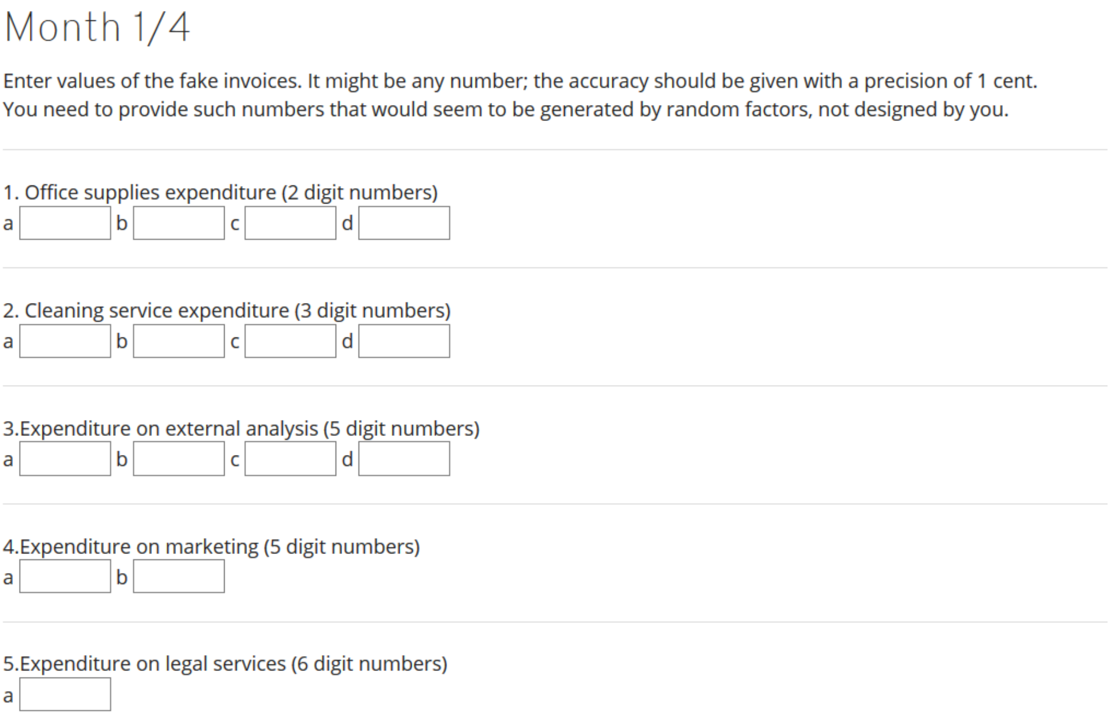
Your numbers would be analyzed using the algorithm for finding out the cases of tax evasion used by the tax officers. Your main aim is not to be caught. You will be graded 2 $ FBEE for participation in the experiment and 3 $ FBEE, if you manage to deceive on the tax officers.

I will publish the experimental results on the site (as an animation). Enter a unique nickname, thanks to it i) you can recognize your result i) you stay anonymous.
Nickname

Source: Own experiment programmed in labsee.com

The experiment is quite short and can be conducted as a first part of the lecture or as a prior homework. The students have to fill the gaps with the numbers on all the four screens representing the hypothetical monthly accounts (see Fig.3). To make the task more difficult, the students must enter the numbers with a pre-defined number of digits.

