The Global Component of Inflation Volatility

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NBP Workshop on Forecasting

Warsaw, November 26-27, 2018

1 Introduction

2 The Model

3 Estimation & Data

4 Results

5 Forecasting Evaluation

6 Conclusions

How much global is the Inflation process?

- Rogoff (2003), Rogoff (2006), Carney (2017), Miles, Paniza, Reis and Ubide (2017): **Globalisation, Inflation and Central Banks.**
- Borio and Filardo (2007), Bianchi and Civelli (2015) and Auer, Borio and Filardo (2017): **effects of global economic conditions on inflation.**

Borio and Filardo (2007)

"*...proxies for global economic slack add considerable explanatory power to traditional benchmark inflation rate equations[...]. Moreover,* ***the role of such global factors has been growing over time, especially since the 1990s.***"

- Ciccarelli and Mojon (2010), Mikolajun and Lodge (2016): **a substantial amount of variation** in a large set of national inflation rates **is explained by global factors** that capture the most persistent component (slow moving trends).

Inflation and Stochastic Volatility

- A large literature on volatilities originated by Engle (1982).
- **Stock and Watson (2007) introduces stochastic volatility in modeling US inflation rate**, emphasizing the importance and good out of sample properties of this feature.

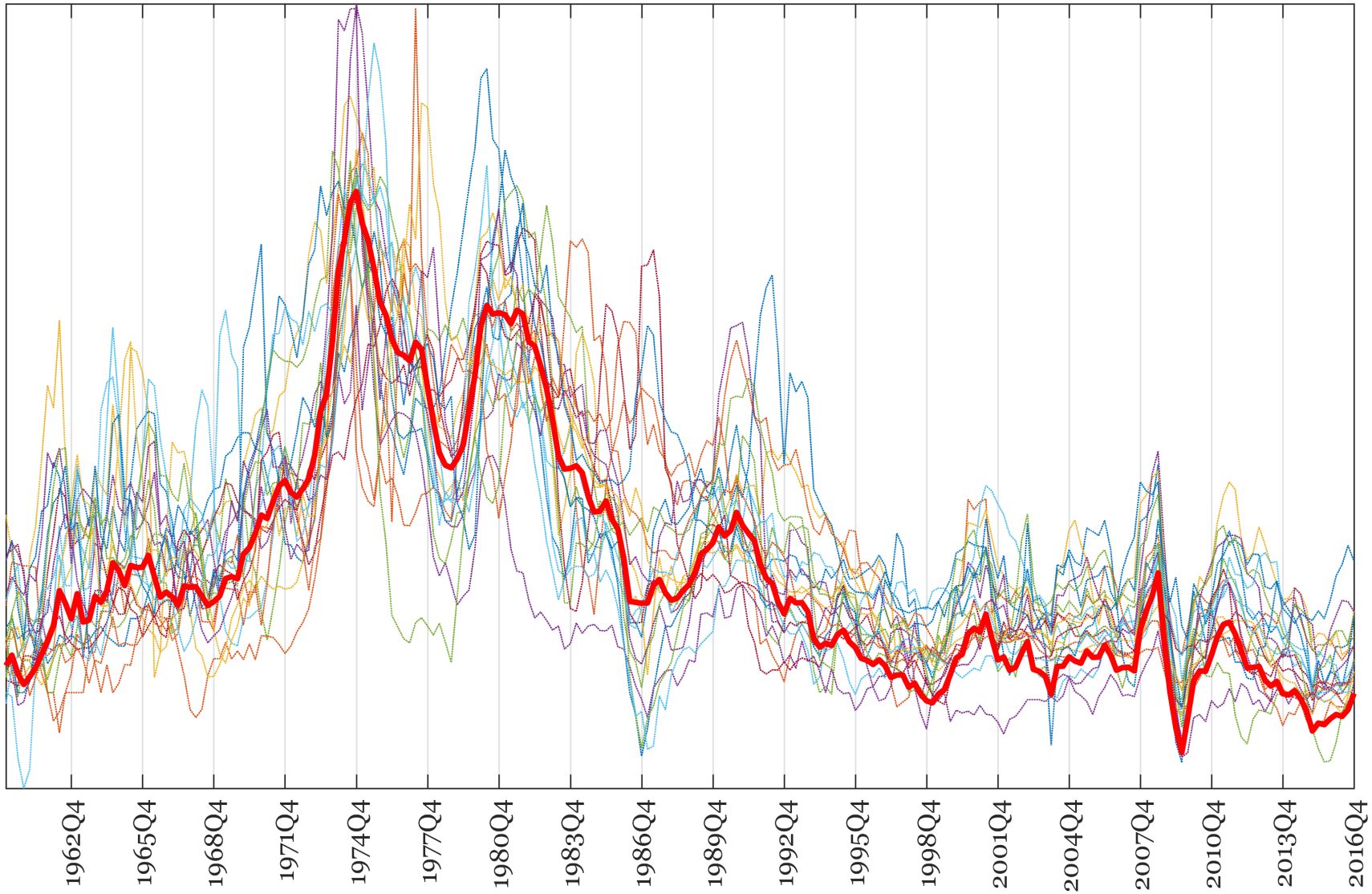
Stock and Watson (2007)

"*Since the mid-1950s, there have been large changes in the variance of the permanent disturbance[...]. But the inflation process has changed in the past, it could change again [...].* ***We therefore consider it imprudent to adopt a fixed-parameter inflation forecasting model either as a benchmark or for real-time forecasting.***"

- Mumtaz and Surico (2008) use a large dynamic factor model that incorporates time varying coefficients and stochastic volatility to model a panel of inflation rates. They document a prominent role of country-specific components, and that SV drop has not been synchronized across nations.

