# Monetary-Fiscal Interactions and Household Heterogeneity: an Analytical Characterization Paweł Kopiec, Narodowy Bank Polski 

## Introduction

## What I do?

- I develop an analytical formula for the government spending multiplier in economy populated with heterogeneous agents
- In my analysis I use the Bewley-Huggett-Aiyagari model extended to capture product market frictions
- The derived formula clearly shows the character of monetary-fiscal interactions
when households are heterogeneous
- Calibrated model is used to estimate the multiplier and its components under 3 monetary-fiscal scenarios:
- tax-financed stimulus
-debt-financed stimulus
- liquidity trap


## Frictional product market:

- A decentralized, search-and-matching product market is crucial for solving the model with paper and pencil
- Non-Walrasian market for goods in the literature: Diamond (1982), Michaillat and Saez (2015), Kaplan and Menzio (2016), Bai, Rios-Rull and Storesletten (2018)


## Technical contribution:

I relax 3 restrictive assumptions made in the literature to derive closed-form expressions in heterogeneous agent models:

1. Extreme illiquidity: used by Werning (2015), Ravn and Sterk (2016), McKay and Reis (2016)
2. Constant real interest rates: used by Auclert, Rognlie and Straub (2018), Patterson (2018)
3. Partial equilibrium: used by Auclert (2018)

## Theoretical analysis: Bewley-Huggett-Aiyagari model with frictional product market

## Self-employed households:

$$
\begin{gathered}
V(z, b)=\max _{c, v, b^{\prime}}\left\{\tilde{u}(c, v)+\beta \mathbb{E}_{z^{\prime} \mid z} V\left(z^{\prime}, b^{\prime}\right)\right\} \\
\text { subject to: } \\
c+\tau(z) \cdot \Theta+\frac{b^{\prime}}{1+i}=\frac{b}{\Pi}+z \cdot f \\
c=q \cdot v \\
b^{\prime} \geq-\xi
\end{gathered}
$$

## Fiscal and monetary policy:

$$
\begin{gathered}
\Theta+\frac{\bar{B}^{\prime}}{1+i}=\frac{\bar{B}}{\Pi}+G \\
G=q \cdot v_{G} \\
i=\max \left\{\bar{i}+\phi_{Y} \cdot\left(\frac{Y-\bar{Y}}{\bar{Y}}\right)+\phi_{\Pi} \cdot(\Pi-\bar{\Pi}), 0\right\}
\end{gathered}
$$

Matching process and price-setting:

$$
\begin{gathered}
x \equiv \frac{\int_{B \times Z} v(b, z) d \mu(b, z)+v_{G}}{\int_{B \times Z} z d \mu(b, z)} \\
f(x)=M(x, 1), q(x)=M\left(1, \frac{1}{x}\right) \\
\Pi=\Pi(x), \Pi^{\prime}(x)>0
\end{gathered}
$$

## Resource constraint:

$$
\int_{B \times Z} c(b, z) d \mu(b, z)+G=\underbrace{f(x) \cdot \int_{B \times Z} z d \mu(b, z)}_{\equiv Y(x)}
$$

## Additional notation:

$\lambda \equiv \frac{d \bar{B}_{t+1}}{d G_{t}}$ - proportion of $d G_{t}$ financed with public debt
$\alpha \equiv \frac{d \Gamma / d x}{d Y / d x}$ - a demand-driven comovement between prices and output
$\Omega \equiv \phi_{\Pi} \cdot \alpha+\phi_{Y}-$ strength of the monetary policy reaction
$\Lambda$ - fiscal rule that determines the way in which stimulus is financed

## Multiplier:

An unexpected rise in fiscal purchases arrives in period $t$ and is followed by a perfect-foresight transition path. The multiplier's formula is:

$$
\frac{d Y_{t}}{d G_{t}}=\frac{1+\frac{\partial C_{t}}{\partial G_{t}}}{1-\frac{\partial C_{1}}{\partial x_{t}} \cdot \frac{1}{f^{\prime}\left(x_{t}\right)}}
$$

where:

$$
\frac{\partial C_{t}}{\partial G_{t}} \equiv-\underbrace{\left(1-\frac{\lambda}{1+i}\right) \cdot \mathbb{E}_{\mu}(M P C \cdot \tau)}_{\text {Taxation channel }(-)}+\underbrace{\beta \cdot(1+i) \cdot \mathbb{E}_{\mu}\left(M P S \cdot \frac{1}{u_{c c}(c)} \cdot \mathbb{V}_{b G}^{\Lambda}\right)}_{\text {Precautionary motives channel }(-/+)}
$$

and:

$$
\begin{aligned}
& \frac{\partial C_{t}}{\partial x_{t}} \cdot \frac{1}{f^{\prime}\left(x_{t}\right)} \equiv-\underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}(\text { MPS } \cdot c)}_{\text {Intertemporal substitution channel }(-)}+\underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}(\text { MPC } \cdot \text { URE })}_{\text {Unhedged interestexposure channel }(-/+)} \\
& \quad+\underbrace{\mathbb{E}_{\mu}(M P C \cdot z)}_{\text {Income channel }(+)}-\underbrace{\left(\frac{\Omega}{(1+i)^{2}}-\alpha\right) \cdot \bar{B} \cdot \mathbb{E}_{\mu}(\text { MPC } \cdot \tau)}_{\text {Debt service cost channel }(-/+)}-\underbrace{\alpha \cdot \mathbb{E}_{\mu}(\text { MPC } \cdot b)}_{\text {Fisher channel }(-/+)}
\end{aligned}
$$

## Empirical analysis: a calibrated model is used to decompose the multiplier under 3 alternative scenarios

Empirical counterparts of model's cross-product terms (SHIW survey, Italy):


Key calibration target - MPC over cash-in-hand deciles:


## Three scenarios:

1. Benchmark: unconstrained monetary policy and tax-financed stimulus
2. Liquidity trap: monetary policy constrained by the ZLB and tax-financed stimulus
3. Debt-financed stimulus: accompanied by unconstrained monetary policy

## Simulation results:

| Channel $\backslash$ Scenario | Benchmark | Liquidity trap | Debt-financed |
| :--- | :---: | :---: | :---: |
| Taxation | -0.57 | -0.57 | 0 |
| Precautionary motives | -0.02 | -0.02 | -0.28 |
| Intertemporal substitution | -0.05 | 0 | -0.05 |
| Interest rate exposure | 0.16 | 0 | 0.16 |
| Income | 0.58 | 0.58 | 0.58 |
| Debt service costs | -0.10 | 0.06 | -0.10 |
| Fisher | -0.06 | -0.06 | -0.06 |
| MUTLIPLIER | 0.88 | 0.97 | 1.55 |