Figure 3: The Benford’s Law Experiment design



Source: Own experiment programmed in labsee.com

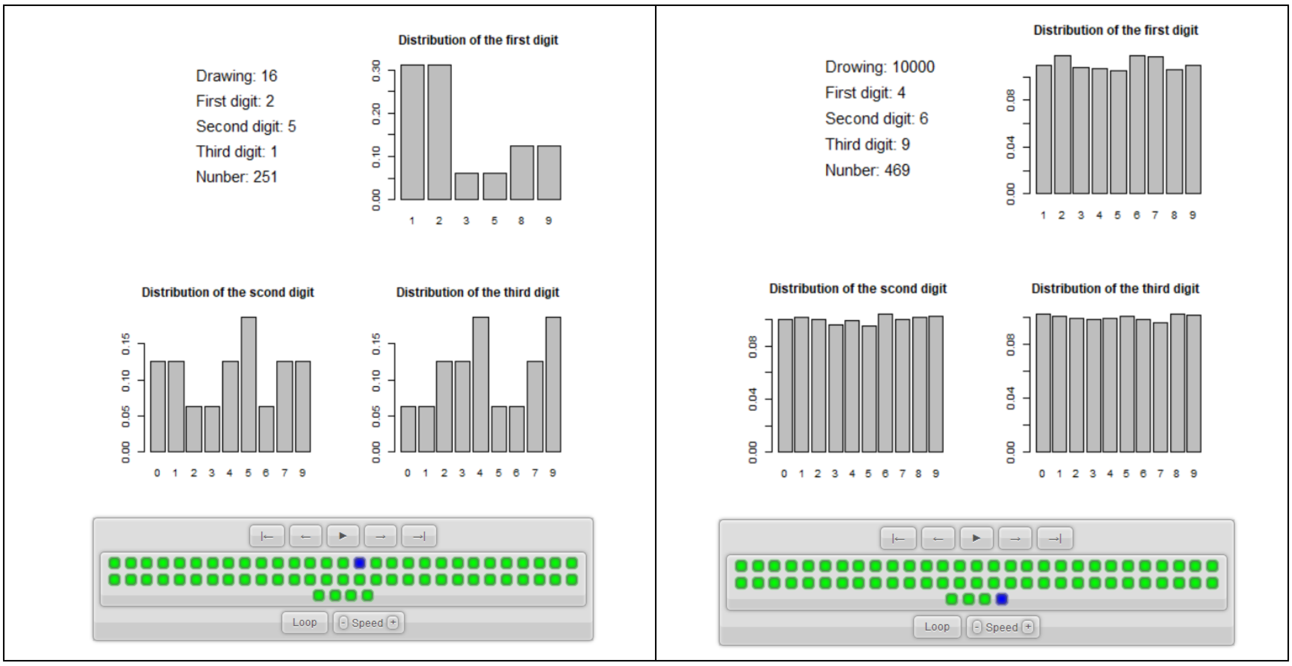
The experiment was conducted using the labsee.com platform[3] for the on-line experiments; however, the experiment is asynchronous and one can use any other online platform (the technical issues one can find in the supplementary on-line resources).

Introducing the BL by the simulation and the analysis of the real data

To introduce the BL, the lecturer presents the Monte Carlo simulation as an illustration of a simple thought experiment, which shows the misleading perception of a probability based on the prior student’s experience about the distribution of the values around the mean. In the first part of the experiment, they should imagine one bag containing nine ping-pong balls, labeled from 1 to 9, and the other two bags containing ten ping-pong balls labeled from 0 to 9. If the participant is asked to close his eyes and pull out a random ball from the first bag, what would be the probability of pulling out a ball labeled as 7? According to theory and common sense, the probability should be 1/9. The same would be applicable to any other number. For the second and third bag, the probability of pulling out a ball with a random number should be 1/10. The MC simulation *ball_drawings.html*[4] presents the visualization of the sequential drawings, which mimics the process of the random choosing of 60 samples out of three digits numbers (see Fig.4).

[3] Labsee.com is a platform designed for conducting the large-scale online economic experiments.
[4] You can find the R-project to generate the presented visualizations on the online annex or you can use the ready visualizations which is included there.