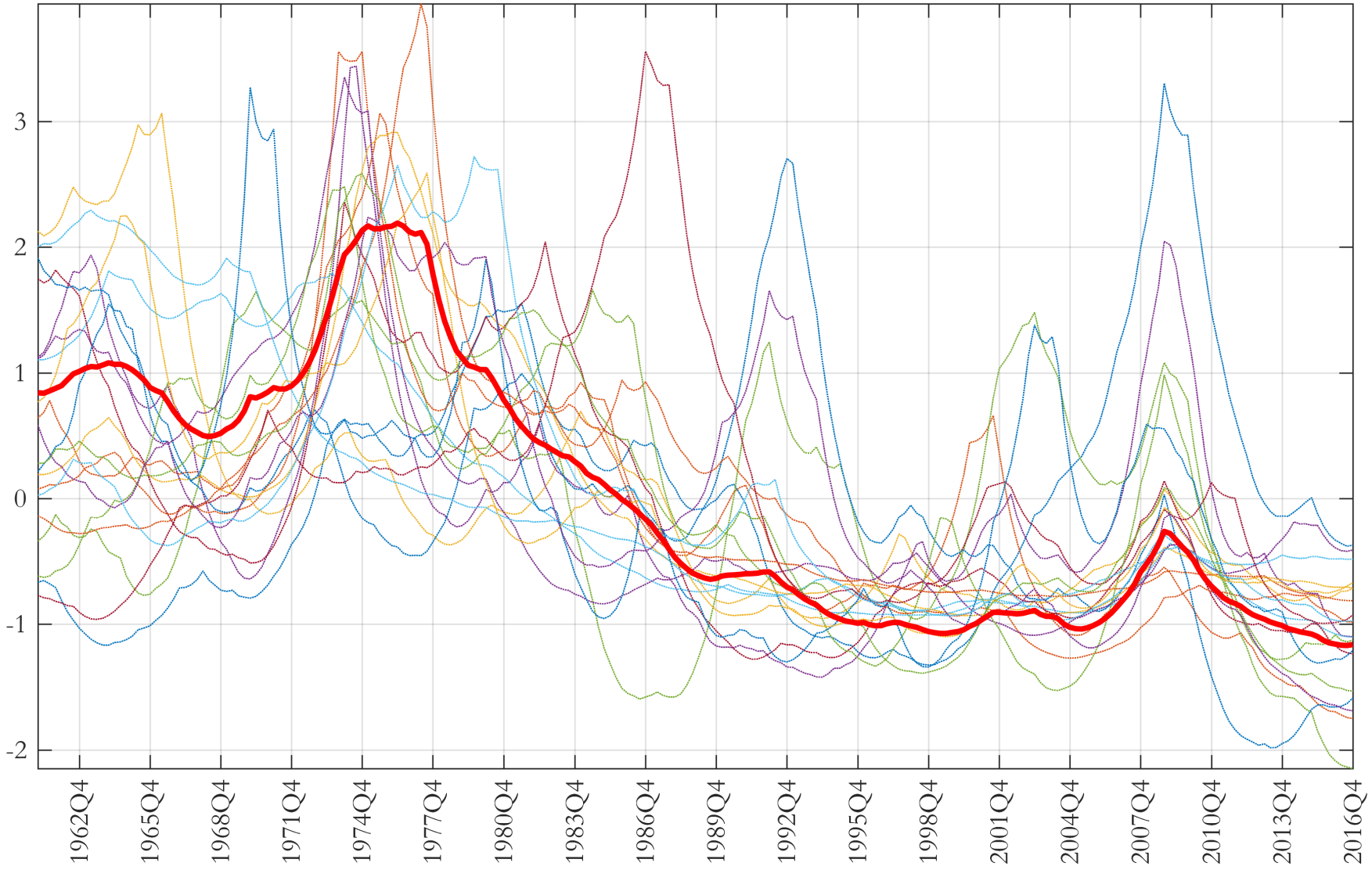
CPI inflation rates and PCA

Data for 20 OECD countries. The first PC explains almost 75%



CPI inflation rates SVs from univariate AR-SV

Data for 20 OECD countries. The first PC explains almost 60%



What this paper does

- We include **Stochastic Volatility in modeling multi-country inflation rates** (20 OECD countries since the 1960s).
- We investigate cross-country commonality **not only in inflation levels, but also in inflation volatilities**.
- We build a Multivariate Autoregressive Index model with Autoregressive components and Stochastic volatility (**MAI-AR-SV**), and derive a fully-fledged **Bayesian MCMC algorithm**.
- We decompose both levels and volatilities so to disentangle **contributions of a single global component** and the idiosyncratic components.
- We run a **point and density forecasting evaluation** to test the out of sample performance of the model.

Main Results

- The estimated **global factor explains roughly 70%** of the variability **of CPI inflation levels**.
- Significantly time-varying **global inflation volatility** since the 1960s.
- **Important evidence of commonality in volatilities**, increased in the last two decades. **A large fraction** of headline CPI inflation volatility can be **attributed to the global factor**.
- The same decompositions conducted on **Non-Food&Non-Energy inflation** show a **smaller and more stable** degree of commonality.
- Point and density forecasting evaluation shows that the MAI-AR-SV model has **very good out of sample performance** for inflation rates.

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The MAI-AR-SV model

Introducing SV in the Multivariate Autoregressive Index with AR components

- Reinsel (1983), Carriero Kapetanios and Marcellino (JoE, 2016)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} \underbrace{A_\ell \cdot B_0}_{\downarrow \; \underbrace{A_\ell}_{n\times r} \cdot \underbrace{B_0}_{r\times n}} \cdot y_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + u_t$$

- Rank reduction from n to r

$$F_t \equiv \underbrace{B_0}_{r\times n} \cdot y_t$$

- F_t, i.e. the "Index", will be interpreted as Global Inflation ($r = 1$)

The mai-AR-sv model

Introducing SV in the Multivariate Autoregressive Index with AR components

- Cubadda and Guardabascio (2017)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot y_{t-\ell} + \sum_{\ell=1}^{q} \underbrace{\Gamma_\ell}_{\downarrow} \cdot y_{t-\ell} + u_t$$

$$\underbrace{\Gamma_\ell}_{n\times n} = \begin{bmatrix} \gamma_{1,\ell} & 0 & \dots & 0 \\ 0 & \gamma_{2,\ell} & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_{n,\ell} \end{bmatrix}$$

