

Preventing Self-fulfilling Debt Crises*

Michal Szkup[†]
University of British Columbia

February 15, 2015

Abstract

This paper develops a model of self-fulfilling debt crises and uses it to study the effectiveness of various government policies in preventing such crises. In the model, a crisis is a result of the interaction between bad fundamentals and self-fulfilling expectations of domestic firms and lenders. I solve the model using the global games approach and analyze policy proposals directed at preventing debt crises, such as, an increase in taxes, spending cuts and fiscal stimulus. I explain the costs and benefits associated with each policy, provide conditions under which these policies decrease or increase probability of default, and investigate their welfare implications. I find that tax increase or spending cuts tend to decrease the likelihood of crisis but may result in lower welfare. On the other hand, a well-timed fiscal stimulus can improve welfare, but it tends to increase the probability of a crisis. The above conclusions depend crucially on the timing and credibility of the government's policies, as well as on the initial state of the economy.

Key words: debt crisis, self-fulfilling expectations, credibility, austerity, fiscal stimulus, global games

JEL codes: C72, D82, D84, E44, E62, F34

*I would like to thank Douglas Gale, Tim Kehoe and Tom Sargent for their advice and invaluable comments and suggestions. I also thank Anmol Bhandari, Jaroslav Borovicka, Jess Benhabib, Harold Cole, Patrick Kehoe, David Kohn, John Leahy, Enrique Mendoza and Cecilia Parlatore Siritto for their insightful comments and suggestions.

[†]Email: michal.szakup@ubc.ca

1 Introduction

Sovereign debt crises are a recurrent phenomenon. After the turbulent 1980s and a series of defaults in the late 1990s and early 2000s, sovereign defaults once again became a hotly debated topic. Indeed, one of the most important issues troubling the global economy for the last several years has been the sovereign debt crisis in Europe.

One of the leading views on the European debt crisis is that it is the result of an interplay between poor economic fundamentals and self-fulfilling expectations.¹ According to that view, the governments in the Eurozone defaulted, or suffered from high interest rates on their debt, in part because of low confidence among market participants. This raised a number of questions: Is it possible for the government to implement policies that help to restore confidence and, hence, avert a crisis? If it is possible, then which policies are the most effective in achieving this? Should the government introduce austerity measures or, on the contrary, engage in fiscal stimulus to lift the economy out of the recession? Are these policies credible? And, finally, is it important to communicate policies to the market participants in advance?

The above questions are hotly debated among both economists and politicians. While most economists and politicians believe that the government has access to policy tools that can prevent a crisis, there is considerable disagreement regarding which policies are the most effective. A popular view, which initially won support in policy circles, is that the best way to avert a crisis is to introduce austerity measures such as tax hikes or spending cuts. A competing view, one that has gained more support recently, is that austerity is harmful. According to that view, in order to avoid a crisis, the government should do the opposite and engage in fiscal stimulus, even at the expense of higher budget deficits.^{2,3}

Despite the importance of the above questions (and lack of consensus regarding the answers to them), there exists little theoretical work that analyzes and compares the effectiveness of these policies within a model of sovereign default. The goal of this paper is to fill this gap in the literature by answering the above questions. In particular, I develop a two-period model of sovereign debt crisis in which self-fulfilling crises can arise. I then use this framework to analyze both analytically and quantitatively the effectiveness of various policies in preventing debt crisis. Finally, I investigate how these policies' effectiveness depends on their timing and credibility.

In the model, a crisis arises as a result of an interplay between poor fundamentals, lenders' pessimistic expectations and households' pessimistic expectations. If lenders expect a debt crisis, the government finds itself unable to roll over its maturing debt. If the amount of debt due today is high, the cost of repaying the debt in terms of forgone government consumption is also high, and the government chooses to default. On the other hand, if households expect a debt crisis, they reduce their investment because the return on capital is lower in default. This negatively affects future output and, thus, future tax revenues. The drop in the present value of tax revenues increases the government's relative debt burden, which makes defaulting

¹See Conesa and Kehoe (2012), Corsetti and Dedola (2011), De Grauwe and Ji (2013), or Krugman (2011).

²These questions have been discussed extensively, for example, on VoxEU.org - a policy portal set up by the Centre for Economic Policy Research. See Corsetti (2012) for a summary of this discussion.

³This recommendation is based on recent empirical and theoretical findings suggesting that the fiscal multiplier is particularly large in recession when the interest rate is at the zero lower bound. See Auerbach and Gorodnichenko (2011), Christiano, Eichenbaum, and Rebelo (2011) or Ilzetzki, Mendoza, and Vegh (2013) for the recent contributions to this literature.

more attractive. Therefore, if the fundamentals are already weak, a crisis can be caused by a shift in households' or lenders' expectations.

Models of self-fulfilling crises have a long tradition in the literature on sovereign default, beginning with Sachs (1984) and Calvo (1988).⁴ Typically, however, these models feature multiple equilibria and, thus, are open to a number of criticisms (see Morris and Shin (2000)). Most importantly, it is hard to conduct a policy analysis in such an environment. This difficulty stems from the fact that the outcome of the model is either undetermined or determined by a “sunspot variable” that does not have economic meaning and is determined outside the model. This leads to another problem: the difficulty of conducting comparative statics analysis within such a framework. In order to deal with the issue of indeterminacy of equilibrium, I use the global games technique. In particular, I assume that households and lenders do not know the average productivity in the economy and, instead, only observe noisy signals. I show that this small departure from common knowledge is enough to restore the uniqueness of equilibrium within the class of monotone equilibria.

Applying global games within a standard sovereign default framework is a challenging task. The difficulty stems from three facts. First, the equilibrium has to be calculated in an environment where most of the payoff-relevant variables are endogenous. Second, the global game is played among three different groups of agents (households, lenders and the government), each with different preferences and objectives. Finally, the payoffs of the lenders do not satisfy global strategic complementarities but only a weak single-crossing property, a much weaker condition. This last feature of the model requires extending the result of global games to such environments.⁵

The uniqueness of equilibrium allows me to study the effects of various policies on the equilibrium probability of default. While the model allows me to analyze a wide range of policies, I focus on three policy measures: (1) changes in taxes, (2) fiscal stimulus, and (3) spending cuts (modeled as a commitment by the government to use a fraction of its revenues to repay its maturing debt). I find that a change in the probability of default implied by each policy is equal to the “direct effect” - an initial effect of the policy change on the government incentive to default holding households' and lenders' beliefs constant - times the “multiplier effect” - a change in the government's default decision implied by the adjustment in households' and lenders' beliefs. Moreover, I show that the “multiplier effect” is always positive and independent of the considered policy measure. Using this insight, and under some additional assumptions, I find that spending cuts always decrease the probability of default; that increases in tax rates decrease the probability of default if either the initial tax rate or capital share of output is low (or both); and that fiscal stimulus increases the probability of default unless the marginal product of capital in the economy is initially very high.

The complexity of the model and lack of a closed-form solution prevents me from analytically determining the welfare implications of the above policies. Therefore, at this point, I turn to numerical simulations. I set the parameters of the model to standard values commonly used

⁴Other early papers that study self-fulfilling debt crises include Alesina, Prati, and Tabellini (1990), Giavazzi and Pagano (1990) or Detragiache (1996).

⁵Morris and Shin (2003) were the first to point that global games that satisfy only a strict single-crossing condition generically have a unique equilibrium in monotone strategies. I extend their result to environments with a weak single crossing condition and show that their result holds in a more complex environment of my model.

in the quantitative models of sovereign default. Interestingly, I find that while fiscal stimulus increases the probability of default, it also leads to a substantial increase in welfare. This is because expanding capital increases consumption in both periods and in both the repayment and default states. On the other hand, whether an increase in the tax rate leads to an increase or decrease in welfare depends on the initial level of taxes in the economy. If taxes are low, then an increase in taxes leads to welfare improvement; if they are already high, a further increase reduces welfare.

In the final part of the paper, I analyze what happens if the government policy change is unexpected or if the government cannot commit to implement the announced policy. I find that both assumptions alter the above conclusions significantly. When policy adjustment is unexpected, then a “multiplier effect” is absent and each policy leads to smaller change in the equilibrium probability of default. Moreover, fiscal stimulus leads to a much smaller increase in welfare, while spending cuts turn out to be welfare-reducing. In contrast, an increase in tax always leads to improved welfare. I also find that, lacking the ability to commit, the government will never implement spending cuts and will implement a tax increase only if the initial tax rate in the economy is low enough.

The model used in this paper builds on the framework developed in an influential paper by Cole and Kehoe (2000).⁶ The main difference between my model and theirs is in the information structure. While in Cole and Kehoe (2000) households and lenders have perfect knowledge of economic fundamentals, in my model, they face uncertainty regarding the current state of the economy. As explained above, introducing uncertainty results in a unique equilibrium that contrasts with the multiplicity of equilibria in their setup. This allows me to use comparative statics to analyze effects of various government policies. However, the added complexity of the incomplete-information structure requires me to focus on a two-period model, while they analyze an infinite-horizon problem. The models also differ slightly in terms of timing and the bond market structures.

The uniqueness of equilibrium is the result of applying the global games methodology as developed by Carlson and Damme (1993) and Morris and Shin (1998) (see Morris and Shin (2003) for an excellent survey of the global games literature). Corsetti, Guimaraes, and Roubini (2006) and Morris and Shin (2006) use global games to study the effectiveness of IMF assistance in preventing a self-fulfilling debt crisis and the moral hazard it creates.⁷ In contrast to these papers, I analyze a broader set of policies that include both austerity and fiscal stimulus and focus on the importance of their timing and their credibility. Moreover, to the best of my knowledge, the model developed here is the first attempt to bridge the gap between the highly stylized models used in the global game literature and the standard framework used in macroeconomics and international finance.

In global games, the unique outcome is typically determined by a realization of a random variable referred to as “underlying economic fundamentals”, but which otherwise does not have a clear interpretation within the model. This led some researchers to question the usefulness of global games models since they cannot be used to identify the relevant fundamentals that

⁶Cole and Kehoe (1996) use that model to quantitatively analyze the Mexican crisis. See, also, Conesa and Kehoe (2012), who use this framework to study the European governments’ debt dynamics during the recent debt crisis.

⁷See, also, Zwart (2007) for the signaling effects of IMF policy choices in the global game model of sovereign debt crisis.

determine the unique equilibrium outcome.⁸ The model developed in this paper builds on the setup commonly used in the quantitative literature on sovereign default and, therefore, identifies economic fundamentals that are relevant to the debt crisis. Thus, my approach is less vulnerable to the above critique. However, this comes at the price of extra complexity in the model and a lack of closed-form solutions that are typically obtained in the global games literature.

This paper is also related to the literature on sovereign debt in the spirit of Eaton and Gersovitz (1981), which is summarized well by Eaton and Fernandez (1995) and Panizza, Sturzenegger, and Zettelmeyer (2009). More recently, this line of research has focused on developing quantitative models of sovereign default that can account for the observed dynamics around the default episodes. (See Aguiara and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009) or Mendoza and Yue (2012) and references therein for more on quantitative models of sovereign default.)

Finally, a large body of work studies the recent recession in Europe, and the policy response to it, in the context of closed-economy models.⁹ Several papers use DSGE models to evaluate the effectiveness of various policies. For example, Eggertsson, Ferrero, and Raffo (2012) study the effects of structural reforms, while Corsetti, Kuester, Meier, and Müller (2013) investigate the effects of expansionary fiscal policy. Austerity is the main theme of a discussion volume, Corsetti (2012). In addition, Auerbach and Gorodnichenko (2011), Auerbach and Gorodnichenko (2012) and Ilzetzki, Mendoza, and Vegh (2013), among others, have done empirical studies on the effect of fiscal policy and fiscal multiplier. My work complements these papers by providing an exhaustive analysis of different policies including austerity and fiscal stimulus in an environment with self-fulfilling debt crisis.

2 Model

There are two periods, $t = 1, 2$, and three types of agents: a continuum of identical households, a continuum of identical lenders, and the government. The economy is characterized by the average productivity level A , which is distributed according to a normal distribution with mean A_{-1} and standard deviation σ - i.e. $A \sim N(A_{-1}, \sigma^2)$. Here, A_{-1} denotes the past average productivity level in the economy which all agents know. The current average level of productivity, A , is realized at the beginning of period 1 and is constant across time, but it is initially unobserved by the agents. A is revealed to everyone at the end of period 1.

2.1 Households

There is a continuum of identical households indexed by $i \in [0, 1]$. Households are risk averse and have preferences given by

$$\sum_{t=1,2} [\log(c_t) + \log(g_t)],$$

⁸See the “Discussion” section in Morris and Shin (2000).

⁹See De Grauwe (2010) for an early account of the crisis in Europe and proposed remedies.

where c_t is private consumption and g_t is government spending. Each household initially is endowed with the same amount of capital k_1 and has access to a production function:

$$y_t^i = \tilde{Z} e^{A_i} f(k_t^i),$$

where $f(k) = k^\alpha$, $0 < \alpha < 1$. Here A_i is a household-specific productivity level; \tilde{Z} is the aggregate productivity level that depends on the government's default decision; and f is a production function that takes as inputs capital and, implicitly, inelastically supplied labor. Households' proceeds from production are taxed at a rate $\tau > 0$ and are the only source of income for the household. Finally, capital is assumed to fully depreciate each period.

Households' idiosyncratic productivity is constant across time and given by

$$A_i = A + \varepsilon_i,$$

where ε_i is *i.i.d.* across households and is uniformly distributed on $[-\varepsilon, \varepsilon]$, $\varepsilon > 0$. Note that this implies that A is the average level of productivity in the economy, and knowing A is equivalent to knowing the aggregate output. Since A is initially unobserved, the average level of productivity is the key variable that agents would like to learn about.

Households receive their idiosyncratic productivity shocks A_i at the beginning of $t = 1$. The idiosyncratic productivity is each household's private information and is unobserved by other households, lenders or the government.¹⁰ Since households do not observe the average productivity level, they use realizations of their idiosyncratic productivity to update their beliefs regarding A .

After observing their respective productivity realizations, each household i makes its investment decision - i.e. it choose its capital stock, k_2^i , for period 2. They make these choices before \tilde{Z} is determined (and before the actual production takes place), and, thus, when making their investment decisions, households face uncertainty regarding their future income.¹¹ Households are committed to their investment decisions and they cannot adjust them later.

The production takes place at the end of period 1, after \tilde{Z} is determined, at which point the households invest the amount chosen earlier and consume the rest of their income, that is:

$$c_1^i = \tilde{Z} (1 - \tau) e^{A_i} f(k_1) - k_2^i$$

Households make no decisions in period 2. They simply use their capital to produce, and they consume all of their after-tax income: $c_2^i = \tilde{Z} (1 - \tau) e^{A_i} f(k_2^i)$.

2.2 The Government

The government is benevolent and maximizes utility of a median household.¹² In each period t , it provides households with public consumption goods, g_t , and finances its expenditure by

¹⁰I assume, however, that later, the government observes the output produced by the household so that it can collect appropriate taxes.

¹¹While non-standard, this assumption captures two realistic features of an investment process. First, investment takes time and often requires earlier planning. Second, investment decisions are made under uncertainty regarding future economic conditions (in this case, uncertainty about \tilde{Z}). See Section 2.5 for more discussion of this assumption.

¹²This simplifies the government's problem - otherwise the government's would need to take into account the distribution of private consumption among households. This simplification is not key to any results.

taxing households' income and (in period 1) by borrowing in the bond market. The government enters period 1 with a legacy debt, B_1 , which is due later in this period, and it initially does not observe the average level of productivity in the economy, A .

At the beginning of period 1, the government announces an interest rate $r > 0$ at which it is willing to borrow in the bond market. Once the households and lenders make their choices, the government observes A and decides, sequentially, how much to borrow, B_2 ; whether to default or not, d_1 ; and how much public good to provide to households, g_1 . In period 2, the government repays its debt B_2 , if it did not default on it earlier, and provides g_2 to households. The government can default only in period 1, in which case it defaults on all of its debt.¹³

Following the large literature on sovereign default, I assume that default is costly and associated with a drop in aggregate productivity (and, hence, in output) by a factor Z . In particular, when the government defaults \tilde{Z} takes value $Z < 1$ while $\tilde{Z} = 1$ otherwise.¹⁴

There is also an additional cost of default - if the government issues a positive amount of debt at $t = 1$ - i.e. $B_2 > 0$ - and then decides to default, it faces a further cost of default equal to ξB_2 , $0 < \xi \leq 1$. ξB_2 can be interpreted as a "litigation cost" associated with the legal battles between bondholders and the government following a default.¹⁵

Denote the government default decision by $d_1 \in \{0, 1\}$, where $d_1 = 0$ if the government decided to repay its debt and $d_1 = 1$ if there is a default. Then, the government faces the following sequence of budget constraints:¹⁶

$$\begin{aligned} g_1 &= \tau Z^{d_1} Y_1^R - (1 - d_1) \times B_1 + (1 - \xi)^{d_1} B_2 & \text{at } t = 1 \\ g_2 &= \tau Z^{d_1} Y_2^R - (1 - d_1) \times (1 + r) B_2 & \text{at } t = 2, \end{aligned}$$

where Y_t^R is the aggregate output at time t if the government repays the legacy debt, B_1 .