Figure 4: Simulation of drawing the random numbers

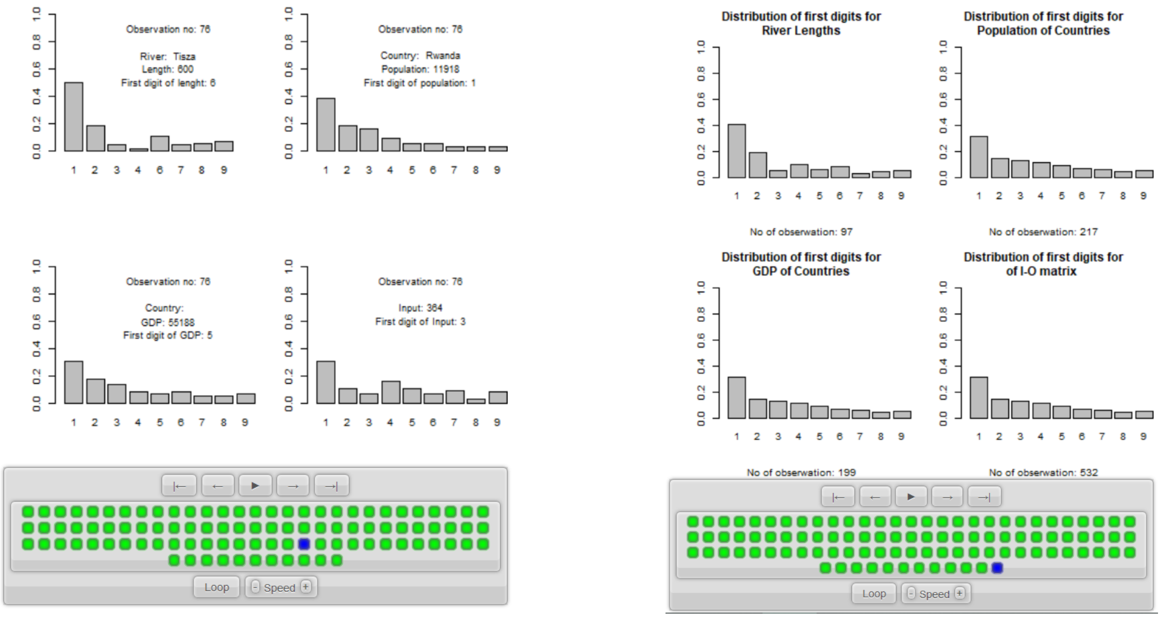


Source: Own analysis in R in *animation* package (Xie, 2013)

The main observation that should be stressed based on the simulation is that for a small number of repetitions, it is difficult to find any patterns in the generated data. However, as the number of drawings grows, the probability is consistent with the expected uniform distribution (the last screen shows the results of 10 000 drawings) .

After this simulation, it is the right the time to introduce the BL using the suspense technique. The teacher should ask students whether they did use the kind of reasoning presented in Monte Carlo simulation. The majority of students will probably confirm this. Then the teacher should follow the narrative structure and create the tension by announcing that if it was their strategy, then they are caught on cheating on the invoice experiment, because the distribution of the digits is significantly different. The teacher can present the visualizations of the distributions of the first digit in the empirical datasets from the different subjects to show that the real-life data follow the BL. The visualization *empirical_data.html* includes four data samples. The first three of them are the real data describing: i) lengths of 98 longest rivers in the world; ii) the population of the majority of countries; iii) the GDP of the majority of countries. The fourth data set is the Input-Output (I-O) matrix used in the used in estimation of DGSE model (Kiuila, 2018). The visualization shows (see Fig.5) that even for a relatively small number of observations, it is easy to see the common pattern of the distribution of first digits (Fig. 5a). The first digits are definitely not uniformly distributed. For any digit, the frequency of at the first position is inversely related to its rank: for instance, the digit 1 is much more frequent than the other digits. For the full sets of the observations (Fig. 5b), the distribution of all the data look nearly identical and follows the BL.

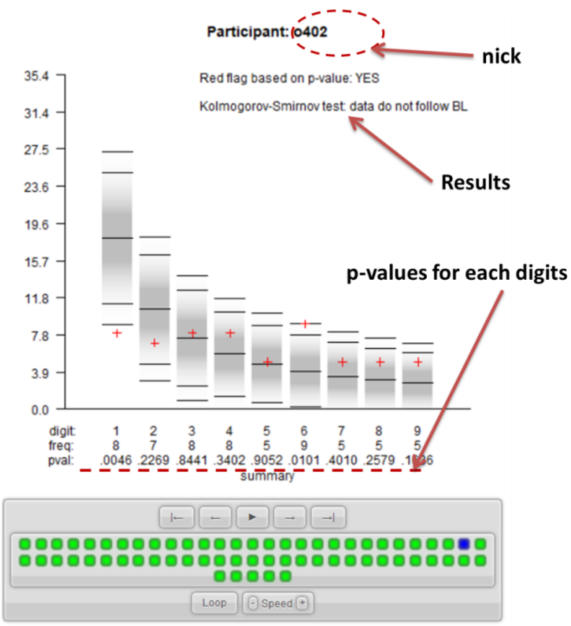
Figure 5: The frequencies of the first numbers of the analysed datasets (rivers, populations, GDP, I-O matrix): a) sample 1 to N ; b) full dataset



Source: own analysis in R in *animation package* (Xie, 2013).