- q Univariate AutoRegressive Coefficients (q potentially larger than p) in diagonal Γ_ℓ

The mai-ar-SV model

Introducing SV in the Multivariate Autoregressive Index with AR components

- Cogley and Sargent (2005) and Primiceri (2005)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot y_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + \underbrace{u_t}$$

$$u_t \overset{i}{\sim} \mathcal{MN}(\mathbf{0}, \Omega_t), \qquad \underbrace{\Omega_t}_{n\times n} = G^{-1}\Sigma_t\Sigma_t\left(G^{-1}\right)'$$

- Log-volatilities law of motion

$$\Sigma_t = Diag(\sigma_t), \qquad \log\sigma_t = \log\sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0}, \underbrace{Q_\sigma}_{n\times n}\right)$$

An Alternative Representation (1)

- Borrowing from Johansen (1995), construct matrix $B_{0\perp}$ orthogonal to B_0:

$$\underbrace{B_0}_{r\times n} \cdot \underbrace{B'_{0\perp}}_{n\times(n-r)} = \mathbf{0}_{r\times(n-r)}$$

- Consider then the TV decomposition (CKM16) that divides $\mathbb{R}^n$ into the subspaces spanned onto the rows of $B_0 \cdot \Omega_t^{1/2}$ and $B_{0\perp}\Omega_t^{-1/2}$:

$$I_n = \Omega_t B_0' \left(B_0 \Omega_t B_0'\right)^{-1} B_0 + B_{0\perp}' \left(B_{0\perp}\Omega_t^{-1} B_{0\perp}'\right)^{-1} B_{0\perp}\Omega_t^{-1}$$

- Using this decomposition, we can rewrite the model as a FAVAR:

$$y_t = \sum_{\ell=1}^{\max(p,q)} (\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + u_t$$

where $F_t = B_0 \cdot y_t$, $G_t = B_{0\perp}\Omega_t^{-1} y_t$, $\Xi_t = B_0 \Omega_t B_0'$ and $\Xi_{\perp,t} = B_{0\perp}\Omega_t^{-1} B_{0\perp}'$

An Alternative Representation (2)

- Multiplying both sides of the previous representation by B_0 and $B_{0\perp}\Omega_t^{-1}$:

$$F_t = \sum_{\ell=1}^{q} B_0 \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_0(\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \omega_t$$

$$G_t = \sum_{\ell=1}^{q} B_{0\perp}\Omega_t^{-1} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_{0\perp}\Omega_t^{-1}(\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \psi_t$$

where $$\begin{bmatrix} \omega_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} B_0 u_t \\ B_{0\perp}\Omega_t^{-1} u_t \end{bmatrix} \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \Xi_t & 0 \\ 0 & \Xi_{\perp,t} \end{bmatrix} \right)$$

- so that $\begin{bmatrix} F_t \\ G_t \end{bmatrix}$ evolves as VAR with block uncorrelated errors, since

$$\mathbb{E}(\omega_t \psi_t') = \mathbb{E}(B_0 u_t u_t' \Omega_t^{-1} B_{0\perp}') = B_0 \Omega_t \Omega_t^{-1} B_{0\perp}' = \mathbf{0}_{r\times(n-r)}$$

Levels Decomposition

- Due to temporal independence and orthogonality of ω_t and ψ_t, common and idiosyncratic components are orthogonal at all leads and lags
- We can regress of y_t on contemporaneous and lagged values of ω_t, and consider the regression residual as idiosyncratic component

$$y_t = B_1(L)\omega_t + B_2(L)\psi_t.$$

- As in Jorda's (2005) local projections method, the same regression can be used to compute the dynamic response of y_t to impulses in the common shocks ω_t.

Stochastic Volatility Decomposition

- Having decomposed the innovations in **two orthogonal sets of components**:

$$u_t = \Omega_t B_0' \Xi_t^{-1} \cdot \underbrace{\omega_t}_{\text{Common}} + B_{0\perp}' \Xi_{\perp,t}^{-1} \cdot \underbrace{\psi_t}_{\text{Idiosyncratic}}$$

- We can exploit the orthogonality of ω_t and ψ_t to decompose the SV:

$$\Omega_t = \Omega_t^{com} + \Omega_t^{idio} \Leftrightarrow \begin{cases} \Omega_t^{com} = \Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t \\ \Omega_t^{idio} = B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \end{cases}$$

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Prior on B_0

- Block structure, r blocks of variables

$$\underbrace{y_t}_{n\times 1} = \begin{bmatrix} y_t^{1\prime} & y_t^{2\prime} & \dots & y_t^{r\prime} \end{bmatrix}', \qquad \forall j \in \{1,\dots,r\} \ \underbrace{y_t^j}_{n_j\times 1}, \qquad n = \sum_{j=1}^{r} n_j$$

- Normalization of the first variable of each block (identifying restriction)

$$\underbrace{B_0}_{r\times n} = \begin{bmatrix} 1 & \widetilde{B}_{0,1} & 0 & \mathbf{0}_{1\times(n_2-1)} & \dots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 1 & \widetilde{B}_{0,2} & \dots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 0 & \mathbf{0}_{1\times(n_2-1)} & \dots & 1 & \widetilde{B}_{0,r} \end{bmatrix}, \quad \forall j \ \underbrace{\widetilde{B}_{0,j}}_{1\times(n_j-1)}$$

- $n-r$ separate univariate regressions to calibrate independent priors of $B_{0,j,k}$ using the first principal components of each j-th block $\left(S_t^j\right)_{j=1}^{r}$

$$\forall j \in \{1,\dots,r\}, \quad \forall k \in \left\{2,\dots,n_j\right\}, \quad S_t^j = B_{0,j,k} \cdot y_{t,k}^j + u_{j,k,t}, \quad u_{j,k,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{j,k}^2\right)$$