2.3 Lenders and the Bond Market

There is a continuum of identical, risk-neutral lenders, indexed by $j \in [0, 1]$, each with finite wealth $b > 0$. Lenders choose at $t = 1$ whether to participate in the bond market or invest in storage. The net return on storing funds is zero, while the return from participating in the bond market is endogenous and determined in equilibrium.

Lenders do not observe the realization of the average productivity, but, instead, each lender j observes a private signal x_j of A where

$$x_j = A + v_j, \quad v_j \sim N(0, \sigma_x^2)$$

¹³I allow for default only in period 1 because of an inherent asymmetry between the two periods in the model. Since the period 2 is the last period of the model, it is hard to support repayment as an equilibrium outcome in that period - compared to period 1, the government faces much smaller costs of default and lacks the ability to roll over part of its debt. It is possible to extend the model to allow for default in period 2 at the costs of an increase in complexity and additional assumptions regarding the model's parameters.

¹⁴One can think of this drop in output as resulting from a disruption in credit markets following a sovereign default. See Mendoza and Yue (2012) for microfoundations of this channel.

¹⁵Following a default creditors tend to fill a substantial number of lawsuits against a defaulting government. For example, in the case of default by Argentina in 2001, there were over 140 lawsuits filled abroad, including 15 class action lawsuits, in addition to a large number of lawsuits filled in Argentine courts (Panizza, Sturzenegger, and Zettelmeyer (2009)). I interpret ξ as the costs to the government associated with these legal battles.

¹⁶Without loss of the generality I assume that the interest rate on the legacy debt, B_1 , is zero.

with v_j being *i.i.d.* across lenders and independent of A .¹⁷

Only the government and lenders have access to the bond market. I assume that the government has all the market power in the bond market, and, therefore, the government sets an interest rate r at which it is willing to borrow new funds. Taking r as given, lenders decide whether to supply their funds to the bond market, determining the total funds available in the bond market, S . The government then chooses its new borrowing, B_2 , where $B_2 \in [0, S]$. After the government raises new funds, the bond market shuts down and lenders invest the funds not borrowed by the government in storage.

2.4 Timing

The timing of period 1 is summarized in Figure 1.

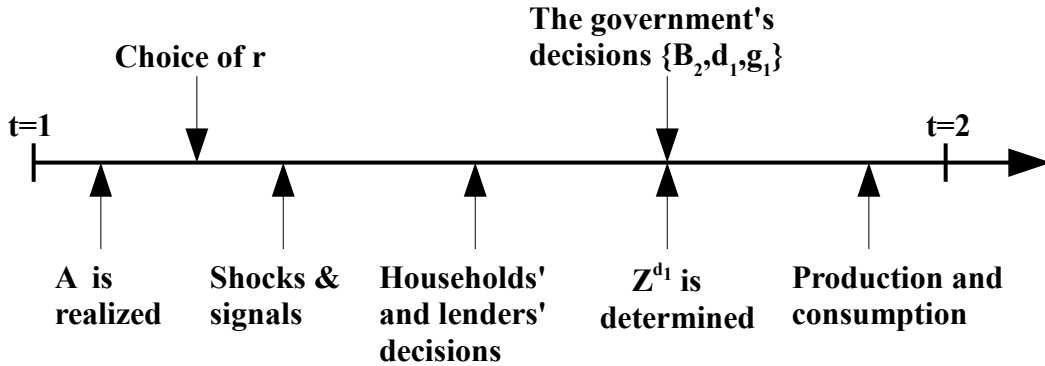


Figure 1: Timeline

At the beginning of period 1, nature draws the productivity level A , which is initially unobserved by the government, households and lenders. Then, based only on the information contained in the prior belief, the government sets an interest rate, r , at which it is willing to borrow from the lenders. Once r is announced, households receive their idiosyncratic productivity shocks and lenders observe private noisy signals of A . Given their productivity shocks, households choose how much they want to investment, while lenders, using their private signals, decide whether to supply their funds in the market. At this point, the government learns the true A , and based on lenders' and households' decisions and the realization of A , it decides sequentially how much to borrow today, B_2 , whether to default or not, d_1 , and how much of public good to provide to households, g_1 . Once the government borrows its desired amount, the bond market shuts down and remaining lenders' funds are invested in storage. Finally, at the end of the period, production, actual investment and consumption take place and the average productivity level is revealed to all the agents.

Period 2 is much simpler. At the beginning of the period production takes place. Then, the government collects the taxes and provides public consumption, g_2 . Finally, households consume their after-tax output.

¹⁷Moreover, v_j 's are independent of ε_i 's so that realization of x_j does not provide information regarding idiosyncratic households' productivity shocks, and vice versa.

2.5 Timing and Information Structure

The timing and the information structure of the model is chosen to satisfy four separate objectives: (1) to induce an information structure that is conducive to the existence of a unique equilibrium; (2) to make the government's new borrowing, default and spending decisions simple to solve; (3) to avoid any issue of learning or signaling in the model; and (4) to make households' expectations matter for the equilibrium decisions.

In order to achieve the first goal, following the large literature on global games, I assume that households and lenders do not observe the average productivity level but, rather, the private and noisy signals of A . This breaks common knowledge of fundamentals among agents which is key to the uniqueness result.

To achieve the second goal, I let the government observe A before making its new borrowing, default and spending decisions in period 1 - under complete information, the government's optimal decisions can be solved, to a large extent, in a closed-form.

Allowing the government to observe A creates a tension within the model. Since the government has an informational advantage it can influence households' and lenders' decisions by signaling its private information through its choices. In order to avoid this problem and achieve the third objective, I make two assumptions. First, I assume that the government sets the interest rate r at the beginning of period 1 based only on its prior belief, and, thus, a choice of r is uninformative about A . Second, I assume that households and lenders make their respective decisions simultaneously and before the government's choices of $\{d_1, g_1, B_2\}$. It follows that households and lenders make their choices based only on the information contained in the prior belief and their respective signals.¹⁸

Finally, I assume that households decide how much to invest (and are committed to their choices) before the government's default decision and before production takes place. This implies that households, when making their investment decisions, have to form beliefs regarding the government's default decision, opening up the possibility of a debt crisis caused by households' expectations.

3 Equilibrium Analysis

An equilibrium in the model is defined as follows:

Definition 1 *An equilibrium is a set of government policy functions $\{r, d_1, g_1, g_2, B_2\}$, a profile of households' consumption and investment choices $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{k}_2\}$, a profile of lenders' supply decisions β , and a supply function S such that:*

1. $\{r, d_1, g_1, g_2, B_2\}$ solves the government's problems at $t = 1, 2$, taking households' and lenders' decisions as given;
2. $\forall i, \{c_1^i, c_2^i, k_2^i\}$ solves household i 's problems at $t = 1, 2$, taking as given other agents' decisions;

¹⁸Dropping these two assumptions would lead to a complex signaling game between the government and other agents and, as shown by Angeletos, Hellwig, and Pavan (2006) and Angeletos and Pavan (forthcoming), could reintroduce multiplicity of equilibria into the model.

3. $\forall j, \beta^j$ solves lender j 's problem, taking as given other agents' decisions;

4. $S = \int_{i \in [0,1]} \beta^i di$.

The above definition of an equilibrium is standard and it requires that all the agents behave optimally in each subgame, taking as given the actions of others. It also requires that the supply of funds in the bond market is consistent with lenders' supply decisions.

The equilibrium can be computed by backward induction starting with period 2 and then moving to period 1. The key, and the most difficult step, is to solve simultaneously for households' investment and lenders' supply decisions. In what follows I will focus on equilibria in monotone strategies. This greatly simplifies the task of solving the model and makes the analysis more tractable.

3.1 Additional Assumptions

To simplify the analysis and ensure that the government problem is well-posed, I make the following assumptions.

Assumption 1 *The legacy debt is large enough compared to the initial stock of capital, $B_1 > \bar{B}(k_1)$.*

Assumption 1 ensures that, if the government decides to repay its "legacy debt", it will find it optimal to borrow a positive amount. Otherwise, a debt crisis can only be a result of households' pessimistic expectations or poor fundamentals; lenders stop playing any role in the model. Since the focus of this paper is to analyze how to prevent a self-fulfilling debt crisis, I rule out this possibility.

Assumption 2 *The wealth of lenders is bounded above by \bar{b} - i.e., $b < \bar{b}$, and the "litigation costs" are large - i.e., $\xi \rightarrow 1$.*

Assumption 2 is required to ensure that the government's incentive to default is decreasing as the supply of funds in the market increases, a property that substantially simplifies the analysis.¹⁹

Assumption 3 *The output cost of default is small enough so that $(1 - Z^A) > \alpha$.*

Finally, Assumption 3 implies that the government's optimal unconstrained borrowing, the amount it would like to borrow if it repays the debt, is monotone in A .

Given the above assumptions, I now analyze the equilibrium of the model. I compute the equilibrium using backward induction. Note that once the government makes its choices for $\{B_2, d_1, g_1\}$, no agent makes any decision and the equilibrium outcomes are determined. Therefore, I start the analysis by describing the government's new borrowing, default and spending decisions in period 1.

¹⁹To see why an upper bound on wealth is needed to ensure that this is, indeed, the case, an interested reader should consult Section B.5 of the Appendix.

3.2 Period t=1: The Government's Decisions

The government decides how much to borrow, whether or not to default and how much to spend to maximize the median household's utility, internalizing how each of these decisions affects consumption, aggregate productivity and future tax revenues. The government makes these decisions after observing households' investment decisions, the supply of funds in the market and the average level of productivity in the economy.²⁰

Let $V_1^R(A, \mathbf{k}_2, S)$ be the value of repaying when the average productivity is equal to A , the households' investment profile is \mathbf{k}_2 , and the supply of funds in the bond market is S . Then $V_1^R(A, \mathbf{k}_2, S)$ is given by:

$$\begin{aligned} V_1^R(A, \mathbf{k}_2, S) &= \max_{B_2 \in [0, S]} \sum_{t=1,2} \log(\bar{c}_t^R) + \log(g_t^R) \\ \text{st. } g_1^R &= \tau Y_1^R - B_1 + B_2 \\ g_2^R &= \tau Y_2^R - (1+r)B_2 \\ \bar{c}_1^R &= (1-\tau)e^A f(\bar{k}_1) - \bar{k}_2 \\ \bar{c}_2^R &= (1-\tau)e^A f(\bar{k}_2) \end{aligned}$$

When the government decides to repay its debt, it chooses its new borrowing B_2 to maximize the median household's utility subject to the available funds in the market, S , and its budget constraints. The government also takes the median household's consumption and investment choices $\{\bar{c}_1^R, \bar{c}_2^R, \bar{k}_2\}$ as given.

Since at this point, S is already determined, it is possible that the optimal (unconstrained) amount the government would like to borrow, denoted by B_2^u ,²¹ is larger than S ; in this case, the government is constrained in the bond market and borrows $B_2 = S$. On the other hand, if the supply of funds exceeds the government's unconstrained demand for funds, B_2^u , then the government sets $B_2 = B_2^u$. As shown below, the unconstrained demand for funds by the government, B_2^u , plays an important role in the lenders' problem.

Let $V_1^D(A, \mathbf{k}_2, S)$ be the value associated with defaulting - that is:

$$\begin{aligned} V_1^D(A, \mathbf{k}_2, S) &= \sum_{t=1,2} \log(\bar{c}_t^D) + \log(g_t^D) \\ \text{s.t. } g_1^D &= \tau(ZY_1^R) + (1-\xi)S \\ g_2^D &= \tau(ZY_2^R) \\ \bar{c}_1^D &= (1-\tau)Ze^A f(\bar{k}_1) - \bar{k}_2 \\ \bar{c}_2^D &= (1-\tau)Ze^A f(\bar{k}_2) \end{aligned}$$

If the government defaults, it borrows the maximal possible amount in the market - i.e., $B_2 = S$ - and then repudiates all of its debt, and both of these actions tend to increase

²⁰The government chooses $\{B_2, d_1, g_1\}$ sequentially, but since no other event takes place during that time, in equilibrium, this is equivalent to a situation where the government makes these choices simultaneously. Since solving the government's problem sequentially requires additional notation, for expositional clarity, I define the value of defaulting and repaying as if $\{B_2, d_1, g_1\}$ were chosen simultaneously.

²¹That is B_2^u is the optimal solution to the above maximization problem when $S \rightarrow +\infty$.

government spending in period 1. When $\xi \rightarrow 1$, this first effect vanishes and the main benefit of default is an increase in the g_1 due to repudiation of the “legacy debt” B_1 . However, default leads to a drop in aggregate productivity by factor Z , which constitutes the main cost of default. Note that in the limit as $\xi \rightarrow 1$, the value of default becomes independent of S . Thus, in what follows, I denote the value of defaulting by $V_1^D(A, \mathbf{k}_2)$.²²

When deciding whether or not to default, the government compares $V_1^R(A, \mathbf{k}_2, S)$ with $V_1^D(A, \mathbf{k}_2)$ and chooses to repay its debt if and only if the value associated with repaying is larger than the value associated with defaulting - i.e.:

$$\Delta V(A, \mathbf{k}_2, S) \equiv V_1^R(A, \mathbf{k}_2, S) - V_1^D(A, \mathbf{k}_2) \geq 0 \quad (1)$$

Equation (1) indicates that the government’s default decision depends crucially on the average productivity level A , lenders’ supply decisions $\beta = \{\beta_i\}_{i \in [0,1]}$ and household’ investment decisions $\mathbf{k}_2 = \{k_2\}_{i \in [0,1]}$. This, in turn, opens up the possibility that for some values of A the government is exposed to the self-fulfilling debt crisis.

3.2.1 Default Decisions and the Fragility Region

Now, consider the government’s default decision. For sufficiently low productivity levels, the government finds it optimal to default regardless of the households’ and lenders’ actions - when A is low, defaulting leads to an increase in government spending in period 1 since $B_1 > (1 - Z)\tau Y_1^R$. Moreover, if productivity is small enough, an increase in government spending outweighs the costs associated with a drop in private consumption due to default. On the other hand, when the average level of productivity is high, the government always finds it optimal to repay the debt. Intuitively, for high A , we have $B_1 < (1 - Z)\tau Y_1^R$ and, therefore, defaulting not only leads to a drop in private consumption, but also results in less government spending. Accordingly, for each interest rate r , there exist two thresholds, $\underline{A}(r)$ and $\bar{A}(r)$, such that the government always defaults if $A < \underline{A}(r)$ and never default if $A > \bar{A}(r)$.

For all $A \in [\underline{A}(r), \bar{A}(r)]$ the government’s default decision depends on the households’ and lenders’ choices. If households expect a default, they reduce their investment, leading to a drop in the government’s future tax revenues. The presence of default costs implies that the government’s revenues decrease faster in repayment than in default, which makes defaulting relatively more attractive. For the intermediate values of productivity, this effect is strong enough that the government finds itself in a position where it is optimal to default, even though if investment were high, it would choose to repay the debt.

Similarly, a shift in lenders’ expectations can lead to a debt crisis. If lenders expect default, they choose to invest all their funds in storage, which prevents the government from smoothing debt repayment across time. Now, in order to avoid default, the government has to repay a large fraction of its maturing debt today, which is very costly in terms of the forgone utility from the government spending. For $A \in [\underline{A}(r), \bar{A}(r)]$, this cost is large enough that the government finds it optimal to default.

²²From the definition of V_1^D we can see that if $\xi < 1$, then the value of defaulting is an increasing function of S . When ξ is small, the value of defaulting increases with S at a faster rate than value of repaying, and equilibrium in monotone strategies does not exist. The assumption that $\xi \rightarrow 1$ is needed to ensure the existence of equilibrium in monotone strategies while keeping default costly for the lenders.

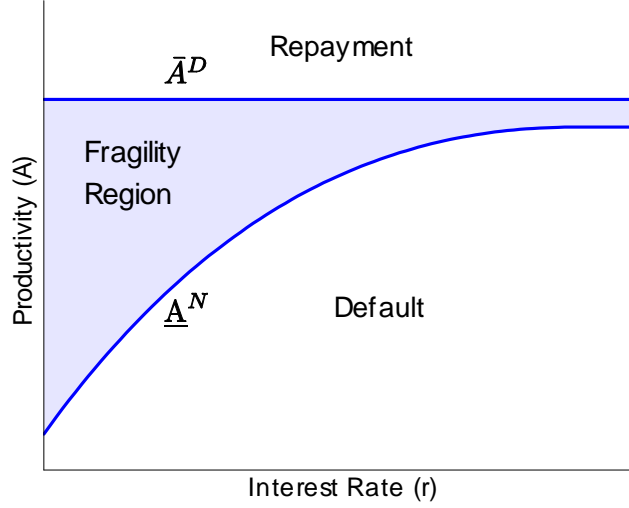


Figure 2: Fragility Region

Figure 2 depicts the fragility region $[\underline{A}(r), \bar{A}(r)]$. For all A in the fragility region, the government's default decision depends on households' and lenders' expectations. If we were free to choose equilibrium beliefs, as in the case of complete-information models, both default and no default could be supported as equilibrium outcomes. However, given the assumed information structure, beliefs are not free objects but, rather, are determined in equilibrium. As I show below, introducing uncertainty into the model (sometimes referred to as "the global game technique") allows me to pin down beliefs and prove the uniqueness of the equilibrium in monotone strategies.