After the introduction of the main idea of the BL, the teacher should present the applications of it in detecting the tax frauds (Nigrini, 2012; Simkin, 2010) and show the summary visualization of the results of the antifraud procedures applied to the tax experimental data. The analysis was performed using the R-CRAN package *BenfordTests* (Joenssen, 2015). The first procedure is a "*red flag indicator*" based on a digit's p-value for each of the digits. If any p-value shows the inconsistency with the BL (i.e. if p-value > 0.1) the flag is set. The second is the Kolmogorov-Smirnov goodness-of-fit test. With the use of the automatizing R codes, the visualization is ready immediately after finishing the experiment (see supplementary on-line materials).

Figure 6: The visualisation example from the application



Source: Own application based on R and BenfordTests (Joenssen, 2015), available in on-line materials.

An important part of the visualization is to place the nickname of the participants on the screen. The students can see their results of the antifraud procedures publicly, but they are still anonymous. In psychology, the experience can be personally transformative, if it changes one's point of view (Paul, 2013). Being caught on cheating can be this kind of experience, even if it is only a game. If the future choice of cheating or not is based on a perception of a probability of not being caught, then the obtained results leave no illusions. Even simple statistics procedures can indicate the fraud extremely effectively. The experiment was conducted twice (with groups of 10 and 60 people), and all the students were flagged.

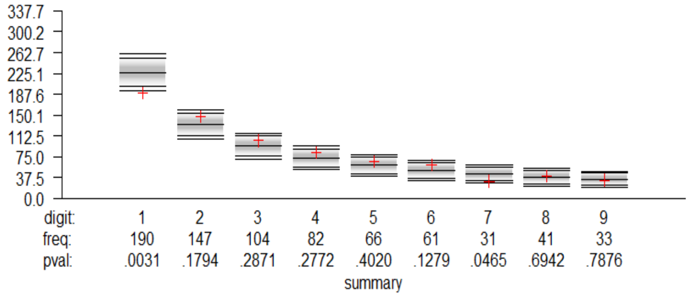
The simplicity of the test can is a good motivation for students to conduct an investigation by themselves. The students dispose with the ready-to-use R codes, which means that they are able to conduct the test with no prior experience in coding.

```
###### Check, installation and loading of required packages #######
requiredPackages = c("readr", "BenfordTests") # list of required packages
for(i in requiredPackages){ if(!require(i,character.only = TRUE)) install.packages(i)
library(i,character.only = TRUE) }

# loading data set - with one variable / you can change data set and conduct tests without coding
simple_data <- as.data.frame(read_csv("simple_data.csv"))

# tests - do not change
ks.benftest(simple_data[,1], digits = 1, pvalmethod = "simulate", pvalsims = 100000)
signifd.analysis(simple_data[,1],graphical_analysis=TRUE,alphas=50,freq=TRUE,tick_col="red",
ci_col="gray", ci_lines=c(.01,.05))
```

The rest of the attached codes are more complicated, mainly because of the publication purposes, but even students and teachers with no or little coding experience are able to use and modify them. Codes are free and available for anyone.

There are numerous sets of the economic data, which were not tested on a possibility of the mistakes, errors, tax frauds, or even the scientific frauds. In the international trade, the importers usually avoid reporting the large volumes, as this may imply the higher tariff due. From this perspective, the international trade statistics may be an interesting test area for applying the BL in order to identify the fraud.

Figure 7. An example of the empirical distribution of the first digit in the volumes of export between EU Member States for the year 2015.