Prior on other elements

- The prior on $a = vec\,(A')$ is $a \sim \mathcal{MN}\,(\,\mathbf{0},\, V_a\,)$:

$$V_a = \begin{bmatrix} \widehat{\sigma}^2_{y,1} & 0 & \dots & 0 \\ 0 & \widehat{\sigma}^2_{y,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \widehat{\sigma}^2_{y,n} \end{bmatrix} \otimes \begin{bmatrix} \Upsilon_1 & 0 & \dots & 0 \\ 0 & \Upsilon_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \Upsilon_p \end{bmatrix}, \quad \Upsilon_\ell = \frac{\lambda_a^2}{\ell^d} \cdot I_r$$

- Prior on SV are calibrated as in Primiceri(2005).
- Prior on the AR coefficients

$$\bar{\gamma} = \begin{bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \\ \vdots \\ \bar{\gamma}_q \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{n\times 1} \\ \mathbf{0}_{n\times 1} \\ \vdots \\ \mathbf{0}_{n\times 1} \end{bmatrix}, \qquad V_\gamma = \lambda_\gamma \cdot \begin{bmatrix} 1^{-d} & 0 & \dots & 0 \\ 0 & 2^{-d} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q^{-d} \end{bmatrix} \otimes I_n$$

MAI-AR-SV, Gibbs Sampler

1. **Draw the AR coefficients** $\gamma \left| A, B_0, G, (\sigma_t)_{t=1}^T \right.$
   Transform the model, standardize, and perform a Bayesian Regression.
2. **Draw the loadings** $A \left| B_0, G, \gamma, (\sigma_t)_{t=1}^T \right.$
   Bayesian Multivariate Regression with heteroskedastic innovations. Use the orthogonalization approach of CCM (2016) to handle large n.
3. **Draw the factor weights elements** in $B_0 \left| \gamma, A, G, (\sigma_t)_{t=1}^T \right.$
   Metropolis step similar to CKM2016 but adapted to take into account SV.
4. **Draw the off-diagonal elements** in $G \left| \gamma, A, B_0, (\sigma_t)_{t=1}^T \right.$
   Transform the model as in Primiceri (2005) and perform a Bayesian Regression with heteroskedastic innovations.
5. **Draw a history of volatilities** $(\sigma_t)_{t=1}^T \left| \gamma, A, B_0, G \right.$
   As amended by Del Negro and Primiceri (2013), and using the Omori, Chib, Shephard and Nakajima (2007) approximation for the $\log \chi_1^2$.

Specification and Dataset

- **CPI inflation: Consumer Price Index, year on year growth**
- The analysis is performed for **both headline and core CPIs changes**
- Source: OECD *Main Economic Indicators*
- Quarterly frequency dataset:
  - **All Items**: 228 observations, 1960-Q1 → 2016-Q4
  - **Non-food & non-energy items**: 152 observations, 1979-Q1 → 2016-Q4
- Data for 20 OECD countries:
  USA, Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK
- Single Index (global common factor), as Ciccarelli and Mojon (2010)
- 4 lags used