3.2.2 Households' problem

Households make their investment choices simultaneously with lenders' supply decisions. Consider household i with an idiosyncratic productivity shock A_i that chooses how much to invest. Then, household's problem can be written as

$$\begin{aligned} \max_{k_2} E \left[\sum_{t=1,2} [\log(c_t) + \log(g_t)] \middle| A_i, \sigma \right] \\ \text{s.t. } c_1 = (1 - \tau) Z^{d_1(\sigma)} e^{A_i} f(k_1) - k_2 \\ c_2 = (1 - \tau) Z^{d_1(\sigma)} e^{A_i} f(k_2) \\ \sigma = \{k_2, \beta, r, d_1, g_1, g_2, B_2\}, \end{aligned}$$

where σ is the strategy profile of all players and the expectations are taken over the government default decisions, $d_1(\sigma)$. Household i chooses k_2 to maximize its utility subject to the budget constraint, taking σ as given.

The households' problem has two key features. First, investment choices are made before the government's default decision and before production takes place. As explained above, this opens the door to the self-fulfilling debt crisis fueled solely by households' expectations.

Second, households observe neither the idiosyncratic shocks of other households nor the average productivity level. That prevents households from coordinating their beliefs regarding future default. Even though, in equilibrium, each household has correct beliefs regarding other agents' strategies, due to assumed information structure, households are unable to predict their choices perfectly, which is the key to the uniqueness result.

Without imposing an additional structure, households' problem is difficult to solve. It turns out, however, that if the government follows a monotone default strategy (i.e., it defaults if and only if $A < A^*$), then households' optimal investment is easy to characterize.

Lemma 1 *Suppose that the government follows a monotone default strategy with threshold A^* . Then, household i 's optimal investment is given by*

$$k_2 = (1 - \tau) e^{A_i} f(k_1) \Lambda(A^*, A_i),$$

where $\Lambda(A^*, A_i)$ is decreasing in the default threshold and increasing in idiosyncratic productivity realization.²³

3.2.3 Lender's problem

Simultaneously with households' investment choices, lenders decide whether to supply their funds to the bond market or to invest their funds in storage. Lenders base their decisions on the prior belief about A and their private signals, x_i . Let $\mathcal{R}(\boldsymbol{\sigma})$ be the government repayment set for a fixed strategy profile $\boldsymbol{\sigma}$. Then, the expected payoff to lender j from supplying the funds to the bond market is given by

$$\int_{A \in \mathcal{R}(\boldsymbol{\sigma})} \left(1 + r \min \left\{ 1, \frac{B_2^u(A; \boldsymbol{\sigma})}{S(A; \boldsymbol{\beta})} \right\} \right) f(A|x_j) dA,$$

where $f(A|x_j)$ is lender's j posterior belief, $B_2^u(A; \boldsymbol{\sigma})$ is the unconstrained desired borrowing by the government; and $S(A; \boldsymbol{\beta})$ is the supply function implied by the lenders' supply strategy profile $\boldsymbol{\beta}$. Finally, $\min \{1, B_2^u(A; \boldsymbol{\sigma})/S(A; \boldsymbol{\beta})\}$ is the amount that lender i expects to lend to the government given that the average productivity level is A .²⁴

Lender j supplies the funds to the bond market if and only if the expected return from supplying the funds is higher than the return from investing in storage, or

$$\int_{A \in \mathcal{R}(\boldsymbol{\sigma})} \left(1 + r \min \left\{ 1, \frac{B_2^u(A; \boldsymbol{\sigma})}{S(A; \boldsymbol{\beta})} \right\} \right) f(A|x_j) dA \geq 1.$$

As in the case of households' problem, restricting attention to monotone strategies leads to a simple characterization of lenders' optimal behavior.

Lemma 2 *Suppose that the government defaults if and only if $A < A^*$ and restrict attention to monotone strategies for lenders. Then, an optimal strategy for each lender j is to supply*

²³See the Appendix for the exact definition of $\Lambda(A^*, A_i)$.

²⁴For all $A \notin \mathcal{R}(\boldsymbol{\sigma})$, the government borrows all available funds in the market and then defaults implying that in this case lender i earns nothing.

the funds to the bond market if and only if he receives a signal $x_j \geq x^*$. Moreover, x^* is the unique solution

$$\int_{A^*}^{\infty} \left(1 + r \min \left\{ 1, \frac{B_2^u(A; \boldsymbol{\sigma})}{S(A; \mathbf{x}^*)} \right\} \right) f(A|x^*) dA = 1,$$

where $S(A; \mathbf{x}^*)$ is the supply function when all lenders follow this strategy.

In other words, if we restrict attention to monotone strategies, then all lenders find it optimal to follow a monotone strategy with the same threshold x^* .

3.3 Equilibrium Default Threshold and Optimal Agents' Strategies

Above I characterized optimal behavior of each type of agents. This, in turn, allows me to prove the following proposition, which states that for any interest rate r there exists a unique equilibrium in monotone strategies.

Proposition 1 *Let $\varepsilon \rightarrow 0$. Then for all $\sigma_x < \bar{\sigma}_x$ and for each interest rate r the model has a unique equilibrium in monotone strategies such that:*

1. *The government defaults if and only if $A < A^*$;*
2. *Each lender j supplies the funds to the market if and only if $x_j \geq x^*$;*
3. *Households' investment rules, k_2 , are monotone in their signals.*

Proof. See Appendix B ■

The proof of the above result consists of two steps. First, I show that each agent's best response function, when faced with other agents following monotone strategies, is itself monotone. Then, I establish that there exists a unique strategy profile of monotone strategies such that no agent has incentives to deviate from his prescribed strategy. In light of Lemmas 1 and 2, to prove the first part, it is enough to show that given monotone households' and lenders' strategies, the government strategy is also monotone. The second part can then be transformed into a fixed-point problem in the space of monotone strategy profiles.

The proof of Proposition 1 builds on the insights and results of Athey (1996) and Athey (2001); Morris and Shin (2003) use a similar approach. The above result, however, is non-trivial for several reasons. First, the environment considered in this paper is very different compared to a typical environment considered in the global games literature, including that of Morris and Shin (2003), and a priori it is not clear that it can be reduced to a "global game". Second, difficulty comes from the fact that in the model, the global game is played by three different types of agents, each with different preferences and different choice sets. Finally, lenders' payoff function satisfies only a weak single-crossing condition rather than global strategic complementarities, as in typical global games, or a strict single-crossing condition, as

in Morris and Shin (2003).^{25,26}

Figure 3 depicts the boundaries of the “fragility region” (the dashed lines) and default threshold $A^*(r)$ when A_{-1} is low (the left panel) and when A_{-1} is high (the right panel). We see that, regardless of A_{-1} , the default threshold $A^*(r)$ is a non-monotone function of the interest rate r . To understand this, note that when the interest rate is equal to zero no lender will ever supply the funds. Faced with no funds in the bond market, the government finds it optimal to default for most productivity values in the “fragility region”. As r increases, the supply of funds increases since higher r compensates lenders for exposing themselves to default risk. At the same time, households’ investment rules shift upwards since they anticipate that the government will choose to repay the debt for a larger set of productivity levels. This decreases the government’s incentives to default and leads to a lower $A^*(r)$. A higher interest rate, however, increases the costs of rolling over the debt, discouraging the government from smoothing debt repayment over time. This tends to decrease the value of repaying debt to the government. For sufficiently high r , this negative effect dominates, implying that $A^*(r)$ becomes an increasing function of r .

Figure 3 also indicates that the past value of productivity has a strong effect on the shape of the default threshold. In particular, when the last-period average productivity A_{-1} is low, the default threshold is high and relatively insensitive to changes in the interest rate. The reason behind that is the following. Holding everything constant, as A_{-1} falls, lenders assign a higher probability to government default and, thus, decrease their supply of funds. This leads to lower liquidity in the bond market, which, in turn, results in an increase in $A^*(r)$. Moreover, given higher $A^*(r)$, lenders expect to earn return r with lower probability. Therefore, their decisions become less sensitive to changes in the interest rate.

It is important to stress that, while the default threshold is unique, the outcome of the model in the “fragility region” is driven fully by households’ and lenders’ expectations. For all productivity levels in the “fragility region”, both repayment and default could be supported as an equilibrium outcomes, if we had the freedom to choose lenders’ and households’ expectations. However, households’ and lenders’ expectations are not free objects. An incomplete-information structure transforms beliefs into equilibrium objects and requires them to be sequentially rational and consistent with agents’ strategy profiles. This imposes requirements on the beliefs that are not present in the complete-information game. The approach used in global games is, then, to use an appropriate information structure that will lead to a unique equilibrium outcome.

²⁵We say that a function $f : X \times T \rightarrow \mathbb{R}$ satisfies the weak single-crossing condition if for all $x', x'' \in X$ such that $x'' \geq x'$ and all $t'', t' \in T$ such that $t'' > t'$ we have $f(x'', t') - f(x', t') > 0$ implies that $f(x'', t'') - f(x', t'') \geq 0$. Function f satisfies the strict single-crossing property if $f(x'', t') - f(x', t') > 0$ implies $f(x'', t'') - f(x', t'') > 0$ (Milgrom and Shannon and Athey). While innocuous, such an extension is important since in many games such as market games or auctions only weak single crossing property is satisfied by the payoff functions (for more details, see Athey (1996)).

²⁶Extending global games results to an environment in which payoff functions satisfy only a version of a single-crossing condition, rather than global strategic complementarities, is not without cost. Similar, to Morris and Shin (2003) I am unable to prove uniqueness of equilibrium in general strategies. A potential approach to obtain uniqueness would be to follow Karp, Lee, and Mason (2007) and impose additional assumptions on the size of the noise in the prior and signals to weaken strategic substitutabilities present in lenders’ problem.

3.4 Optimal Choice of r

It remains to characterize the government's optimal choice of interest rate, r . The government chooses the interest rate based on the current and past fundamentals of the economy, $\{B_1, k_1, A_{-1}\}$. The government also knows its future policy functions $\{d_1, g_1, g_2, B_2\}$ and realizes that it can affect consumption, investment and supply of funds through its choice of interest rate.²⁷ To choose the optimal interest rate, the government solves the following problem:

$$W(A_{-1}, B_1, \mathbf{k}_1; \boldsymbol{\sigma}) = \max_r E \left[\sum_{t=1,2} \log(\bar{c}_t) + \log(g_t) \middle| A_{-1} \right]$$

s.t. policy functions $\{c_1, c_2, d_1, B_2, g_1, g_2\}$
lenders' and households' strategies $\{\boldsymbol{\beta}, \mathbf{k}_2\}$.

When choosing the interest rate, the government faces the following trade-off. On the one hand, a higher r decreases the default threshold by increasing the supply of funds in the bond market. A lower default threshold also encourages higher investment since households are less worried about default. On the other hand, high r increases the cost of borrowing at $t = 1$, making it more costly to roll over the maturing debt. The government weighs the positive effect of a lower default threshold against the increase in the borrowing costs.

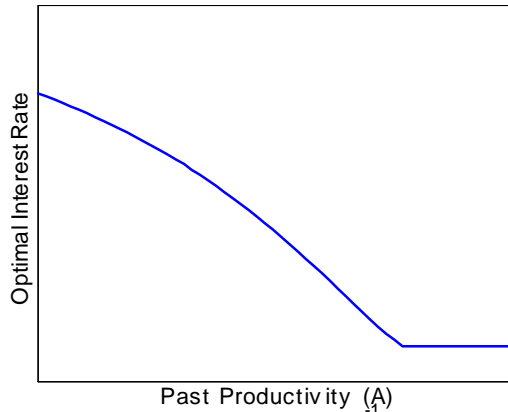


Figure 3: Optimal choice of r

Figure 5 depicts the optimal choice of interest rate as a function of past productivity, A_{-1} . When A_{-1} is very high, the probability that the current productivity will fall into a fragility region is low. At that point, a higher interest rate has almost no effect on the probability of a crisis, but it increases the costs of borrowing. Therefore, the government sets the interest rate close to zero for high values of past average productivity. At lower values of A_{-1} , the government assigns a positive probability to the possibility that current productivity will fall into the fragility region. In this case, a higher interest rate can substantially decrease the probability of debt crisis. Therefore, the government starts to use the interest rate as a way of

²⁷The government's choice of interest rate affects consumption and investment indirectly, through its effect on default threshold $A^*(r)$, default decision d_1 , and government spending g_1 and g_2 .

avoiding default. As past productivity falls, the government finds it optimal to set higher and higher interest rates. This is because, as Figure 3 shows, for low values of past productivity, a high interest rate is needed to decrease a default threshold by a significant amount.

4 Preventing Self-fulfilling Debt Crises

In this section, I analyze which government policies can the decrease probability of default (or “prevent a debt crisis”). I assume that each policy change is: announced at the beginning of the model and that the government is committed to implementing the announced policies. Later, in Section 5, I analyze briefly what happens if a policy change is unexpected or the government lacks commitment.

4.1 Equilibrium Effects of Policy Adjustments

Before analyzing specific policies, it is useful to understand the equilibrium forces are at play when the government adjusts its policy. For this purpose, consider an abstract policy adjustment, captured by a change in a parameter ψ .²⁸ I am interested in the way a change in ψ affects the ex-ante probability of default.

For a given interest rate r , an equilibrium is determined by three sets of equations: the government’s default equation, which says that at the productivity level is A^* the government is indifferent between repaying the debt and defaulting:

$$\Delta V(A^*, k_2, x^*; \psi) = 0;$$

households’ first-order conditions for optimal investment,

$$H(A^*, k_2, x^*; \psi) = 0;$$

and lenders’ indifference conditions, which say that at a signal x^* , each lender j is indifferent between supplying his funds to the bond market and investing in storage, given by

$$L(A^*, k_2, x^*; \psi) = 0.$$

The effect of policy adjustment on the ex-ante probability of default is then captured by $dA^*/d\psi$ implied by this system of equations.²⁹ Applying the Implicit Function Theorem yields:

$$\frac{dA^*}{d\psi} = \frac{\left[\frac{\partial A^*}{\partial \psi} \right]^{\Delta V} + \left[\frac{\partial A^*}{\partial k_2} \right]^{\Delta V} \left[\frac{\partial k_2}{\partial \psi} \right]^H + \left[\frac{\partial A^*}{\partial x^*} \right]^{\Delta V} \left(\left[\frac{\partial x^*}{\partial \psi} \right]^L + \left[\frac{\partial x^*}{\partial k_2} \right]^L \left[\frac{\partial k_2}{\partial \psi} \right]^H \right)}{1 - \left[\frac{\partial A^*}{\partial k_2} \right]^{\Delta V} \left[\frac{\partial k_2}{\partial A^*} \right]^I - \left[\frac{\partial A^*}{\partial x^*} \right]^{\Delta V} \left(\left[\frac{\partial x^*}{\partial A^*} \right]^L + \left[\frac{\partial x^*}{\partial k_2} \right]^L \left[\frac{\partial k_2}{\partial A^*} \right]^I \right)} \quad (2)$$

where, to avoid confusion, a superscript indicates which equation determines a given partial effect (e.g., $\left[\frac{\partial A^*}{\partial k_2} \right]^{\Delta V}$ denotes a partial derivative of A^* with respect to k_2 implied by the default equation).

²⁸For concreteness, one can think of this policy as an increase in taxes, in which case $\psi = \tau$.

²⁹By the envelope theorem the equilibrium interest rate is unaffected by the marginal change in ψ .

To understand the above expression, first keep households' and lenders' beliefs about A^* constant. Then, a change in ψ affects the government's default decision directly through its effect on the default equation (captured by $[\partial A^*/\partial\psi]^{\Delta V}$), and indirectly through adjustments in households' investment choices (term $[\partial A^*/\partial k_2]^{\Delta V} [\partial k_2/\partial\psi]^H$ in the numerator) and lenders' supply decisions ($[\partial A^*/\partial x^*]^{\Delta V} [\partial x^*/\partial\psi]^L$).³⁰ Thus, the numerator of $dA^*/d\psi$ is equal to the change in the default threshold, keeping households' and lenders' beliefs about A^* constant. I refer to this change as the “direct effect” of policy adjustment.

In response to this initial change in the default threshold, households and lenders adjust their strategies, which leads to a further change in A^* inducing another round of adjustment in households' and lenders' strategies, and so on. The change in A^* implied by the adjustments in agents' beliefs is then captured by the denominator in equation (2). I refer to this as the “multiplier effect”.

In light of the above discussion, we can write the total change in the default threshold implied by the adjustment in policy ψ as

$$\begin{aligned} \frac{dA^*}{d\psi} &= [\text{Multiplier Effect}] \times [\text{Direct Effect}] \\ &= [\text{Multiplier Effect}] \times \left[-\frac{\partial\Delta V(A^*, k_2(\psi), S(\psi, k_2(\psi)), \psi)/\partial\psi}{\partial\Delta V(A^*, k_2, S)/\partial A^*} \right] \end{aligned}$$

Note that the size of the multiplier effect is independent of ψ and is determined instead by how sensitive households' and lenders' strategies are to their beliefs about A^* . Moreover, under the assumptions made in Section 3.1, the “multiplier effect” is always positive.³¹ Thus, to determine whether a policy decreases the probability of default, it is enough to focus on the “direct effect”. However, since ΔV is increasing in A^* holding households' and lenders' strategies constant (i.e., $\partial\Delta V(A^*, k_2, S)/\partial A^* > 0$), it follows that the sign of the “direct effect” is determined by the sign of $\partial\Delta V(A^*, k_2(\psi), S(\psi), \psi)/\partial\psi$ - the change in the government's incentives to default keeping households' and lenders' beliefs constant. Remark 1 summarizes the above discussion.