K-S Test for Benford Distribution

data: simple_data[, 1]
D = 1.3567, p-value = 0.01506

Source: Own elaboration based on IMF Direction of Trade statistics and R package BenfordTests (Joenssen, 2015), available in on-line materials.

The empirical distributions of the first digits are not consistent with the BL distribution for some years (see Fig. 7 for 2015 and supplementary on-line materials). For such a big sample, the presented deviation from the BL can be a good motivation for the further detailed investigation.

For more advanced students in quantitative methods, it is worth to present two papers which in a comprehensive way present: i) one of the simplest explanations of the BL (Fewster, 2009.); ii) why the presented explanations of the BL still are not satisfying (Berger & Hill, 2011). The quotation from the second paper is a good way to present the main problem with the BL – it is still a mystery.

> *Although some experts may still feel that "like the birthday paradox, there is a simple and standard explanation" for BL [AP2 , p.6] and that this explanation "occurs quickly to those with appropriate mathematical background", there does not appear to be a simple derivation of BL that both offers a "correct explanation" [AP2 , p.7] and satisfies Speed's goal to provide insight. A broad and often ill-understood phenomenon need not always be reduced to a few theorems. Although many facets of BL now rest on solid ground, there is currently no unified approach that simultaneously explains its appearance in dynamical systems, number theory, statistics, and real-world data. In that sense, most experts seem to agree with [Few] that the ubiquity of BL, especially in real-life data, remains mysterious* (Berger & Hill, 2011, p. 91).

Playing with the BL accidentally creates the mystery and a crime puzzles atmosphere. In one group the influence of this framework was so intense that students spontaneously created the idea of a contest in which they divided themselves into two teams, playing the roles of "creative accountants" and "tax authority". The first team's goal was to generate such datasets, which would not be detectable by the second group. After some modifications, the contest seems to be a very promising tool to inspire students to do the additional effort during quantitative methods course.

The mystery of the BL touch the Authors too. How to explain the result of a one simple MC experiment? The experiment can be conduct in N-steps.

Step 1: From the sample of the natural numbers ranked from 1 to 999, a number p_1 is randomly picked using the random numbers generator with the uniform distribution.

Step 2: From the sample of the natural numbers ranked from 1 to p_1, a number p_2 is drew using the same technique as in a step one.