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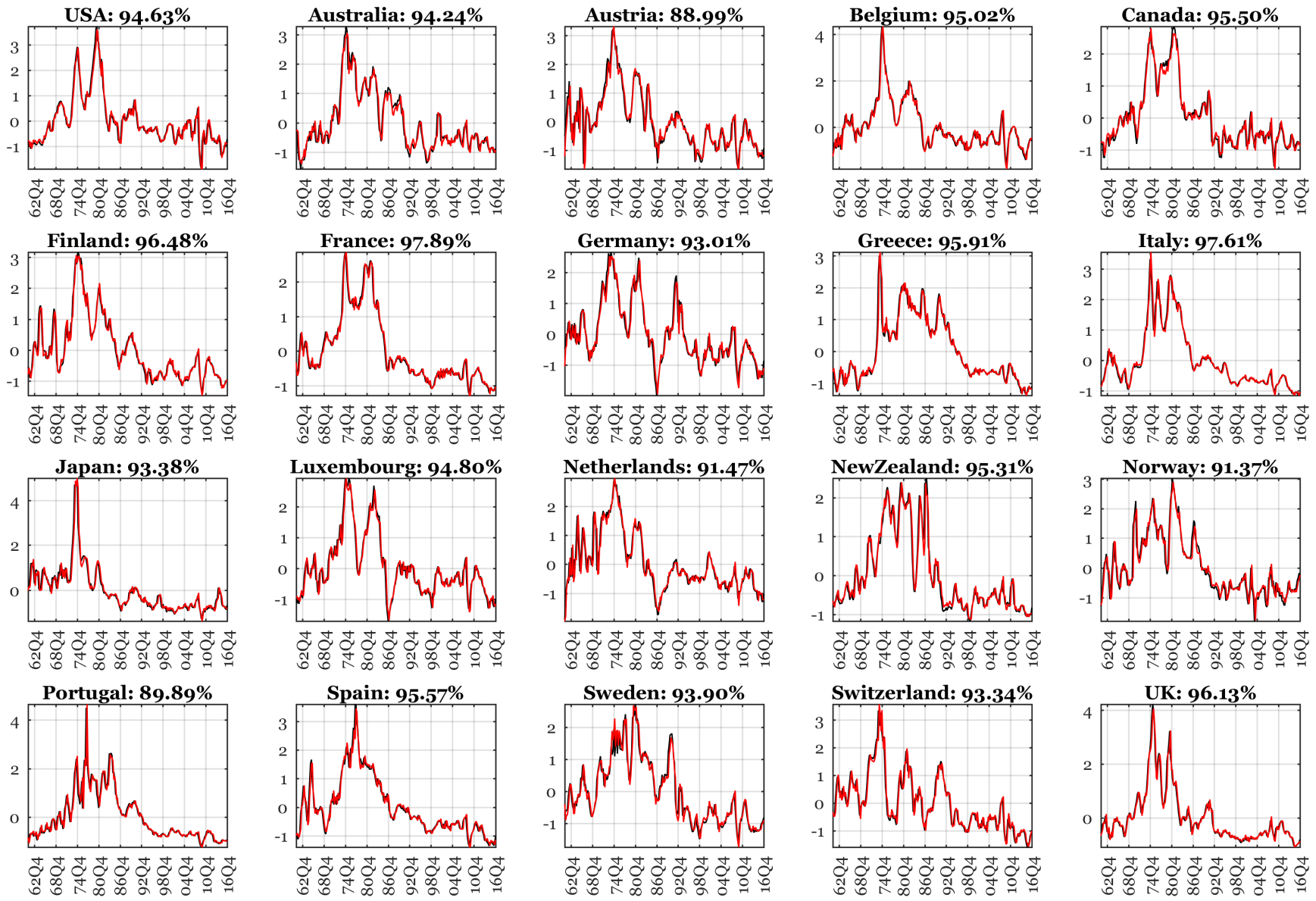