Remark 1 *In order to determine whether a given policy measure decreases or increases probability of a crisis, it is enough to focus on the change in the government's incentives to default, keeping households' and lenders' beliefs constant as captured by $\partial\Delta V(A^*, k_2(\psi), S(\psi, k_2(\psi)), \psi)/\partial\psi$.*

4.2 Overview of Policies

Using the above insights, I now analyze specific government policies. I focus on three policy measures: (1) increase in taxes, (2) fiscal stimulus (financed with debt), and (3) spending cuts. Below, I describe how each of these policies is introduced into the model.

Increase in Taxes In the model, a rise in the tax rate is captured by an increase in τ , the fraction of output that the government takes away from households. I assume that once adjusted, τ is kept constant across periods and is the same regardless of whether or not the government defaults.

³⁰The last term in the numerator captures the fact that a change in households' investment decisions affects also the government borrowing choice which in turn may induce a further change in lenders strategies.

³¹See Appendix section ??.

Fiscal Stimulus I model fiscal stimulus as an increase in the capital stock of each of the households from k_1 to $(1 + s)k_1$ financed by the government.³² I assume that the government faces the same technological constraint as households and it needs one period to transform resources into capital. Thus, to be effective in period 1, the stimulus takes place at the end of a previous period, $t = -1$.

I do not explicitly model the government’s financing decision. Instead, I allow the government to finance a stimulus with either short-term debt that matures at $t = 1$ together with B_1 and carries a net interest rate $r^{ST} \geq 0$ (short-term debt financing), or with long-term debt that matures at $t = 2$ and carries net interest rate $r^{LT} \geq 0$ (long-term debt financing). The government has an option to default on this debt in period 1.

Spending Cuts I model spending cuts as the government’s commitment to spend, at most, $(1 - c)$ of its revenue for public consumption at $t = 1$ - in other words, the government commits to repay at least $c\tau Y_1$ of the debt maturing at $t = 1$.³³

The timing of the policy adjustment is depicted in Figure 5, where austerity measures include both tax increases and spending cuts.

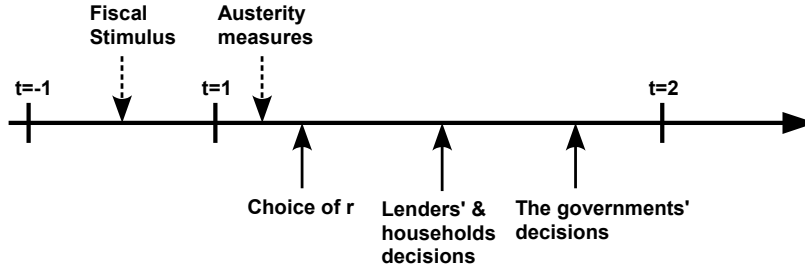


Figure 4: Timing of Policy Adjustments

4.3 Policy Effects

4.3.1 Assumptions

In order to obtain analytical results, I need to make additional assumptions. These assumptions simplify lenders’ problem and allow the model to be solved to a large extent in closed-form. Since these assumptions are strong and, in general, may not be satisfied at a given set of parameters, in the Appendix I provide results of numerical simulations for a wide range of parameters.

³²Here, s measures the size of the stimulus in percentage points of the initial capital stock.

³³A natural way to model spending cuts would be to restrict government spending to $(1 - c)g_1$, where g_1 is the government equilibrium spending without any policy adjustment. Unfortunately, g_1 is an endogenous variable that depends on both lenders’ and households’ strategies, which makes it difficult to ensure that given spending cuts are attained. For example, a change in the government borrowing aimed at lowering spending will affect lenders’ supply strategies and may very well result in a drop in spending that far exceeds announced spending cuts.

Assumption 4 *The interest rate r satisfies $r < \frac{B_1}{b-B_1}$.*

Assumption 5 *Lenders' signals are infinitely precise, $\sigma_x \rightarrow 0$.*

Assumption 6 *Z is large enough so that for all $A > \underline{A}$, the government's desired borrowing exceeds the supply of funds in the market.*

Assumptions 4 and 5 ensure that at the default threshold, the supply of funds in the bond market is bounded from above by B_1 . Assumption 6 simplifies the lenders' problem substantially, assuming away the issue of competition between lenders. It follows that changes in the government's desired borrowing, B_2^g , have no effect on lenders' decisions.

4.3.2 Increase in taxes

Consider the effect of increasing the tax rate τ on the probability of default. The change in the government's default incentives implied by a small increase in τ is given by:³⁴

$$\begin{aligned} \frac{\partial [\Delta V(A^*, k_2(\tau), S, \tau)]}{\partial \tau} &= \underbrace{Y_1^R (u_{g_1}^R - u_{g_1}^D) + Y_2^R (u_{g_2}^R - u_{g_2}^D)}_{\text{Concavity effect}} + \underbrace{Y_1^R (1-Z) u_{g_1}^D + Y_2^R (1-Z) u_{g_2}^D}_{\text{Differential increase in tax revenues}} \\ &\quad - \underbrace{\frac{\alpha}{1-\tau} \tau Y_2^R (u_{g_2}^R - Z u_{g_2}^D)}_{\text{Investment distortion}} \end{aligned} \quad (3)$$

where $u_{g_t}^D$ and $u_{g_t}^R$ are the median household's marginal utilities from government spending in default and in repayment at time t , respectively.

Equation (3) tells us that an increase in the tax rate affects the government's default incentives through three channels. First, higher τ implies higher tax revenues. Since at A^* , government spending is lower in repayment than in default, the concavity of the utility function implies that the same increase in government spending leads to a greater increase in value of repaying than in the value of defaulting, thus decreasing the government's default incentives ("concavity effect").

Second, because of the output cost of defaults, the same increase in the tax rate translates into a greater increase in tax revenues in repayment than in default. Thus, government spending increases more in repayment, further decreasing the government's default incentives ("differential increase in tax revenues").

Finally, the last term captures the negative effect of higher taxes on households' investment decisions, where $\alpha/(1-\tau)$ is the rate at which output decreases with higher taxes; τY_2^R indicates the extent to which this translates into a drop in government spending; and $u_{g_2}^R - Z u_{g_2}^D$ measures how "painful" this decrease in spending is to the median household in repayment compared to default.

While the first two effects tend to decrease the governments' default incentives, the negative effect of taxes on investment tends to encourage default. The following lemma states, however, that if the rate at which taxes decrease investment or/and if tax revenues are a small fraction of output, then an increase in the tax rate will lead to a decrease in probability of default.

³⁴Under assumptions 4 – 6, supply of funds depends on τ only indirectly through A^* .

Lemma 3 *A sufficient condition for taxes to reduce the probability of default is that*³⁵

$$1 > \frac{\alpha}{1 - \tau} \tau.$$

4.3.3 Fiscal Stimulus

Now, consider the effect of fiscal stimulus on the probability of default. The change in the government’s default incentives implied by a fiscal stimulus is given by:

$$\begin{aligned} \frac{\partial \Delta V(A^*, k_2, S, s)}{\partial s} &= \underbrace{\tau \frac{\partial Y_1}{\partial s} (u_{g_1}^R - u_{g_1}^D) + \tau \frac{\partial Y_2}{\partial s} (u_{g_2}^R - u_{g_2}^D)}_{\text{Concavity effect}} + \underbrace{\frac{\partial Y_1}{\partial s} \tau (1 - Z) u_{g_1}^D + \frac{\partial Y_2}{\partial s} \tau (1 - Z) u_{g_2}^D}_{\text{Differential increase in tax revenues}} \\ &\quad - \underbrace{u_{g_t}^R (1 + r^{stim}) k_1}_{\text{Increase in debt}}, \end{aligned} \quad (4)$$

where r^{stim} is the interest rate on the debt issued to finance the stimulus.

Equation (4) tells us that a fiscal stimulus affects the government’s default incentives through three channels: (1) the “concavity effect”; (2) a differential increase in government tax revenues in repayment and default (both of which were also present in the case of a tax increase); and (3) a negative effect due to an increase in the government’s debt burden (equal to $u_{g_1}^R (1 + r^{ST}) k_1$ if the stimulus is financed with short-term debt or $u_{g_2}^R (1 + r^{ST}) k_1$ if financed with long-term debt).

The typical argument for the fiscal stimulus is that it leads to an increase in the output and that, due to the “fiscal multiplier”, the positive effects of such an increase exceed the costs of the stimulus. According to that view, fiscal stimulus is beneficial because by expanding output, it actually lowers debt-to-GDP ratio and hence decreases the government incentives to default. Equation (4) reveals that, to the extent that the government maximizes households’ welfare, this logic is not correct. Abstracting from the “concavity effect”, we see that the positive effect of fiscal stimulus is determined not by the fiscal multiplier but, rather, by the differential increase in the government’s revenues in repayment versus default implied by the stimulus, given by $\tau (1 - Z) \sum_t \partial Y_t / \partial s$. Thus, in order for stimulus to decrease the government’s incentives to default “fiscal multiplier” has to be very large. Thus, we should expect that, within the model, a stimulus will typically lead to an increase rather than a decrease in the probability of default. The next result provides a sufficient condition when fiscal stimulus will increase likelihood of a crisis.

Lemma 4 *Consider fiscal stimulus and let $g_t^R(A^*)$ be the government spending in repayment at time t when the average productivity is equal to A^* .*

1. *If $(1 + r^{ST}) k_1 > \alpha B_1$, then a stimulus financed with short-term debt necessarily increases the probability of default.*
2. *If $(1 + r^{LT}) k_1 \frac{g_1^R(A^*)}{g_2^R(A^*)} > \alpha B_1$, then a stimulus financed with long-term debt necessarily increases the probability of default.*

³⁵Even if this condition is violated, an increase in the tax rate tends to decrease probability of default. See, robustness section in the Appendix.

The above result provides only the sufficient conditions under which fiscal stimulus will lead to an increase in the probability of default. Even if these conditions are not satisfied, a fiscal stimulus tends to increase the probability of default. Intuitively, in order for fiscal stimulus to decrease probability of default the marginal product of capital has to be very high (so that $\partial Y_t / \partial s$ is very large). This can be achieved only if the initial capital stock, k_1 , is small and the capital share of output, α , is high.

4.3.4 Spending Cuts

Finally, consider the government's commitment to devote a small fraction of its revenues at $t = 1$ to repay its "legacy debt", B_1 . This policy always leads to a reduction in government spending in default, since now if the government default, its spending is given by $(1 - c) \tau Z Y_1$. On the other hand, this policy has no direct effect on the government spending in repayment around default threshold. To see that, note that the government's budget constraint under the new policy, after a simple rearrangement, is exactly the same as before:

$$\begin{aligned} g_1^R &= (1 - c) \tau Y_1 - (B_1 - c \tau Y_1) + B_2 \\ &= \tau Y_1 - B_1 + B_2. \end{aligned}$$

Moreover, note that "spending cuts" have no direct effect on the households' or lenders' problem. It follows that the only effect of this policy is to decrease government spending in default, and, hence,

$$\frac{\partial \Delta V(A^*, k_2, S, c)}{\partial c} = \frac{1}{1 - c} > 0.$$

Therefore, we obtain the following result:

Lemma 5 *Spending cuts always lead to a decrease in the probability of default.*

4.3.5 Summary

Above I considered three different policies (increase in taxes, fiscal stimulus, and spending cuts). I explained how these policies affect the government's default incentives and provided sufficient conditions under which these policies decrease or increase the probability of default. In general, the model suggests that whether or not a given policy helps the government avoid a debt crisis, may depend on the initial state of the economy: the initial tax rate τ , initial stock of capital k_1 , the capital share of output α and the output costs of default Z . This was to be expected - in a complex environment, does rarely one policy always dominate the others. Nevertheless, the above analysis suggests that if the government wants to avoid debt crisis, it should focus on austerity measures (such as spending cuts or a tax increase) rather than on fiscal stimulus.

5 Extensions

In Section 4 I analyzed how various government policies affect the probability of default. This analysis abstracted from the welfare implications of each policy. Since the government cares about median household welfare, it may choose not to implement a given policy if it

leads to a reduction in the median household’s expected utility. In section 5.1, I explain the “Welfare-Default Probability” trade-off present in the model. The analysis in Section 4 was also conducted under two important assumptions: (1) expected policy change and (2) commitment. In Sections 5.2 and 5.3 I discuss what happens if each of these assumptions is relaxed. When investigating the above issues, I rely mostly on numerical examples since in many cases analytical results are not available. However, I emphasize which of the results can also be shown analytically.

I calibrate the model to imply a ten-percent ex-ante probability of a debt crisis. The most important parameters to calibrate are τ , the tax rate, Z , the output costs of default, and α , the capital share of output. I set $\tau = 0.4$, the average level of tax revenue over GDP in the Eurozone in 2011, as reported by Eurostat, and $Z = 0.92$, implying that in the case of a debt crisis, output declines by eight percent (the observed output decline in Greece after it defaulted in 2010). I choose $\alpha = 0.4$, which is in line with recent estimates of capital share of output for most Eurozone members (see Arpaia, Perez, and Pichelmann (2009) or Jones (2003)).³⁶

5.1 “Welfare-Default Probability” Trade-off

I measure the welfare gains (or losses) associated with a given policy adjustment as percentage change in consumption under the original policy required to make the median households indifferent between the new and the old policy. Specifically, let \bar{c}_t and \bar{g}_t be the median households’ consumption of private and public good under the original policy and denote by \tilde{c}_t and \tilde{g}_t the consumption of private and public good under new policy. Finally, let ω be defined as a solution to the following equation:

$$E[u((1 + \omega)\bar{c}_t, (1 + \omega)\bar{g}_t)] = E\left[u\left(\tilde{c}_t, \tilde{g}_t\right)\right].$$

Then, ω measures the benefits of a policy change in terms of consumption units and, in what follows, I refer to ω as the welfare gain of a policy adjustment.³⁷

Figure 6 depicts a change in the probability of default (horizontal axis, denoted by $\Delta \text{Pr} \%$) and a change in welfare (vertical axis, denoted by ω) implied by continuously varying a tax rate (blue solid line), size of spending cuts (green dashed line), fiscal stimulus financed with short-term debt (orange dash-dot line) and with long-term debt (pink dotted line) starting from their baseline values. The markers show the change in default probability and welfare implied by a two-percent adjustment in each policy.

Figure 6 confirms the analytical results established in Section 4. In particular, we see that an increase in the tax rate and spending cuts decrease the probability of default, while fiscal stimulus increases it. This suggests that the assumptions made in Section 4 are not essential for the established results.

Regarding welfare, we see that at the benchmark calibration, all policies lead to an increase in welfare. In particular, note that fiscal stimulus raises welfare despite increasing the

³⁶Other parameters are $B_1 = 0.5$, $b = 1$, $\sigma_x = 1/30$, and $\sigma = 1/10$. The interest rate on the short-term and long-term debt used to finance a stimulus is set to be $r^{ST} = r^{LT} = 0$. We will see that even under this extreme assumption, it is hard to make stimulus decrease the probability of default.

³⁷This is a standard way of computing welfare effects of policy change in microfounded models (see Lucas (2003)).

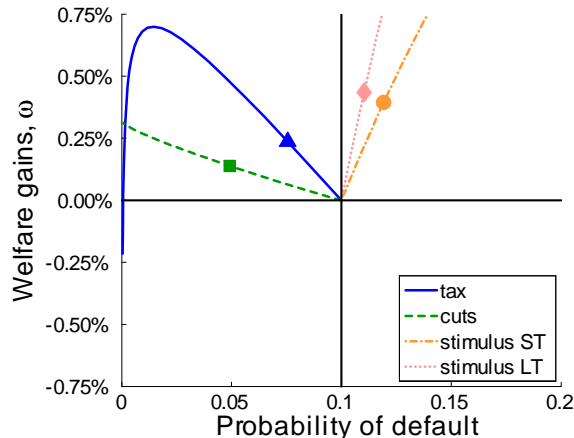


Figure 5: “Probability of default-Welfare” trade-off

probability of default. This is because an increase in k_1 leads to a higher private and public consumption in both periods and in both repayment and default states. While an increase in the probability of default tends to decrease welfare, this effect is dominated by an overall increase in consumption. This result is general and, as shown in the Appendix, holds under different calibrations. On the other hand, whether an increase in taxes increases welfare depends on the initial value of τ . As we can see from Figure 6, there exists a level of taxes that maximizes welfare. If the initial tax rate is already at this level, then an increase in τ will be detrimental to the median household’s welfare.

Finally, note that Figure 6 can also help us to deduce the effects of different policy combinations. In particular, the figure suggests that accompanying fiscal stimulus with spending cuts or increase in taxes can achieve both a substantial decrease in the probability of default and a significant welfare improvement.