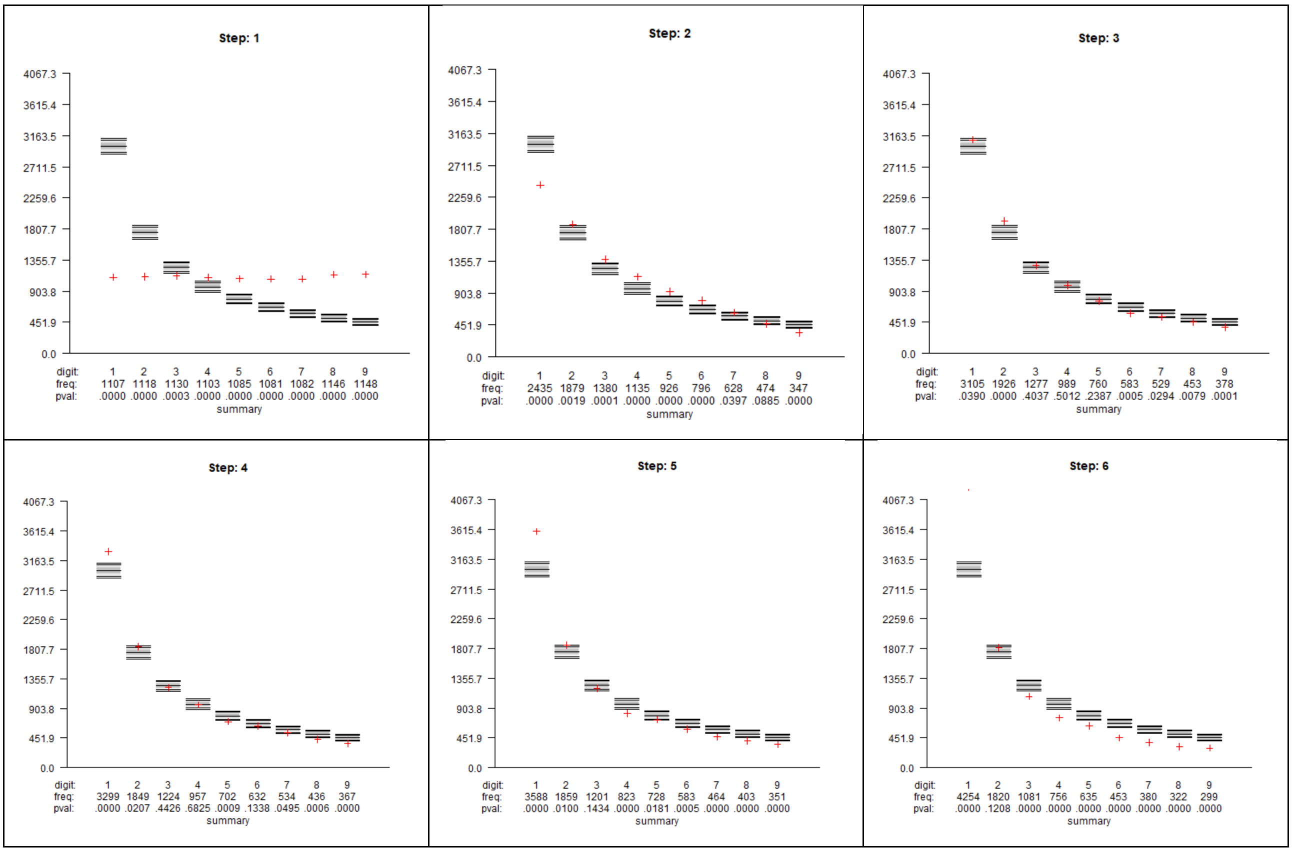
Step 3: From the sample of the natural numbers ranked from 1 to p_2, a number p_3 is drew using the same technique as in step one.

…..

Step N: From the sample of the natural numbers ranked from 1 to p_n, a number p_{n+1} is drew using the same technique as in step one.

The entire procedure should be replicated 10 000 times, which should result in the N vectors containing all the randomly picked numbers p for each step. For each such vector, the first-digit law is tested. The results of the step 1 are similar to the ball drawing simulation with uniform distribution. More steps caused that the number 1 is more frequent (see Fig. 8) and the distribution is approaching the Benford's distribution. But there is the puzzle is: why there are only 3 steps to generate the numbers which follow the BL? For more than 3 steps the simulated distribution moves away from the theoretical distribution. Digit 1 is much more frequent.

Figure 8. The visualization of the first six steps of Monte-Carlo simulation.



Source: Own application based on R and BenfordTests (Joenssen, 2015), available in on-line materials.

The experiment and the prepared MC simulations can be used in the wide range of the academic courses, such as accounting, applied microeconomics, and quantitative methods. The scheme of presenting material depends on the subject of study. The presented at the paper schema is adjusted to an applied microeconomic course for the data science students, but the materials can be used as Lego bricks and easily adjusted to one needs.

Conclusions

This paper presents a simple in-class experiment and the Monte Carlo simulations. Its goals are twofold. First, it is to familiarize the students with the existence and the properties of the Benford law, which is one of the "*gems of statistical folklore*" (Berger & Hill, 2011). Second, it is to achieve the important teaching goals. One of the main teaching goals of this experiment was to train the students' skills of the realistic perception of the probability distributions, which is currently not the part of microeconomics courses. The experiment, therefore, attempted to show that the simplified perception of the probabilities may result in a fail during the real-life data analysis. The additional goal was to train the students' skills of the critical assessment of the data, as a common use of the unreliable or fake data is one of the problems of the social sciences. The demonstration how easily the fraud may be detected, is expected to diminish the motivation of students to cheat in their courses, or, perhaps, in the further professional career. Finally, the goal of the experiment was to familiarize the students with the basic IT tools, necessary for the simple data analysis.

Authors will continue to create new materials concerning the Bendford's Law and generally probability and risk perception. We will be happy to share new experiments and simulations at our site.

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