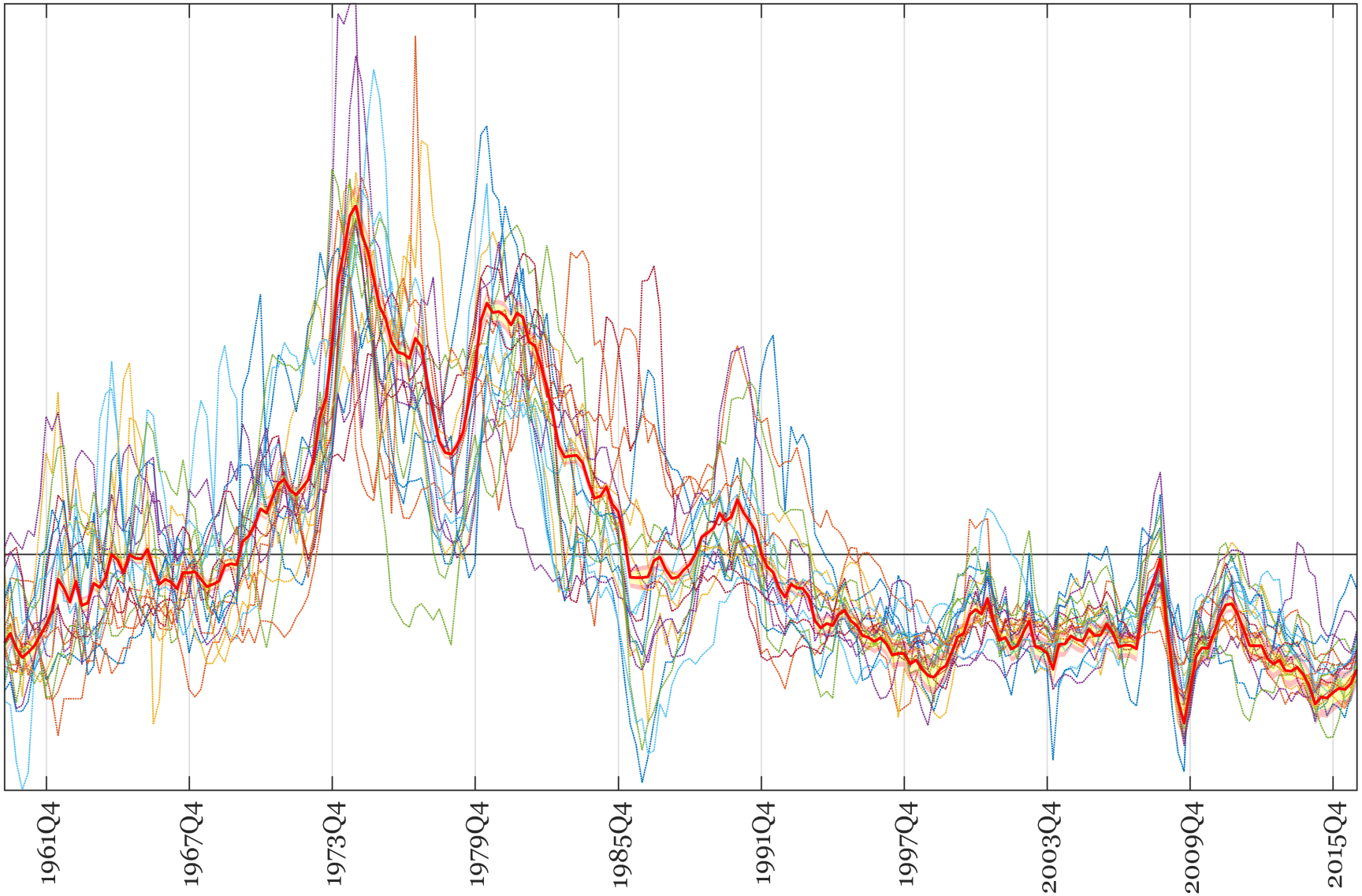
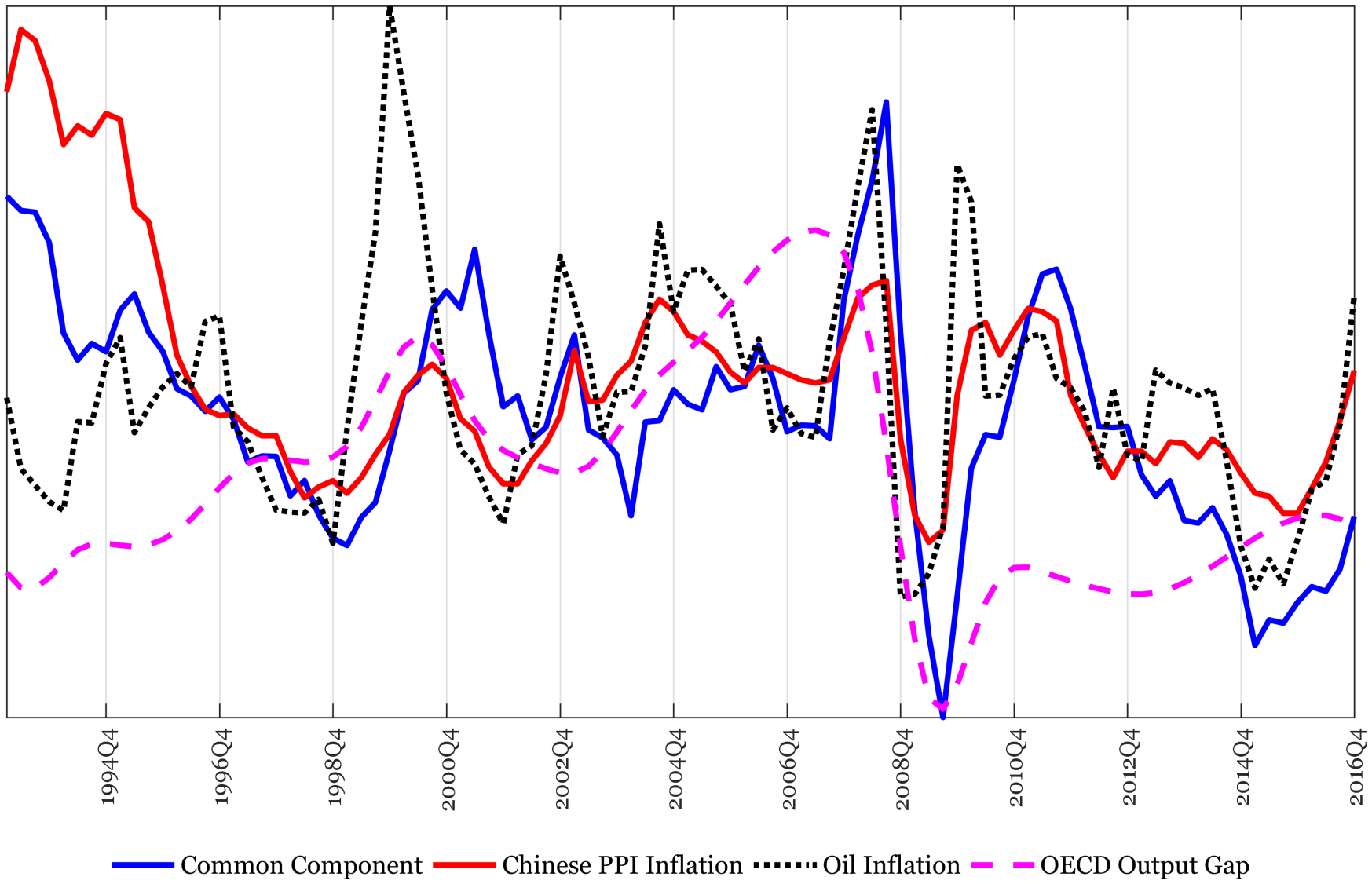
In Sample Fit: headline inflation