5.2 Unexpected Policy Change

Until now, I have assumed that households and lenders expect a policy change. In this section, I investigate what happens if such a policy adjustment is unexpected. In particular, suppose that there is strong disagreement among top government officials regarding the implementation of a policy. Further, assume that households and lenders observe this lack of agreement and conclude that there will be no policy adjustment in the near future (thus, any policy change will be unexpected from agents’ perspective). However, after observing the actual productivity level, households’ investment decisions and lenders’ supply choices, the government finds itself on the verge of defaulting, and policymakers, now up against the wall, decide to act. At this point, the government can still use any of the described policies to try to avoid defaulting. But would these measures still be effective?

The first important implication of an unexpected policy change is the lack of the “multiplier effect”. Since agents expect no policy adjustment, their beliefs regarding default are unchanged. Moreover, since the policy change takes place after households and lenders have made their decisions, a policy change has no effect on their choices implying that the change in the default

threshold is simply equal to the change in the government default incentives, i.e.:

$$\frac{dA^*}{d\psi} = \frac{\partial A^*}{\partial \psi}.$$

Second, since fiscal stimulus now takes place towards the end of period 1, it leads only to an increase in output in period 2. Moreover, it now has to be financed by borrowing in the markets in period 1, at the time when the government is already struggling to raise desired amount of funds. Thus, we expect that fiscal stimulus leads to a smaller increase in welfare and potentially larger increase in the probability of default.

Figure 7 illustrates how a change in each policy affects the default probability and welfare. As before, the lines trace the change in a equilibrium probability of default and the median household’s welfare, as relevant policy parameters vary and markers illustrate the effects of a 2% change in each policy.

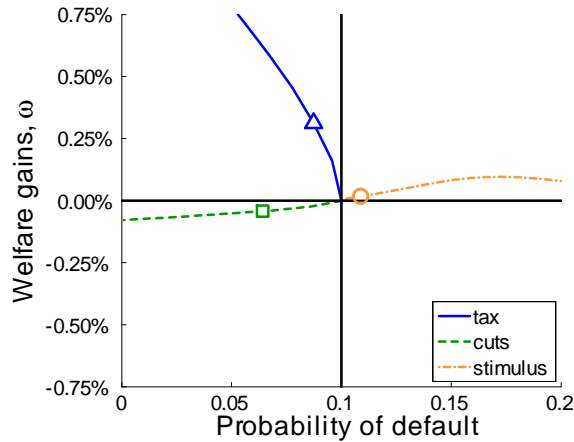


Figure 6: Unexpected Policy Change - “Probability of default-Welfare” trade-off

Compared to the case in which the policy adjustment was announced early, we see that now spending cuts tend to decrease welfare. The reason behind this, is that from the median households’ perspective the government default decision is ex-post optimal. Therefore, changing the default decision without any associated change in households’ and lenders’ behavior has to reduce welfare. As expected, Figure 7 shows that a fiscal stimulus now leads to only a small increase in welfare, but the lack of the “multiplier effect” implies that its negative effect on the probability of default is also smaller. Finally, we see that the winner in this case is the tax increase. While a tax increase leads to a smaller reduction in the probability of default than before (lack of “multiplier effect”), it results in a much larger increase in welfare. This is because, an unexpected increase in the tax rate does not discourage investment today.

5.3 Commitment

The other key assumption made in Section 4 was the government’s ability to commit to implement the announced policies. In this section, I investigate what happens if this assumption is relaxed.

In order to analyze this issue, I assume again that the government announces policy early (as in Figure 5) and that the government will implement its policy if and only if it leads to an increase in welfare ex-post - i.e., given households' and lenders' choices. Note that under this timing, there is no issue of credibility when considering fiscal stimulus since the stimulus takes place before households and lenders make their decisions. Hence, I limit my discussion to spending cuts and changes in the tax rate.

5.3.1 Spending Cuts

It is easy to see that without commitment, the government will never implement spending cuts ex-post in default. This is because if the government defaults, then there is no benefit from reducing spending - a drop in the government expenditures simply lowers the median household's utility. Therefore, under no commitment, the value of defaulting is unchanged by the announcement of spending cuts. Since, as argued above, spending cuts do not affect the value of repayment to the government, the government's problem is unaffected by the announcement of spending cuts. Understanding this, households and lenders keep their strategies unchanged, implying that spending cuts leave equilibrium unchanged.

Lemma 6 *Under no commitment, spending cuts have no effect of the probability of default (and welfare).*

5.3.2 Tax Increase

The case of taxes is more complicated. The optimal ex-post tax rate depends both on the realization of productivity and on households' and lenders' choices, which, in turn, depend on these agents' expectation regarding the whole profile of future tax rates.³⁸ These features make analyzing the impact of tax rate under no commitment difficult to solve, even numerically. However, as argued below, the general intuition is easy to sketch.

What matters for the effect of announcing a change in tax rate on the probability of default is whether the government implements a tax increase around the default threshold. In particular, agents ask themselves whether, at productivity A^* , the government will implement the tax rate increase in the case of debt repayment and in the case of default. Let τ^D and τ^R denote the highest tax rate the government would be willing to implement ex-post (i.e., after lenders and households have made their choices) in default and in repayment, respectively, when the average productivity is A^* .³⁹ Since government spending is always lower around the default threshold if the government repays rather than defaults, we expect that $\tau^R > \tau^D$. Moreover, since a higher tax rate decreases output tomorrow as well as private consumption, and the government cares about the median household's welfare, we know that $\tau^R < 1$.

Given the above discussion, we should expect that if the announced tax rate is below τ^D , then the government will always implement this tax rate at A^* ; thus, the effect of a marginal

³⁸Lenders care about the government's desired borrowing for all realization of productivity above the default threshold, and since desired borrowing is a function of the tax rate, the lenders have to form expectations about the whole profile of taxes as a function of A . Households, on the other hand, have to predict the tax rate for all realizations of productivity around their idiosyncratic productivity. This is true even when considering the case $\varepsilon \rightarrow 0$.

³⁹Assume that both such objects exist.

increase in taxes on the probability of default is the same as in the case when the government commits to a given policy change. If, however, the new tax rate is between τ^D and τ^R , then it is implemented only in repayment. Compared to the case with commitment, the value of defaulting is now higher (implementing the tax rate would actually decrease the value of defaulting) and, thus, the change in the probability of default is smaller than before. Finally, if the new tax rate is above τ^R , then the tax increase will be never implemented around the default threshold, and, hence, has no effect on the probability of default. The above discussion also suggests that when the initial tax rate is already high, then increasing taxes is not a credible policy and will have no effect on equilibrium.

6 Conclusions

In this paper, I develop a model of sovereign debt crises in which a crisis is a result of an interplay between poor fundamentals, and self-fulfilling expectations of households and lenders. Models of self-fulfilling crises typically have multiple equilibria. To deal with this issue, I use the global games methodology and assume that both households and lenders do not perfectly observe the underlying economic fundamentals, but, rather their noisy signals. I show that imposing this structure leads to a global game played among households, lenders and the government.

Solving a global game in a micro-founded model of sovereign default is a challenging task. The main challenge comes from the fact that, in the model, lenders' payoffs satisfy only a weak single-crossing condition rather than the global strategic complementarities typically present in the global games models. In order to show uniqueness, I extend the approach of Morris and Shin (2003) to setups that feature only a weak single-crossing property and in which agents with different preferences play a global game.

I use the model to study the effectiveness and welfare implications of various government policies meant to prevent a debt crisis: an increase in taxes, fiscal stimulus and spending cuts. I explain how these policies affect the equilibrium probability of default and provide sufficient conditions under which they decrease or increase the likelihood of a crisis. In general, I show that a tax increase or spending cuts tend to decrease, while fiscal stimulus tends to increase, default probability. I also analyze numerically the welfare implications of these policies. I find that spending cuts always result in a higher welfare, while whether a tax increase is beneficial or detrimental depends on the initial tax rate. Somewhat surprisingly, the model implies that fiscal stimulus leads to an increase in welfare even though it tends to increase the probability of default. This is because fiscal stimulus expands output in both periods and in both repayment and default, and this effect tends to dominate the negative effect associated with a higher likelihood of a crisis.

The above conclusions are based on the assumptions that a policy change is expected and the government is committed to implementing the announced policy. In the final part of the paper, I investigate what happens when each of these assumptions is relaxed. I find that unanticipated policy changes have weaker impact on probability of default and welfare than anticipated policy adjustments (with the exception of the welfare implications of higher taxes). I also argue that lacking commitment the government will never implement spending cuts, while it will increase the tax rate only if the initial tax rate is low enough.

A word of caution is needed regarding the interpretation of the results. The model presented

in this paper is still very stylized and, therefore, the numerical results should be interpreted with caution. In particular, one should give much more weight to the relative ranking of policies implied by the model than to their relative magnitudes. The aim of the robustness section in the Appendix is to convince the reader that the policy ranking is very stable and does not depend on a particular parameterization of the model or the assumptions made in the analytical part of the paper.

Finally, in this paper, I analyze a situation in which the government finds itself at a point where a debt crisis is possible. Indeed, the main question this paper addresses is how to avoid a debt crisis when one is likely in the near future. For that purposes, that fact that the model presented above is two-period is a minor issue. However, the fact that the model is not dynamic becomes key when trying to answer questions regarding medium-term policies. A question of particular importance is what the government should do to avoid facing another debt crisis in the future once the debt crisis has been averted today. This is the challenge that the European leaders face today, and it remains an important question for future research.

Appendix A Fragility Region

The goal of this section is to derive the “fragility region” - i.e., a region of productivity values A such that both repaying the debt and defaulting are consistent with the fundamentals, and where the government’s default decision depends on households’ and lenders’ choices. For this purpose, I assume that $\xi \rightarrow 1$.

A.1 Preliminaries

I start by establishing a few preliminary results regarding the optimal unconstrained borrowing by the government if it chooses to repay the debt. The government’s optimal unconstrained new borrowing is a solution to

$$\begin{aligned} & \max_{B_2 \in \mathbb{R}} \sum_{t=1,2} \log(\bar{c}_t^R) + \log(g_t^R) \\ \text{s.t. } & g_1^R = \tau Y_1^R - B_1 + B_2 \\ & g_2^R = \tau Y_2^R - (1+r)B_2 \\ & \bar{c}_1^R = (1-\tau)e^A f(\bar{k}_1) - \bar{k}_2 \\ & \bar{c}_2^R = (1-\tau)e^A f(\bar{k}_2) \end{aligned}$$

where Y_1^R and Y_2^R are the output in periods 1 and 2 if the government repays its debt, given by:

$$Y_1 = e^A f(k_1) \quad \text{and} \quad Y_2 = \int_{i \in [0,1]} e^A f(k_2^i) di$$

and \bar{c}_1^R and \bar{c}_2^R correspond to median households’ consumption at times 1 and 2.

Let B_2^y be the government’s optimal borrowing choice implied by the above problem. If the government borrows, then it does so at an interest rate r , while if it lends, it charges lenders a risk-free interest rate of 0. It follows that the government unconstrained borrowing policy is

given by

$$B_2^u = \begin{cases} B_2^{bor} & \text{if } B_2^{bor} \geq 0 \\ 0 & \text{if } B_2^{bor} < 0 \text{ and } B_2^{len} \geq 0 \\ B_2^{len} & \text{if } B_2^{len} < 0, \end{cases}$$

where

$$B_2^{bor} = \frac{(1+r)B_1 + \tau Y_2 - (1+r)\tau Y_1}{2(1+r)} \quad \text{and} \quad B_2^{len} = \frac{B_1 + \tau Y_2 - \tau Y_1}{2}.^{40}$$

Note that if $B_2^u \leq 0$, then lenders' decisions have no effect on equilibrium. Since I am interested in analyzing how lenders' choices affect the government default's decision, I assume that $B_2^u \geq 0$ for all A . In section *D.1* of this Appendix, I provide a sufficient condition under which $B_2^u \geq 0$ in the "fragility region" derived below.

A.2 Fragility Region

first, consider households. When a household expects default, its optimal investment is given by:

$$k_2^D(A) = \frac{Z(1-\tau)e^A f(k_1)}{1 + \frac{1}{\alpha}},$$

where superscript D denotes default, while if a household expects repayment, then it invests

$$k_2^R(A) = \frac{(1-\tau)e^A f(k_1)}{1 + \frac{1}{\alpha}} = \frac{1}{Z} k_2^D(A)$$

where superscript R denotes repayment. Therefore, if households expect default, they decrease their investment for any given A .

Now, consider the government and recall that the government repays the debt if and only if

$$\Delta V(A, \mathbf{k}_2, S) \equiv V_1^R(A, \mathbf{k}_2, S) - V_1^D(A, \mathbf{k}_2) \geq 0.$$

It is easy to see that for all $A \in \mathbb{R}_{++}$ $\Delta V(A, k_2, S)$ is strictly increasing in \mathbf{k}_2 and in S for all $S < B_2^u$. Moreover, from the above discussion, we know that for a given A , the lowest rationalizable investment is given by $k_2^D(A)$, while the highest rationalizable investment is given by $k_2^R(A)$. Therefore, the government has the strongest incentives to default if $\mathbf{k}_2 = \{k_2^D\}_{i \in [0,1]}$ and $S = 0$. Similarly, the government has the weakest incentives to default if it can borrow the unconstrained optimal amount - i.e., $B_2^u < S$, and $\mathbf{k}_2 = \{k_2^R\}_{i \in [0,1]}$. It follows that, for a given interest rate r , the government will repay the debt irrespective of agents' action if the average productivity is greater or equal to $\bar{A}(r)$, where $\bar{A}(r)$ is the unique solution to:

$$\Delta V(\bar{A}(r), \mathbf{k}_2^D, 0) = 0. \tag{5}$$

Moreover, since $B_2^u = 0$, it follows that the upper bound of the fragility region is independent of r and, thus, in what follows, I will denote it simply by \bar{A} .

⁴⁰The expression $\frac{(1+r)B_1 + \tau Y_2 - (1+r)\tau Y_1}{2(1+r)}$ is decreasing in r . Thus, we know that $B_2^{len} > B_2^{bor}$. Therefore, if $B_2^{len} < 0$ then $B_2^{bor} < 0$ and, similarly, if $B_2^{bor} > 0$ then $B_2^{len} > 0$.

Similarly, the government always defaults if the average productivity in the economy is less than $\underline{A}(r)$, where $\underline{A}(r)$ is the unique solution to

$$\Delta V(\underline{A}(r), \mathbf{k}_2^R, B_2^u(A)) = 0. \quad (6)$$

Then, by the monotonicity of $\Delta V(A, \mathbf{k}_2, S)$ in A , \mathbf{k}_2 and S , it follows that for all $A \in [\underline{A}(r), \bar{A}]$ the outcome of the model depends on the households' and lenders' choices.

Finally, define interest rate r^R as a solution to the following equation:

$$B_2^u(\underline{A}(r^R), r^R; \mathbf{k}_2^R) = 0. \quad (7)$$

That is, if the interest rate is $r > r^R$, then it is optimal for the government to borrow nothing. With this definition we can now characterize the ‘‘fragility region’’.

Lemma A.1 (Fragility Region) *For any given interest rate $r \in \mathbb{R}_{++}$, the ‘‘fragility region’’ is given by $[\underline{A}(r), \bar{A}]$, where $\underline{A}(r)$ and \bar{A} are uniquely determined by equations (5) and (6). Moreover,*

1. $\underline{A}(r)$ is strictly increasing for all $r < r^R$ and constant for all $r \geq r^R$;
2. $\underline{A}(r) < \bar{A}$ for all $r \in \mathbb{R}_{++}$;
3. r^R is uniquely determined by equation (7).

As an immediate corollary of the above result, we obtain a useful result for characterization of the optimal interest rate.

Corollary 1 *The government’s optimal interest rate r^* is smaller or equal to r^R .*

Appendix B Unique Equilibrium in Monotone Strategies

In this section, I show that the model has a unique equilibrium in monotone strategies.

Proposition 1 *Let $\varepsilon \rightarrow 0$. Then, for all $\sigma_x < \bar{\sigma}_x$ and for each interest rate r , the model has a unique equilibrium in monotone strategies such that:*

1. *The government defaults if and only if $A < A^*$;*
2. *Each lender j supplies the funds to the market if and only if $x_j \geq x^*$ and*
3. *Households’ investment rules, k_2 , are monotone in their signals.*

The proof consists of five steps:

1. First, I show that if the government follows a monotone default strategy, then households find it optimal to follow monotone investment strategies.
2. Next, I show that if a lender believes that other lenders and the government follow monotone strategies, then he finds it optimal to follow a monotone strategy.

3. Given the above result, I show that for a given monotone strategy by the government, there exists a unique, symmetric profile of lenders' monotone strategies such that each lender's monotone strategy is the best response to the strategies of others.
4. Next, I show that given the above monotone strategies of households and lenders that we found in steps 1 and 3, the government indeed finds it optimal to follow a monotone strategy.
5. Finally, I show that there exists unique fixed point to the above problem.

B.1 Step 1: Households' Investment Problem

Households make their investment decisions after observing their idiosyncratic productivity shocks and before the government's default decision. Since households make their investment decisions simultaneously with lenders, the only sources of information for household i are the prior and its idiosyncratic productivity shock, A_i .