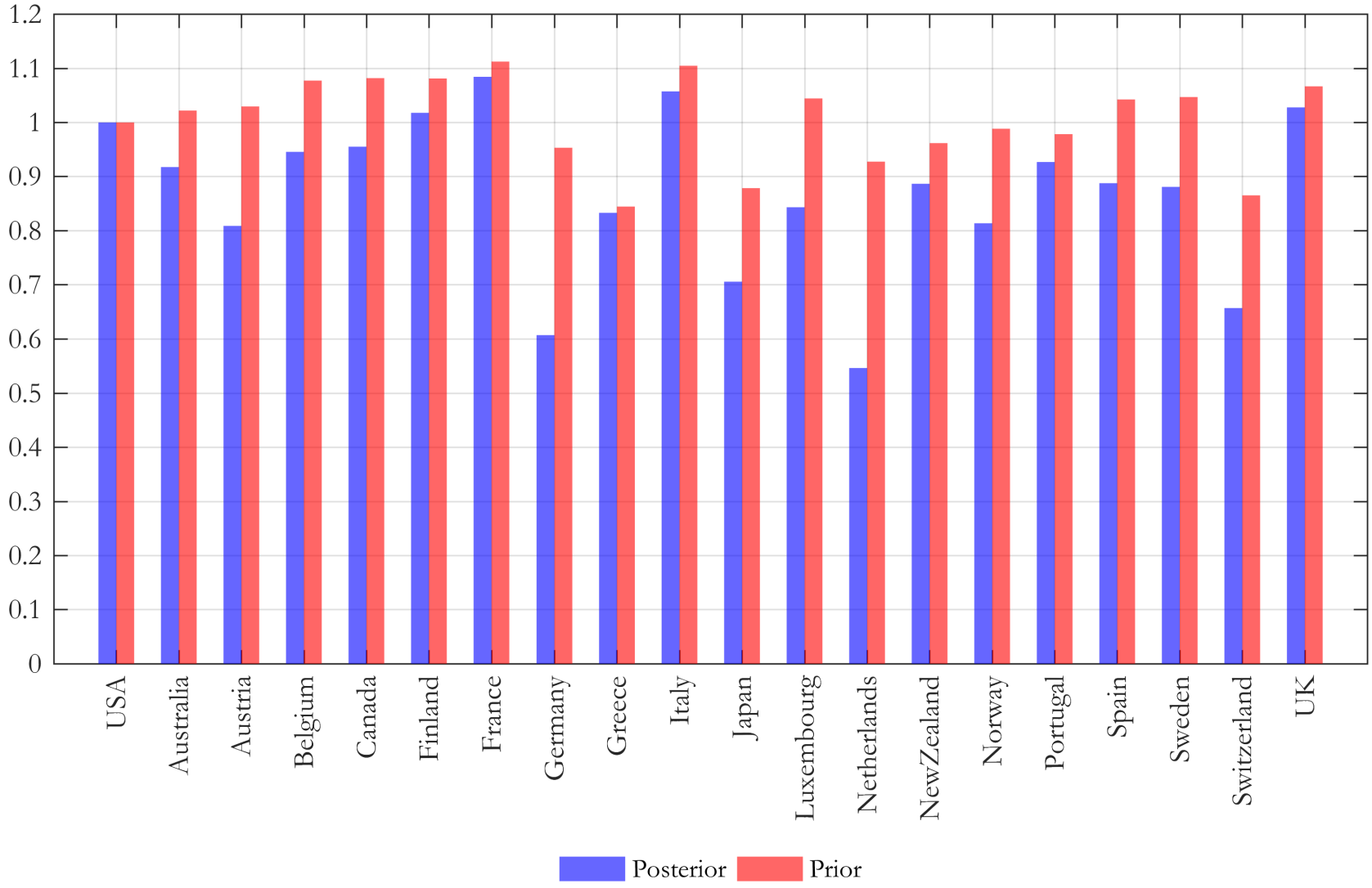
Data Vs Global Factor, Posterior bands



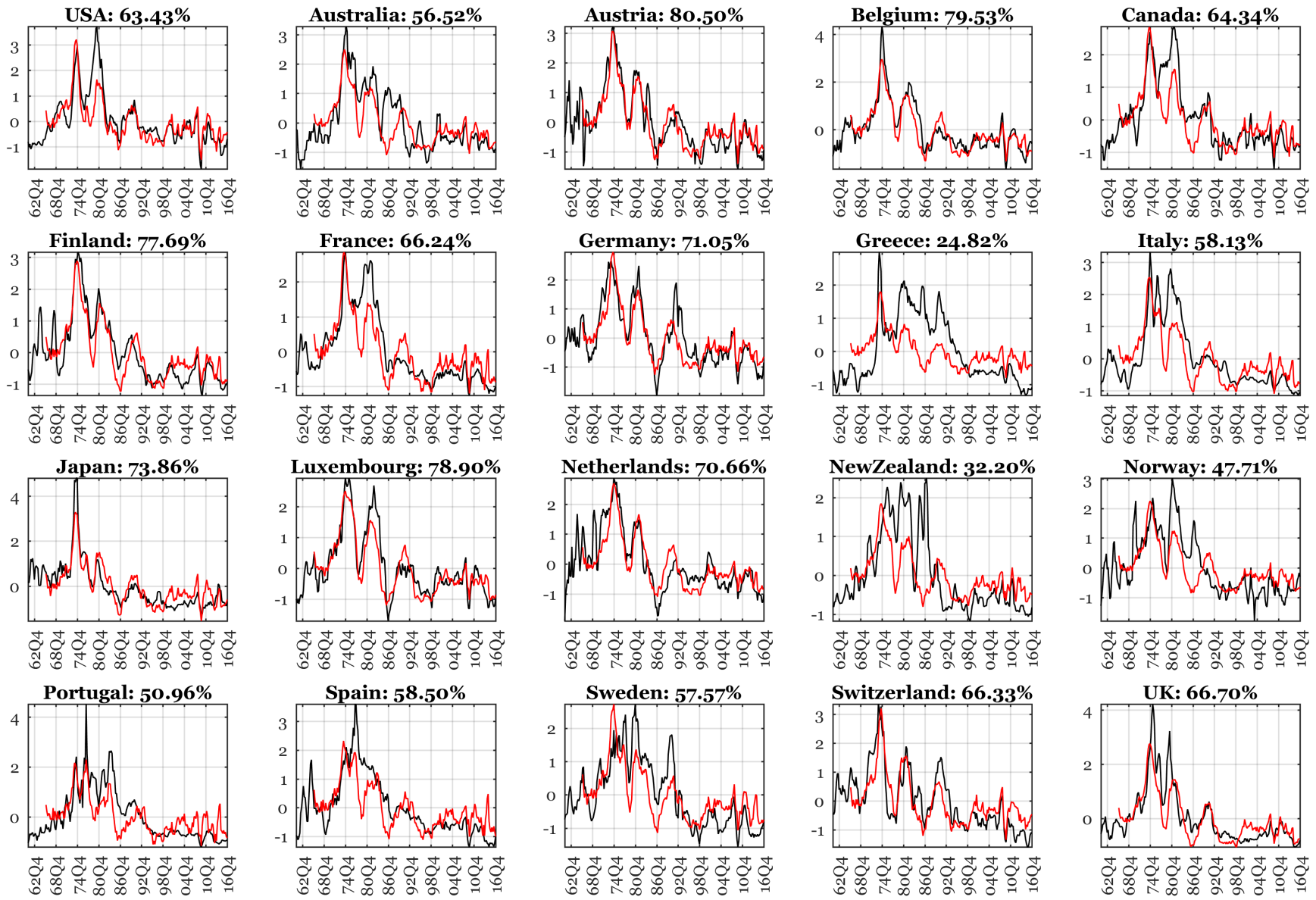
Global Inflation Factor Vs Oil, Chinese PPI, OECD Output Gap