Suppose that households believe that the government's default decision is monotone in A ; that is, the government defaults if and only if the average productivity level is smaller than A^* . Then, household i 's problem can be written as:

$$\begin{aligned} & \max_{k_2^i} \int_{A < A^*} [\log((1 - \tau) Z e^{A_i} f(k_1) - k_2) + \log((1 - \tau) Z e^{A_i} f(k_2))] dP(A|A_i) \\ & + \int_{A \geq A^*} [\log((1 - \tau) e^{A_i} f(k_1) - k_2) + \log((1 - \tau) e^{A_i} f(k_2))] dP(A|A_i), \end{aligned}$$

where $P(A|A_i)$ is the posterior belief of household i regarding aggregate productivity. Household i 's posterior density regarding the aggregate productivity level, denoted by $p(\cdot|A_i)$, is given by a truncated normal distribution:

$$p(A|A_i) = \begin{cases} \frac{\frac{1}{\sigma} \phi\left(\frac{A - A_{-1}}{\sigma}\right)}{\Phi\left(\frac{A_i + \varepsilon - A_{-1}}{\sigma}\right) - \Phi\left(\frac{A_i - \varepsilon - A_{-1}}{\sigma}\right)} & \text{if } A \in [A_i - \varepsilon, A_i + \varepsilon] \\ 0 & \text{otherwise,} \end{cases}$$

where $\frac{1}{\sigma} \phi\left(\frac{A - A_{-1}}{\sigma}\right)$ is the prior belief shared by all the households. It follows that if household's idiosyncratic productivity level is $A_i \geq A^* + \varepsilon$ the household knows that the government will repay its debt and when $A_i \leq A^* - \varepsilon$ then it expects default. However, if the household's idiosyncratic productivity level is $A_i \in (A^* - \varepsilon, A^* + \varepsilon)$ the household is uncertain whether the government will default or not.

Suppose that $A_i \in (A^* - \varepsilon, A^* + \varepsilon)$. Then, the optimal k_2 is given by

$$k_2 = (1 - \tau) e^{A_i} f(k_1) \times \Lambda(A_i),$$

where

$$\Lambda(A_i) \equiv \frac{Z + \Pr(A < A^*|A_i)(1 - Z) + \alpha(1 + Z) + \sqrt{[Z + \Pr(A < A^*|A_i)(1 - Z) + \alpha(1 + Z)]^2 - 4(1 + \alpha)\alpha Z}}{2(1 + \alpha)}.$$

Now, consider the case when $\varepsilon \rightarrow 0$. Recall that $A_i = A + \varepsilon_i$, so that we can express A_i as $A_i = A + \kappa\varepsilon$ for some $\kappa \in [-1, 1]$.⁴¹ Then,

$$\lim_{\varepsilon \rightarrow 0} \Pr(A < A^* | A_i) = \lim_{\varepsilon \rightarrow 0} \frac{\Phi\left(\frac{A^* - A_{-1}}{\sigma}\right) - \Phi\left(\frac{A - (1-\kappa)\varepsilon - A_{-1}}{\sigma}\right)}{\Phi\left(\frac{A + (1+\kappa)\varepsilon - A_{-1}}{\sigma}\right) - \Phi\left(\frac{A - (1-\kappa)\varepsilon - A_{-1}}{\sigma}\right)}$$

Thus, as $\varepsilon \rightarrow 0$, for given κ the households' optimal investment converges to:

$$k_2(\kappa) = \begin{cases} \frac{(1-\tau)e^A f(k_1)}{1+\frac{1}{\alpha}} & \text{if } A > A^* \\ (1-\tau)e^A f(k_1) \times \Lambda(\kappa) & \text{if } A = A^* \\ \frac{(1-\tau)Ze^A f(k_1)}{1+\frac{1}{\alpha}} & \text{if } A < A^*, \end{cases}$$

and the aggregate output at $t = 2$ converges to:

$$Y_2 = \begin{cases} e^A \left[\frac{(1-\tau)e^A f(k_1)}{1+\frac{1}{\alpha}} \right]^\alpha & \text{if } A > A^* \\ \int_{\kappa=-1}^1 \frac{1}{2} e^A [(1-\tau)e^A f(k_1) \times \Lambda(\kappa)]^\alpha d\kappa & \text{if } A = A^* \\ e^A \left[\frac{(1-\tau)Ze^A f(k_1)}{1+\frac{1}{\alpha}} \right]^\alpha & \text{if } A < A^*. \end{cases} \quad (8)$$

B.2 Step 2: Individual Lenders' Problem

Let $u(1, A; x^*, A^*)$ be the payoff to agent i from lending to the government if the government uses threshold strategy with cutoff A^* and other lenders use monotone strategies with cutoff x^* (i.e., each of them lends to the government if and only if he receives a signal greater or equal to x^*). Similarly, denote by $u(0, A; x^*, A^*)$ the payoff to agent i from not lending to the government. Then,

$$\begin{aligned} u(1, A; x^*, A^*) &= \begin{cases} 1 + r \min \left\{ \frac{B_2^u(A)}{S(A; x^*)}, 1 \right\} & \text{if } A \geq A^* \\ 0 & \text{otherwise} \end{cases} \\ u(0, A; x^*, A^*) &= 1 \end{aligned}$$

Define $\Delta u(A; x^*, A^*) \equiv u(1, A; x^*, A^*) - u(0, A; x^*, A^*)$ and note that for any pair (A^*, x^*) , and regardless of government desired borrow function B_2^u , the function $\Delta u(A; x^*, A^*)$ satisfies a weak single-crossing property.⁴²

Let p_x be the precision of lenders' signals - i.e., $p_x = 1/\sigma_x^2$. Then, agent's i posterior density for A is given by $(p_x + p)^{1/2} \phi\left(\frac{A - \frac{p_x x_i + p A_{-1}}{p_x + p}}{(p_x + p)^{-1/2}}\right)$, where x_i is the signal received by agent i . The

⁴¹ Writing the idiosyncratic productivity in this way allows me to analyze the situation when $\varepsilon \rightarrow 0$ since now it is clear that what changes is not the relative distribution of idiosyncratic productivity around A^* but simply the scale of the deviations from A^* .

⁴² A function $f(x)$, where $f: \mathbb{R} \rightarrow \mathbb{R}$, satisfies a weak single-crossing property in x if for all $x_H > x_L$, $f(x_L) > 0$ implies $f(x_H) \geq 0$.

family of density functions parameterized by x_i satisfies the strict monotone likelihood ratio property implying that the posterior density is strictly log-supermodular in (A, x_i) .⁴³

I now use the following lemma:

Lemma 7 (Athey, 1996) *Let $g : S \rightarrow \mathbb{R}$ and $k : S \times \Theta \rightarrow \mathbb{R}$. If g satisfies the weak single-crossing property, k is strictly log-supermodular and $K(s, \theta)$ has constant support in θ ; then, $G(\theta) \equiv \int_S g(s) k(s; \theta) ds$ satisfies the strict single-crossing property in θ .*

By the above lemma, it follows immediately that the lender's i expected payoff, defined as:

$$\Delta \bar{u}(x_i; A^*, x^*) \equiv \int_{A^*}^{\infty} \Delta u(A; x^*, A^*) (p_x + p)^{1/2} \phi \left(\frac{A - \frac{p_x x_i + p A - 1}{p_x + p}}{(p_x + p)^{-1/2}} \right) dA$$

satisfies a strict single-crossing property in x_i . This implies that there exists a unique x_i^* such that if all other agents use monotone strategies with cutoff x^* , then lender i 's best response is also to follow a monotone strategies with uniquely determined cutoff x_i^* . This concludes the second step of our proof.

B.3 Step 3: Uniqueness of Symmetric Threshold x^*

I keep the assumption that the government follows a monotone default strategy- i.e., it defaults if and only if $A < A^*$. I show now that there exists a unique threshold x^* such that if all other agents use a threshold x^* , then x^* is indeed the optimal threshold for agent i . To prove this claim, I use another result from Athey (1996).

Lemma 8 (Athey, 1996) *Suppose that $g(s; \theta)$ is piecewise continuous and is non-decreasing in θ . Further, suppose that $g(s, \theta)$ satisfies a weak single-crossing condition in s , $k(s; \theta)$ is strictly supermodular, $K(s; \theta)$ has constant support and $g(s; \theta) k(s; \theta) \neq 0$ on the set of positive measure. Then,*

$$G(\theta) = \int g(s; \theta) k(s; \theta) d\mu(s)$$

satisfies a strict single-crossing condition.

It is straightforward to show that the conditions of the above results are satisfied and, hence, $\Delta \bar{u}(x^*; x^*, A^*)$, defined as

$$\Delta \bar{u}(x^*; x^*, A^*) \equiv \int_{\mathbb{R}} \Delta u(A; x^*, A^*) (p_x + p)^{1/2} \phi \left(\frac{A - \frac{p_x x^* + p A - 1}{p_x + p}}{(p_x + p)^{-1/2}} \right) dA$$

satisfies a strict single-crossing property in x^* , establishing the claim.

⁴³ A function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is log-supermodular if for all $\{x_L, x_H, y_L, y_H\} \in \mathbb{R}$ such that $x_H > x_L$ and $y_H > y_L$ we have

$$f(x_H, y_H) f(x_L, y_L) \geq f(x_H, y_L) f(x_L, y_H).$$

A function f is strictly log-supermodular if for all $y_H > y_L$ and all pairs (x_H, x_L) in $\text{supp}[F(x, y_L)] \cap \text{supp}[F(x, y_H)]$ such that $x_H > x_L$, $f(x_H, y_H) f(x_L, y_L) > f(x_H, y_L) f(x_L, y_H)$.

Let $F(x; \theta)$ be a distribution function ordered by θ and f be its density. Then, F satisfies the monotone likelihood ratio if for all $\theta_H > \theta_L$ $f(x, \theta_H) / f(x, \theta_L)$ is increasing in x . See Athey (1996).

B.4 Step 4: The Government's Monotone Best Response

Recall that the government repays the debt if and only if $\Delta V(A, \mathbf{k}_2, S) \geq 0$. I want to show that if agents follow the strategies characterized in steps 1 and 3, then the government finds it optimal to follow a monotone default strategy - i.e. default if and only if A is smaller than some A^{**} ; i.e., $\Delta V(A, \mathbf{k}_2, S) \geq 0$ for all $A \geq A^*$ and $\Delta V(A, \mathbf{k}_2, S) < 0$ for all $A < A^{**}$.⁴⁴

Now,

$$\begin{aligned} \Delta V(A, \mathbf{k}_2, S) &= \log\left(\frac{(1-\tau)e^A f(k_1) - \bar{k}_2}{Z(1-\tau)e^A f(k_1) - \bar{k}_2}\right) + \log\left(\frac{\tau e^A f(k_1) - B_1 + B_2}{Z\tau e^A f(k_1)}\right) \\ &\quad + \log\left(\frac{(1-\tau)e^A f(\bar{k}_2)}{Z(1-\tau)e^A f(\bar{k}_2)}\right) + \log\left(\frac{Y_2(A) - (1+r)B_2}{ZY_2(A)}\right) \end{aligned}$$

where

$$B_2 = \min\{B_2^u(A), S(A; x^*)\},$$

Y_2 is given by ?? and \bar{k}_2 is the median household's choice of capital.

In the limit, as $\varepsilon \rightarrow 0$, the first and third terms of expression for $\Delta V(A)$ do not depend on A and, therefore, we have:

$$\begin{aligned} \frac{\partial \Delta V(A)}{\partial A} &= \frac{(B_1 - B_2)}{\tau e^A f(k_1) - B_1 + B_2} + \frac{\frac{\partial B_2}{\partial A}}{\tau e^A f(k_1) - B_1 + B_2} \\ &\quad + \frac{(1+r)B_2(1+\alpha)}{\tau e^A F(\mathbf{k}_2) - (1+r)B_2} - \frac{(1+r)\frac{\partial B_2}{\partial A}}{\tau e^A F(\mathbf{k}_2) - (1+r)B_2}. \end{aligned}$$

If $B_2 = B_2^u$, then the terms including $\partial B_2/\partial A$ sum up to zero (by the first-order condition for the optimal choice of B_2^u). On the other hand, if $B_2 < B_2^u$, then $B_2 = S(A; x^*)$ and since $S(A; x^*)$ is an increasing function of A we have

$$\frac{\frac{\partial B_2}{\partial A}}{\tau e^A f(k_1) - B_1 + B_2} - \frac{(1+r)\frac{\partial B_2}{\partial A}}{\tau e^A F(\mathbf{k}_2) - (1+r)B_2} > 0.$$

Thus, it is enough to show that the sum of the remaining two terms is positive. But

$$\begin{aligned} &\frac{(B_1 - B_2)}{\tau e^A f(k_1) - B_1 + B_2} + \frac{(1+r)B_2(1+\alpha)}{\tau e^A F(\mathbf{k}_2) - (1+r)B_2} \\ &= \frac{(B_1 - B_2) [\tau e^A F(\mathbf{k}_2) - (1+r)B_2] + [\tau e^A f(k_1) - B_1 + B_2] (1+r)B_2(1+\alpha)}{[\tau e^A f(k_1) - B_1 + B_2] [\tau e^A F(\mathbf{k}_2) - (1+r)B_2]} \end{aligned}$$

and, hence, if we can show that the numerator is always positive, the claim will follow immediately. But the numerator is minimized if B_2 is equal to B_2^{\min} where

$$B_2^{\min} = \frac{1}{2}B_1 + \frac{\tau e^A F(\mathbf{k}_2) - (1+\alpha)(1+r)\tau e^A f(k_1)}{(4+2\alpha)(1+r)}$$

⁴⁴For convenience, in the rest of this section, I suppress the dependence of ΔV and \mathbf{k}_2 and S .

Since the second term is always positive, a sufficient condition under which $\partial \Delta V(A) / \partial A > 0$ is

$$\begin{aligned} B_1 - B_2^{\min} &= B_1 - \frac{1}{4+2\alpha} B_2^u - \frac{1+\alpha}{4+2\alpha} B_1 + \frac{\alpha}{(4+2\alpha)} \tau e^{A^*} f(k_1) \\ &> B_1 - \frac{1}{4+2\alpha} b - \frac{1+\alpha}{4+2\alpha} B_1 \\ &= \frac{3+\alpha}{4+2\alpha} B_1 - \frac{1}{4+2\alpha} b \end{aligned}$$

implying that as long as $b < (3+\alpha) B_1$, in response to lenders' and households' monotone strategies, the government finds it optimal to follow a monotone default strategy.

B.5 Step 5: Unique Equilibrium in Monotone Strategies

So far, I have established that a best response of any agent is monotone if all other agents follow monotone strategies. In this section, I show that for sufficiently high precision of private signal, p_x , there exists a unique equilibrium in monotone strategies. To establish the system of equations jointly determining lending threshold, x^* , and default threshold A^* and given by the following two equations,

$$\Delta V(A^*, k_2(A^*), S(A^*; x^*)) = 0 \quad (9)$$

$$\Delta \bar{u}(x^*; x^*, A^*) = 0 \quad (10)$$

has a unique solution.

To establish that, note that equation (9) implicitly defines x^* as a function of A^* . Therefore, it is enough to establish that $d\Delta V(A^*, k_2(A^*), S(A^*; x^*)) / dA^* > 0$ or

$$\frac{\tau e^{A^*} f(k_1) + \frac{\partial B_2(A^*)}{\partial A^*}}{\tau e^{A^*} f(k_1) - B_1 + B_2(A^*)} + \frac{(1+\alpha) \tau e^{A^*} f(k_2) - (1+r) \frac{\partial B_2(A^*)}{\partial A^*}}{\tau e^{A^*} f(k_2) - (1+r)} - (2+\alpha) > 0,$$

where⁴⁵

$$\frac{\partial B_2(A^*)}{\partial A^*} = \begin{cases} \frac{\partial B_2^u(A^*)}{\partial A^*} & \text{if } B_2^u(A^*) \leq S(A^*) \\ \frac{dS(A^*)}{dA^*} & \text{if } B_2^u(A^*) > S(A^*). \end{cases}$$

In section B.4, I showed that

$$\frac{\tau e^{A^*} f(k_1)}{\tau e^{A^*} f(k_1) - B_1 + B_2(A^*)} + \frac{(1+\alpha) \tau e^{A^*} f(k_2)}{\tau e^{A^*} f(k_2) - (1+r)} - (2+\alpha) > \frac{\frac{\alpha}{4+2\alpha} \tau e^{A^*} f(k_1)}{\tau e^{A^*} f(k_1) - B_1 + B_2^{unc}(A^*)} > 0$$

and, thus, if $\partial B_2(A^*) / \partial A^* = \partial B_2^u(A^*) / \partial A^*$, then the claim follows immediately. Hence, in what follows, I focus on the case when $\partial B_2(A^*) / \partial A^* = dS(A^*) / dA^*$.

Note that

$$\frac{dS(A^*)}{dA^*} > -b \frac{p}{p_x^{1/2}} \frac{1}{\sqrt{2\pi}},$$

⁴⁵The fact that $\partial B_2(A^*) / \partial A^* = \partial B_2^{unc}(A^*) / \partial A^*$ when $B_2(A^*) = S(A^*)$ follows from the fact that $B_2(A^* + \varepsilon) > S(A^* + \varepsilon)$ for all $\varepsilon > 0$ as shown below.

implying that

$$\lim_{p_x \rightarrow \infty} \frac{dS(A^*)}{dA^*} \geq 0,$$

and since

$$\frac{\tau e^{A^*} f(k_1)}{\tau e^{A^*} f(k_1) - B_1 + B_2(A^*)} + \frac{(1 + \alpha) \tau e^{A^*} f(k_2)}{\tau e^{A^*} f(k_2) - (1 + r)} - (2 + \alpha)$$

is bounded away from zero, it follows that there exists a precision level \bar{p}_x such that for all $p_x > \bar{p}_x$ there exists a unique equilibrium in monotone strategies.

Appendix C Policy Comparison

C.1 Equilibrium Conditions

Suppose the assumptions 4 to 6 hold. Then, the equilibrium default threshold, A^* , is determined by the following three conditions:

1. The government's default conditions, which can be expressed as:

$$\log\left(\frac{1 - \Lambda(0)}{Z - \Lambda(0)}\right) + \log\left(\frac{\tau e^{A^*} f(k_1) - B_1 + B_2}{Z \tau e^{A^*} f(k_1)}\right) + \log\left(\frac{1}{Z^{1+\alpha}}\right) + \log\left(\frac{\tau Y_2(A^*) - (1+r)B_2}{Z \tau Y_2(A^*)}\right) = 0$$

where $\Lambda(0)$ is constant defined in Section B.1 of this Appendix.

2. The households' optimal investment when the productivity is equal to A^*

$$k_2(\kappa) = (1 - \tau) e^{A^*} f(k_1) \Lambda(\kappa) \quad , \quad \kappa \in (-1, 1),$$

which implies that the aggregate output at $t = 2$ is given by:

$$Y_2(A^*) = \frac{1}{2} \int_{\kappa=-1}^1 e^{A^*} f(k_2(\kappa)) d\kappa;$$

3. Lenders' signal threshold, which under assumption 4 to 6 simplifies to:

$$x^* = A^*$$

Moreover, under assumptions 4 – 6, the supply of funds in the market at A^* simplifies to $S(A^*; x^*) = \frac{r}{1+r} b$. Therefore, a policy change has no “direct” effect on the lenders' problem and the supply of funds in the bond market.

It follows that applying the implicit function theorem to the equilibrium conditions we get

$$\frac{dA^*}{d\psi} = \frac{\left[\frac{\partial \Delta V}{\partial \psi}\right]^{\Delta V} + \left[\frac{\partial \Delta V}{\partial k_2}\right]^{\Delta V} \left[\frac{\partial k_2}{\partial \psi}\right]^H}{\left[\frac{\partial \Delta V}{\partial A^*}\right]^{\Delta V} - \left[\frac{\partial \Delta V}{\partial k_2}\right]^{\Delta V} \left[\frac{\partial k_2}{\partial A^*}\right]^H - \left[\frac{\partial \Delta V}{\partial x^*}\right]^{\Delta V} \left[\frac{\partial x^*}{\partial A^*}\right]^L}$$

or, dividing numerator and denominator by $\partial\Delta V/\partial A^*$

$$\frac{dA^*}{d\psi} = \frac{\left[\frac{\partial A^*}{\partial\psi}\right]^{\Delta V} + \left[\frac{\partial A^*}{\partial k_2}\right]^{\Delta V} \left[\frac{\partial k_2}{\partial\psi}\right]^H}{1 - \left[\frac{\partial A^*}{\partial k_2}\right]^{\Delta V} \left[\frac{\partial k_2}{\partial A^*}\right]^H - \left[\frac{\partial A^*}{\partial x^*}\right]^{\Delta V} \left[\frac{\partial x^*}{\partial A^*}\right]^L} \quad (11)$$

which is a simplified version of equation (2) in Section 4.1.

We see that the denominator is independent of policy change and is always positive. Therefore, to determine how a given policy affects probability of default we only need to focus on the numerator, which is equal to:

$$-\frac{\frac{\partial}{\partial\psi}\Delta V(A^*, k_2(\psi), x^*, \psi)}{\frac{\partial}{\partial A^*}\Delta V(A^*, k_2(\psi), x^*, \psi)},$$

where we take into account the direct effect of a policy change on the households problem but not the indirect effect on households' and lenders' problem through a change in A^* . It is easy to see that when we hold households and lenders beliefs about the default threshold constant then an increase in A^* always decreases the government default incentives (the argument is analogous to the argument in section B.4). Therefore, in what follows we only need to focus on the sign of $\partial\Delta V/\partial\psi$ where if $\partial\Delta V/\partial\psi > 0$ the policy will lead to a decrease in the probability of default and if $\partial\Delta V/\partial\psi < 0$ then it will increase probability of default.

C.2 Direct Effect of Each Policy

In this section I list $\partial\Delta V/\partial\psi$ for each of the policy considered in section 4. I focus on the case when the policy change is marginal.

Tax Increase

$$\frac{\partial\Delta V}{\partial\tau} = \frac{B_1 - B_2}{\tau[\tau e^{A^*} f(k_1) - B_1 + B_2]} + \frac{(1+r)B_2}{\tau[\tau Y_2(A^*) - (1+r)B_2]} - \frac{\frac{\alpha\tau}{1-\tau}(1+r)B_2}{\tau[\tau Y_2(A^*) - (1+r)B_2]}$$

where the first two terms capture the benefit of increase in taxes due to higher government's tax revenues ("concavity effect" and the differential increase in tax revenues) while the last term capture the negative effect of a tax increase has on the capital accumulation.

Spending Cuts Introducing spending cuts c results in the following default equation

$$\begin{aligned} & \log\left(\frac{1 - \Lambda(0)}{Z - \Lambda(0)}\right) + \log\left(\frac{\tau e^{A^*} f(k_1) - B_1 + B_2}{(1-c)Z\tau e^{A^*} f(k_1)}\right) \\ & + \log\left(\frac{1}{Z^{1+\alpha}}\right) + \log\left(\frac{\tau Y_2(A^*) - (1+r)B_2}{Z\tau Y_2(A^*)}\right) = 0 \end{aligned}$$

Differentiating with respect to c we get:

$$\lim_{c \rightarrow 0} \frac{\partial\Delta V}{\partial c} = \lim_{c \rightarrow 0} \frac{1}{1-c} = 1$$

Fiscal Stimulus Consider first fiscal stimulus financed with short-term debt. In that case the default equation becomes

$$\begin{aligned} \log\left(\frac{1 - \Lambda(0)}{Z - \Lambda(0)}\right) + \log\left(\frac{\tau e^A f(k_1(1+s)) - B_1 + B_2 - (1+r^{ST})sk_1}{Z\tau e^A f(k_1(1+s))}\right) \\ + \log\left(\frac{1}{Z}\right) + \log\left(\frac{\tau Y_2(A^*, s) - (1+r)B_2}{Z\tau Y_2(A^*, s)}\right) = 0, \end{aligned}$$

households optimal investment is given by

$$k_2(\kappa, s) = (1 - \tau) e^{A^*} f(k_1(1+s)) \Lambda(\kappa) \quad , \quad \kappa \in (-1, 1)$$

and

$$Y_2(A^*, s) = \frac{1}{2} \int_{\kappa=-1}^1 e^{A^*} f(k_2(\kappa, s)) d\kappa$$

Then the derivative of default equation with respect to s as $s \rightarrow 0$ is given by

$$\frac{\alpha(B_1 - B_2)}{\tau e^A f(k_1) - B_1 + B_2} + \frac{\alpha^2(1+r)B_2}{\tau Y_2(A^*, 0) - (1+r)B_2} - \frac{(1+r^{ST})k_1}{\tau e^A f(k_1) - B_1 + B_2}$$

where the first two terms capture the benefits of the stimulus (“concavity effect” and the differential increase in tax revenues) while the last term captures its costs.

For the case of fiscal stimulus finances with long term debt, the default equation becomes

$$\begin{aligned} \log\left(\frac{1 - \Lambda(0)}{Z - \Lambda(0)}\right) + \log\left(\frac{\tau e^{A^*} f(k_1(1+s)) - B_1 + B_2}{Z\tau e^{A^*} f(k_1(1+s))}\right) \\ + \log\left(\frac{1}{Z}\right) + \log\left(\frac{\tau Y_2(A^*, s) - (1+r)B_2 - (1+r^{LT})sk_1}{Z\tau Y_2(A^*, s)}\right) = 0 \end{aligned}$$

Again, taking the derivative of the above equation with respect to s and taking the limit as $s \rightarrow 0$ we get

$$\frac{\alpha(B_1 - B_2)}{\tau e^A f(k_1) - B_1 + B_2} + \frac{\alpha^2(1+r)B_2}{\tau Y_2 - (1+r)B_2} - \frac{(1+r^{LT})k_1}{\tau Y_2(A^*, 0) - (1+r)B_2}$$

C.3 Deriving Results in Section 4

In this section I provide proves for the results stated in section 4.3.⁴⁶

Proof of Lemma 3. Consider $\partial\Delta V/\partial\tau$ and note first that Assumptions 4 and 5 imply that $B_2 \leq \frac{\tau}{1+\tau}b \leq B_1$ so that the first term is always non-negative and hence we can focus on the remaining two terms. But if $1 > \alpha\tau/(1-\tau)$ then second term is larger than the third term and the result follows immediately. ■

⁴⁶To simplify notation, in what follows I will suppress the dependence of Y_2 on A^* and s and simply write Y_2 instead of $Y_2(A^*, s)$ whenever this does not lead to any confusion.

Proof of Lemma 4. Consider $\partial\Delta V/\partial s$ in the case of short-term stimulus and note that since $B_2 \leq B_2^u$ then $(1+r)(\tau e^A f(k_1) - B_1 + B_2) < \tau Y_2 - (1+r)B_2$ and hence the sum of the first two terms is decreasing in B_2 implying that

$$\partial\Delta V/\partial s \leq \frac{\alpha B_1 - (1+r^{ST})k_1}{\tau e^A f(k_1) - B_1 + B_2}$$

It follows that a sufficient condition for fiscal stimulus financed with short-term debt to increase probability of default is $(1+r^{ST})k_1 > \alpha B_1$.

The condition for fiscal stimulus financed with long-term debt is derived analogously. Simply denote the equilibrium government spending in repayment at $t = 1$ and $t = 2$ by $g_1^R(A^*)$ and $g_2^R(A^*)$, respectively, and follow the same argument as above. ■

Proof of Lemma 5. Immediate from expression for $\partial\Delta V/\partial c$. ■

Appendix D Auxiliary Results

D.1 Condition under which desired borrowing is positive

In this section I derive a sufficient condition under which the government's desired borrowing is positive for all $A \geq \underline{A}^R(0)$.

Recall, that the government desired borrowing is given by

$$B_2^u = \begin{cases} B_2^{bor} & \text{if } B_2^{bor} \geq 0 \\ 0 & \text{if } B_2^{bor} < 0 \text{ and } B_2^{len} \geq 0 \\ B_2^{len} & \text{if } B_2^{len} < 0 \end{cases}$$

It follows that, the government desired borrowing is positive as long as

$$B_2^{len} > 0 \text{ for all } A \geq \underline{A}^R(0)$$

or

$$\frac{B_1 + \tau e^A f(k_2) - \tau e^A f(k_1)}{2} > 0$$

Rearranging and using the definition of $\underline{A}^R(0)$ we get

$$f(k_2(\underline{A}^R(0))) > Z^3(Z(1+\alpha) - \alpha)f(k_1)$$

A necessary condition for the government not to default is that the government consumption at $t = 1$ is strictly positive, i.e.

$$\tau e^{\underline{A}^R(0)} f(k_1) - B_1 + B_2^u > 0$$

If $B_2^u < 0$ then it has to be the case that

$$\tau e^{\underline{A}^R(0)} f(k_1) > B_1$$

or

$$e^{\underline{A}^R(0)} > \frac{B_1}{\tau f(k_1)}$$

Therefore, if I can find a condition under which $f(k_2(A)) > Z^3(Z(1+\alpha) - \alpha)f(k_1)$ at $A = \log(B_1/\tau f(k_1))$ it would follow that the B_2^u cannot be negative. But this is the case if

$$\begin{aligned} f\left(k_2\left(\frac{B_1}{\tau f(k_1)}\right)\right) &> Z^3(Z(1+\alpha) - \alpha)f(k_1) \\ \frac{(1-\tau)\frac{B_1}{\tau f(k_1)}f(k_1)}{1 + \frac{1}{\alpha}} &> [Z^3(Z(1+\alpha) - \alpha)f(k_1)]^{\frac{1}{\alpha}} \\ B_1 &> \frac{\tau(1+\alpha)}{\alpha(1-\tau)} [Z^3(Z(1+\alpha) - \alpha)]^{\frac{1}{\alpha}} k_1 \end{aligned}$$

Thus, we conclude that

$$B_1 > \frac{\tau(1+\alpha)}{\alpha(1-\tau)} [Z^3(Z(1+\alpha) - \alpha)]^{\frac{1}{\alpha}} k_1 \text{ implies } B_2^u(\underline{A}^R(0)) > 0$$

Since under Assumption 3 we have $\frac{\partial B_2^u}{\partial A} \geq 0$ it follows that B_2^u is positive for all $A \geq \underline{A}^R(0)$.

D.2 Fragility Region: Behavior of $\Delta V(A; r)$

In this section I show that the difference between value of repaying and the value of defaulting is strictly increasing in A when $B_2 \in \{0, B_2^u\}$ and if all the households expect or do not expect default. This result is invoked several times when deriving fragility region. Since the argument for the case when households expect default is analogous to the case when households do not expect default I consider only the latter case.

Suppose that households do not expect default. Then

$$k_2^i = k_2^R = \frac{(1-\tau)e^A f(k_1)}{1 + \frac{1}{\alpha}} \text{ for all } i \in [0, 1]$$

The value of repaying the debt at $t = 1$ is given by

$$\begin{aligned} V_1^R(A, r) &= \log((1-\tau)e^A f(k_1) - k_2) + \log(\tau e^A f(k_1) - B_1 + B_2) \\ &\quad + \log((1-\tau)e^A f(k_2)) + \log(\tau e^A f(k_2) - (1+r)B_2) \end{aligned}$$

while the value of defaulting is given by

$$V_1^D(A) = \log(Z(1-\tau)e^A f(k_1) - k_2) + \log(Z\tau e^A f(k_1)) + \log(Z(1-\tau)e^A f(k_2)) + \log(Z\tau e^A f(k_2))$$

Let

$$\Delta V(A, r) = V_1^R(A, r) - V_1^D(A)$$

Taking derivative of $V_1^R(A, r)$ with respect to A I get

$$1 + \frac{\tau e^A f(k_1)}{\tau e^A f(k_1) - B_1 + B_2} + (1+\alpha) + \frac{(1+\alpha)\tau e^A f(k_2)}{\tau e^A f(k_2) - (1+r)B_2}$$

while taking derivative of $V_1^D(A)$ with respect to A yields

$$1 + 1 + (1+\alpha) + (1+\alpha)$$

and, therefore

$$\frac{\partial \Delta V(A, r)}{\partial A} = \frac{\tau e^A f(k_1)}{\tau e^A f(k_1) - B_1 + B_2} + \frac{(1 + \alpha) \tau e^A f(k_2)}{\tau e^A f(k_2) - (1 + r) B_2} - (2 + \alpha)$$

First, set $B_2 = 0$. Then

$$\frac{\partial \Delta V(A, r)}{\partial A} = \frac{\tau e^A f(k_1)}{\tau e^A f(k_1) - B_1} + \frac{(1 + \alpha) \tau e^A f(k_2)}{\tau e^A f(k_2)} - (2 + \alpha) > 0$$

whenever $\tau e^A f(k_1) - B_1 > 0$.

Now, set $B_2 = B_2^u$. Then

$$\begin{aligned} \frac{\tau e^A f(k_1)}{\tau e^A f(k_1) - B_1 + B_2} &= \frac{2(1 + r) \tau e^A f(k_1)}{(1 + r) \tau e^A f(k_1) + \tau e^A f(k_2) - (1 + r) B_1} \\ \frac{(1 + \alpha) \tau e^A f(k_2)}{\tau e^A f(k_2) - (1 + r) B_2} &= \frac{2(1 + \alpha) \tau e^A f(k_2)}{(1 + r) \tau e^A f(k_1) + \tau e^A f(k_2) - (1 + r) B_1} \end{aligned}$$

so that

$$\begin{aligned} \frac{\partial \Delta V(A, r)}{\partial A} &= \frac{2(1 + r) \tau e^A f(k_1) + 2(1 + \alpha) \tau e^A f(k_2) - (2 + \alpha) [(1 + r) \tau e^A f(k_1) + \tau e^A f(k_2) - (1 + r) B_1]}{(1 + r) \tau e^A f(k_1) + \tau e^A f(k_2) - (1 + r) B_1} \\ &= \frac{\alpha \tau e^A f(k_2) - (1 + r) \tau e^A f(k_1) + (2 + \alpha) (1 + r) B_1}{(1 + r) \tau e^A f(k_1) + \tau e^A f(k_2) - (1 + r) B_1} \\ &= \frac{2(1 + r) B_1 + \alpha B_2^{unc}}{(1 + r) \tau e^A f(k_1) + \tau e^A f(k_2) - (1 + r) B_1} > 0 \end{aligned}$$

since $B_2^u > 0$ for all $A \geq \underline{A}^R(0)$.

Consider now derivative of $\Delta V(A, r)$ with respect to r .

$$\frac{\partial \Delta V(A, r)}{\partial r} = \frac{\partial V_1^R(A, r)}{\partial r} = -\frac{B_2}{\tau e^A f(k_2) - (1 + r) B_2} \leq 0$$

with the strict inequality as long as $B_2 > 0$.⁴⁷

D.3 Properties of B_2^u

The unconstrained optimal borrowing by the government is given by

$$B_2^u(A, r) = \frac{(1 + r) B_1 + \tau e^A f(k_2) - (1 + r) \tau e^A f(k_1)}{2(1 + r)}$$

Derivative of B_2^u with respect to r is given by

$$\frac{\partial B_2^u}{\partial r} = -\frac{\tau e^A f(k_2)}{2(1 + r)^2} < 0$$

⁴⁷Recall that if $B_2 < 0$ then $r = 0$ since the government can only lend at the risk-free rate. Therefore, in that case $\partial \Delta V(A, r) / \partial r = 0$.

while derivative with respect to A is given by

$$\frac{\partial B_2^u}{\partial A} = \frac{(1 + \alpha) \tau e^A f(k_2) - (1 + r) \tau e^A f(k_1)}{(1 + r)}$$

and it can be positive or negative, depending on the level of productivity A , the interest rate r , and other parameters of the model.

Note that

$$\frac{\partial^2 B_2^u}{\partial A^2} = \frac{(1 + \alpha)^2 \tau e^A f(k_2) - (1 + r) \tau e^A f(k_1)}{(1 + r)}$$

and, therefore,

$$\frac{\partial B_2^u}{\partial A} > 0 \implies \frac{\partial^2 B_2^u}{\partial A^2} > 0$$

It follows that if $\partial B_2^u / \partial A > 0$ at A' then $\partial B_2^u / \partial A > 0$ for all $A > A'$.

Finally I provide sufficient conditions under which $\partial B_2^u / \partial A > 0$ for all $A > \underline{A}^R(0)$.

Lemma 9 *Suppose that $(1 + \alpha) - \frac{1}{Z^3(Z(1+\alpha)-\alpha)} \geq 0$. Then $\partial B_2^u / \partial A > 0$ for all $A > \underline{A}^R(0)$.*

Proof. Fix $r \leq r^R$. Then we know that $A^* \geq \underline{A}^R(r)$. Substituting $\underline{A}^R(r)$ into the expression for the desired borrowing yields

$$B_2^u(\underline{A}^R(r); r) = \frac{2\tau e^{\underline{A}^R(r)} f(k_2) - 2\tau e^{\underline{A}^R(r)} \sqrt{(1+r) f(k_1) f(k_2) (Z^3(1+\alpha) - \alpha)}}{2(1+r)}$$

We know that for all $r \leq r^R$ we have $B_2^u(\underline{A}^R(r); r) \geq 0$ and therefore, by rearranging above expression for $B_2^u(\underline{A}^R(r); r)$ we get

$$(1+r) \leq \frac{1}{Z^3(Z(1+\alpha) - \alpha)} \frac{f(k_2(\underline{A}^R(r)))}{f(k_1)}$$

implying that for all $r \in [0, r^R]$ we have

$$(1+r) \leq \frac{1}{Z^3(Z(1+\alpha) - \alpha)} \frac{f(k_2(\underline{A}^R(r)))}{f(k_1)}$$

Now consider $\frac{\partial B_2^u(A, r)}{\partial A}$:

$$\frac{\partial B_2^u(A, r)}{\partial A} = \frac{(1 + \alpha) \tau Y_2 - (1 + r) \tau Y_1}{2(1 + r)}$$

Fix r and consider $\frac{\partial B_2^u(A, r)}{\partial A} \Big|_{\underline{A}^R(r)}$:

$$\begin{aligned} \frac{\partial B_2^u(A, r)}{\partial A} \Big|_{\underline{A}^R(r)} &= \frac{(1 + \alpha) \tau e^{\underline{A}^R(r)} f(k_2(\underline{A}^R(r))) - (1 + r) \tau e^{\underline{A}^R(r)} f(k_1)}{2(1 + r)} \\ &\geq \frac{(1 + \alpha) \tau e^{\underline{A}^R(r)} f(k_2(\underline{A}^R(r))) - \frac{1}{Z^3(Z(1+\alpha)-\alpha)} \tau e^{\underline{A}^R(r)} f(k_2(\underline{A}^R(r)))}{2(1 + r)} \end{aligned}$$

and hence

$$\frac{\partial B_2^u(A, r)}{\partial A} \Big|_{\underline{A}^R(r)} \geq 0 \text{ if } (1 + \alpha) \geq \frac{1}{Z^3(Z(1 + \alpha) - \alpha)}$$

Finally, note that if $\frac{\partial B_2^u(A, r)}{\partial A} \geq 0$ at $A = \underline{A}^R(r)$ then $\frac{\partial B_2^u(A, r)}{\partial A} > 0$ for all $A > \underline{A}^R(r)$. This implies that the condition

$$(1 + \alpha) \geq \frac{1}{Z^3(Z(1 + \alpha) - \alpha)}$$

is sufficient for the desired borrowing to be strictly increasing for all $A > \underline{A}^R(0)$. ■

The next result follows immediately from the above lemma and preceding it discussion.

Corollary 10 *If $(1 + \alpha) \geq \frac{1}{Z^3(Z(1 + \alpha) - \alpha)}$ then $B_2^u(A)$ is a convex in A for $A \in [\underline{A}^R(0), \infty)$.*

D.4 Behavior of of $\frac{B_2^u(A)}{S(A)}$

In section D.1 we saw that if $(1 + \alpha) \geq 1 / (Z^3(Z(1 + \alpha) - \alpha))$ then $B_2^u(A)$ is increasing and convex in A for $A \in [\underline{A}^R(0), \infty)$. The supply function $S(A; x^*)$ is given by

$$S(A; x^*) = b \left[1 - \Phi \left(\frac{x^* - A}{p_x^{-1/2}} \right) \right] = bp_x^{1/2} \Phi \left(\frac{A - x^*}{p_x^{-1/2}} \right)$$

and hence it is also an increasing function of A . Moreover, it is easy to show that $S(A; x^*)$ is convex in A for all $A \in (-\infty, x^*]$ and concave in A for all $A \in [x^*, \infty)$.

I am interested in determining how many time $S(A; x^*)$ intersects $B_2^u(A)$. Note that depending on $\{b, x^*, A^*, p_x^{1,2}\}$ $S(A^*; x^*)$ can be smaller or greater than $B_2^u(A^*)$. I consider first the case when $B_2^u(A^*) \geq S(A^*; x^*)$. It is easy to see that in this case either (1) $B_2^u(A) > S(A; x^*)$ for all $A > A^*$, or (2) $S(A; x^*)$ intersects $B_2^u(A)$ twice. So consider the case when $B_2^u(A^*) < S(A^*; x^*)$. Since $B_2^u(A^*)$ is increasing and convex in A and $S(A^*; x^*)$ is increasing and concave in $A \in [x^*, \infty)$, it follows that these two functions have two intersect each other at least once. Moreover, if x^* is very high, and p_x is large so that $\partial S(A; x^*) / \partial A$ is very steep around $A = x^*$ then it is possible that $S(A; x^*)$ intersects $B_2^u(A)$ three times.

In light of the above discussion there are four possible cases to consider when analyzing payoff indifference condition:

1. $B_2^u(A) > S(A; x^*)$ for all $A > A^*$;
2. $B_2^u(A^*) > S(A^*; x^*)$ and $S(A; x^*)$ intersects $B_2^u(A)$ twice.- call these intersection points A_1 and A_2 ;
3. $B_2^u(A^*) < S(A^*; x^*)$ and $S(A; x^*)$ intersects $B_2^u(A)$ only once - call this intersection point A_1 ;
4. $B_2^u(A^*) < S(A^*; x^*)$ and $S(A; x^*)$ intersects $B_2^u(A)$ three times - call these intersection points A_1, A_2 and A_3 .

D.5 Computing $\frac{\partial x^*}{\partial A}$

In this section I prove the following claim:

Lemma 11 *Derivative of x^* with respect to A^* is bounded from above by $\frac{p_x+p}{p_x}$, that is*

$$\frac{\partial x^*}{\partial A^*} \leq \frac{p_x + p}{p_x}$$

Below we will need an expression for the derivative of the unique lending threshold, x^* , with respect to default threshold, A^* . To compute this derivative start with the indifference condition

$$\int_{A^*}^{\infty} \left(1 + r \min \left\{ 1, \frac{B_2^u(A)}{S(A)} \right\} \right) (p_x + p)^{1/2} \phi \left(\frac{A - \frac{p_x x^* + p A - 1}{p_x + p}}{(p_x + p)^{-1/2}} \right) dA - 1 = 0$$

Then

$$\frac{\partial x^*}{\partial A^*} = \frac{\left(1 + r \min \left\{ 1, \frac{B_2^u(A^*)}{S(A^*)} \right\} \right) (p_x + p)^{1/2} \phi \left(\frac{A^* - \frac{p_x x^* + p A^* - 1}{p_x + p}}{(p_x + p)^{-1/2}} \right)}{\frac{\partial}{\partial x^*} \left[\int_{A^*}^{\infty} \left(1 + r \min \left\{ 1, \frac{B_2^u(A)}{S(A)} \right\} \right) (p_x + p)^{1/2} \phi \left(\frac{A - \frac{p_x x^* + p A - 1}{p_x + p}}{(p_x + p)^{-1/2}} \right) dA \right]}$$

To compute denominator we need to establish behavior of $B_2^u(A)/S(A)$. Given that $B_2^u(A)$ is increasing and convex for $A \geq \underline{A}^r(0)$ and that $A^* > \underline{A}^r(0)$ it can be show that there are only possible four cases: (1) $B_2^u(A)/S(A) > 1$ for all $A \geq A^*$, (2) $B_2^u(A^*)/S(A^*) < 1$ and $B_2^u(A)/S(A) > 1$ for all $A > A_1$ and smaller than 1 otherwise, (3) $B_2^u(A^*)/S(A^*) < 1$ and $B_2^u(A)/S(A)$ intersect 1 three times (at A_1, A_2 and A_3), and finally (4) $B_2^u(A^*)/S(A^*) > 1$ and $B_2^u(A)/S(A)$ intersects 1 twice, at A_1 and A_2 , so that $B_2^u(A)/S(A) > 1$ for all $A \notin (A_1, A_2)$. In the first case it is immediate that $\frac{\partial x^*}{\partial A^*} = \frac{p_x+p}{p_x}$. I show now that in the remaining cases we have $\frac{\partial x^*}{\partial A^*} < \frac{p_x+p}{p_x}$.

References

- AGUIARA, M., AND G. GOPINATH (2006): “Defaultable debt, interest rates and the current account,” *Journal of International Economics*, 69(1), 64–83.
- ALESINA, A., A. PRATI, AND G. TABELLINI (1990): “Public Confidence and Debt Management: A Mode and a Case Study of Italy,” in *Public Debt Management: Theory and History*, ed. by R. Dornbusch, and M. Draghi, p. 94. Cambridge University Press.
- ANGELETOS, G.-M., C. HELLWIG, AND A. PAVAN (2006): “Signalling in a Global Game: Coordination and Policy Traps,” *Journal of Political Economy*, 114(3), 452–484.
- ANGELETOS, G.-M., AND A. PAVAN (forthcoming): “Selection-Free Predictions in Global Games with Endogenous Information and Multiple Equilibria,” *Theoretical Economics*.

- ARELLANO, C. (2008): “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 98(3), 690–712.
- ARPAIA, A., E. PEREZ, AND K. PICHELMANN (2009): “Understanding Labour Share Dynamics in Europe,” European Economy Economic Papers 379, European Commission, Brussels.
- ATHEY, S. (1996): “Comparative Statics under Uncertainty: Single Crossing Properties and Log-Supermodularity,” mimeo MIT.
- (2001): “Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information,” *Econometrica*, 69(4), 861–889.
- AUERBACH, A. J., AND Y. GORODNICHENKO (2011): “Fiscal Multipliers in Recession and Expansion,” mimeo University of California, Berkeley.
- (2012): “Measuring the Output Response to Fiscal Policy,” *American Economic Journal: Economic Policy*, 4(2), 1–27.
- CALVO, G. (1988): “Servicing the Public Debt: The Role of Expectations,” *The American Economic Review*, 78(4), 647–661.
- CARLSON, H., AND E. V. DAMME (1993): “Global Games and Equilibrium Selection,” *Econometrica*, 61(5), 989–1018.
- CHRISTIANO, L., M. EICHENBAUM, AND S. REBELO (2011): “When Is the Government Spending Multiplier Large,” *Journal of Political Economy*, 119(1), 78–121.
- COLE, H., AND T. KEHOE (1996): “A self-fulfilling model of Mexico’s 1994-1995 debt crisis,” *Journal of International Economics*, 41(3-4), 309–330.
- (2000): “Self-Fulfilling Debt Crisis,” *The Review of Economic Studies*, 67(1), 91–116.
- CONESA, J. C., AND T. KEHOE (2012): “Gambling for Redemption and Self-Fulfilling Debt Crises,” *Federal Reserve Bank of Minneapolis Staff Report*.
- CORSETTI, G. (ed.) (2012): *Austerity: Too Much of a Good Thing?* voxeu.org.
- CORSETTI, G., AND L. DEDOLA (2011): “Fiscal Crisis, Confidence and Default: A Bare-bone Model with Lessons for the Euro Area,” mimeo Cambridge University.
- CORSETTI, G., B. GUIMARAES, AND N. ROUBINI (2006): “International lending of last resort and moral hazard: A model of IMF’s catalytic finance,” *Journal of Monetary Economics*, 53(3), 441–471.
- CORSETTI, G., K. KUESTER, A. MEIER, AND G. J. MÜLLER (2013): “Sovereign Risk, Fiscal Policy, and Macroeconomic Stability,” *Economic Journal*, 123(566), F99–F132.
- DE GRAUWE, P., AND Y. JI (2013): “Self-Fulfilling Crises in the Eurozone: An Empirical Test,” *Journal of International Money and Finance*, 34, 15–36.
- DETRAGIACHE (1996): “Rational Liquidity Crises in the Sovereign Debt Market: In Search of a Theory,” *IMF Staff Papers*, 43(3), 545–570.

- EATON, J., AND R. FERNANDEZ (1995): “Sovereign Debt,” in *Handbook Of International Economics*, ed. by G. M. Grossman, and K. S. Rogoff, p. 2031. North-Holland.
- EATON, J., AND M. GERSOVITZ (1981): “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *Review of Economic Studies*, 48(2), 289–309.
- EGGERTSSON, G., A. FERRERO, AND A. RAFFO (2012): “Can Structural Reforms Help Europe?,” working paper.
- GIAVAZZI, F., AND M. PAGANO (1990): “Confidence Crisis and Public Debt Management,” in *Public Debt Management: Theory and History*, ed. by R. Dornbusch, and M. Draghi, p. 94. Cambridge University Press.
- HATCHONDO, J. C., AND L. MARTINEZ (2009): “Long-duration bonds and sovereign defaults,” *Journal of International Economics*, 79(1), 117–125.
- ILZETZKIA, E., E. G. MENDOZA, AND C. A. VEGH (2013): “How big (small?) are fiscal multipliers?,” *Journal of Monetary Economics*, 60(2), 239–254.
- JONES, C. (2003): “Growth, Capital Shares and a New Perspective on Production Function,” mimeo University of California, Berkeley.
- KARP, L., I. H. LEE, AND R. MASON (2007): “A global game with strategic substitutes and complements,” *Games and Economic Behavior*, 60(3), 155–175.
- KRUGMAN, P. (2011): “A Self-Fulfilling Euro Crisis?,” VoxEU.org, 18 August.
- LUCAS, R. (2003): “Macroeconomic Priorities,” *American Economic Review*, 93(1), 1–14.
- MENDOZA, E., AND V. YUE (2012): “A General Equilibrium Model of Sovereign Default and Business Cycles,” *The Quarterly Journal of Economics*, 127(2), 889–946.
- MORRIS, S., AND H. SHIN (1998): “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks,” *The American Economic Review*, 88(3), 587–597.
- (2000): “Rethinking Multiple Equilibria in Macroeconomic Modelling,” in *NBER Macroeconomics Annual*, ed. by B. S. Bernanke, and K. Rogoff, p. 139. MIT Press.
- (2003): “Global games: Theory and applications,” in *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress (Econometric Society Monographs) (Volume 1)*, ed. by M. Dewatripont, L. P. Hansen, and S. Turnovsky, p. 56. Cambridge University Press.
- (2006): “Catalytic finance: When does it work?,” *Journal of International Economics*, 70(1), 161–177.
- PANIZZA, U., F. STURZENEGGER, AND J. ZETTELMEYER (2009): “The Economics and Law of Sovereign Debt and Default,” *Journal of Economic Literature*, 47(3), 1–47.
- SACHS, J. (1984): “Theoretical Issues in International Borrowing,” in *Princeton Studies in International Finance*, pp. 94–118. Princeton University.

ZWART, S. (2007): “The Mixed Blessing of IMF Intervention: Signalling vs Liquidity Support,”
Journal of Financial Stability, 3(2), 149–174.