Global Factor: Country Weights

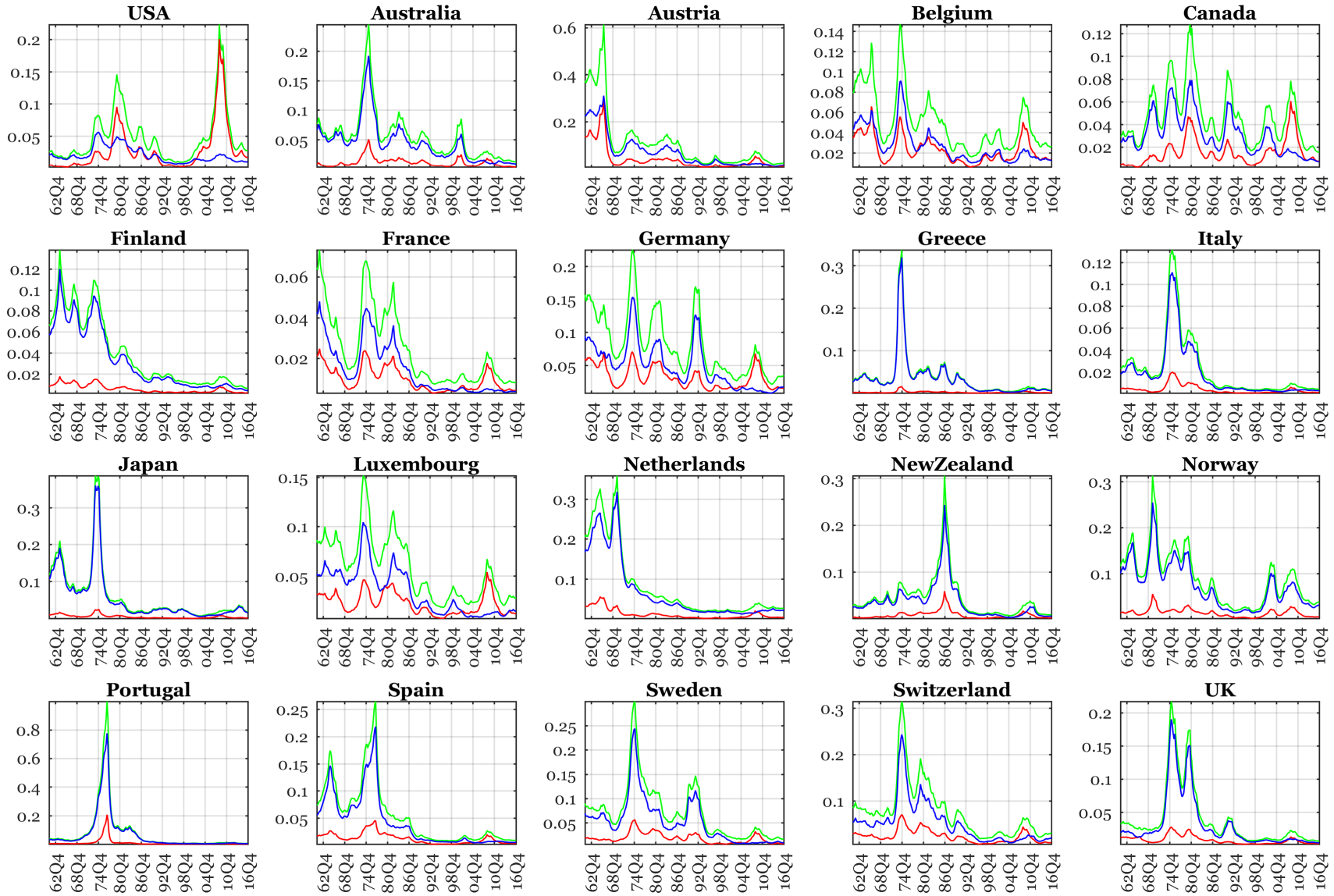


Projections on the global component ω_t



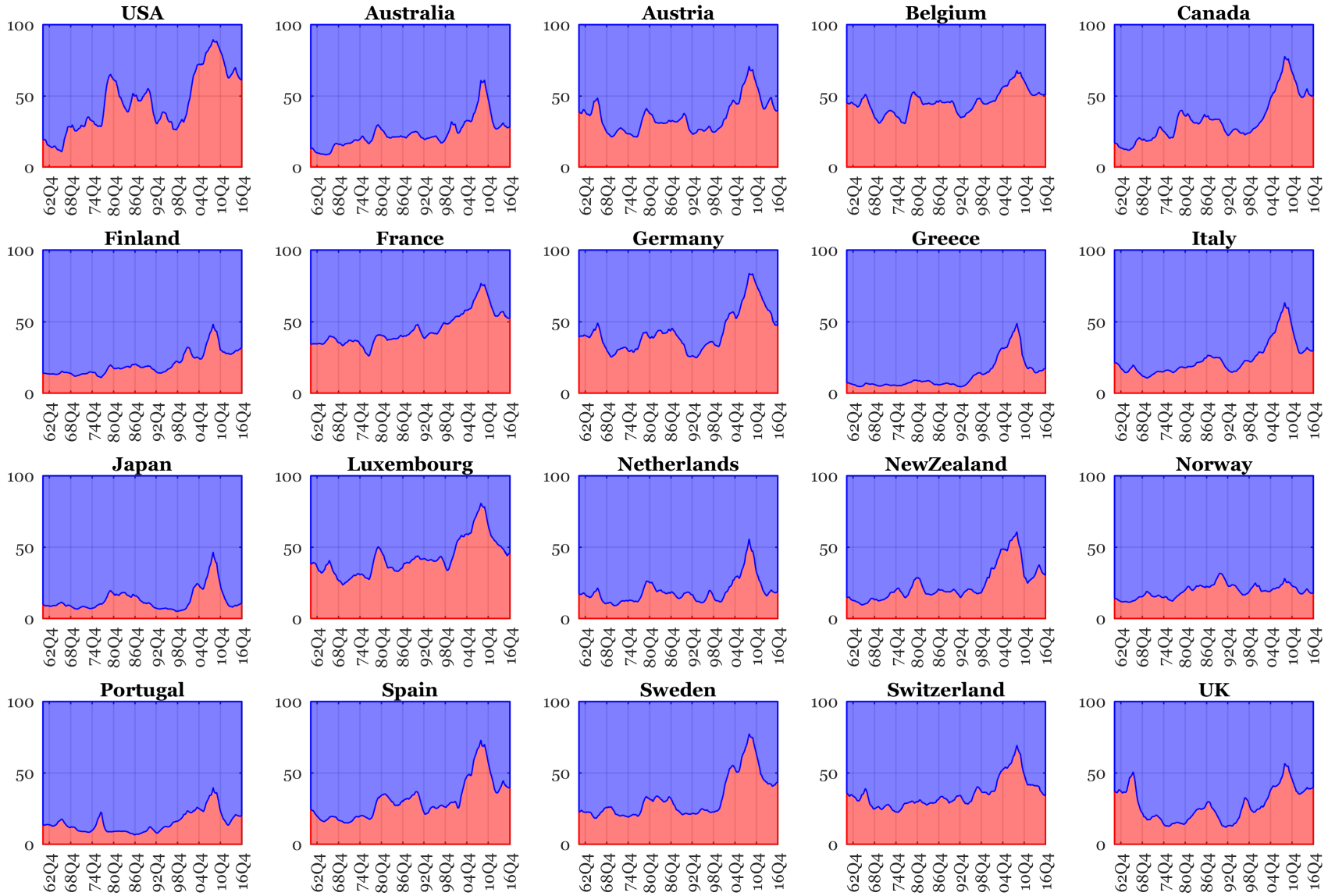
Residuals Volatility Decomposition

total (green), common (red), idio (blue)

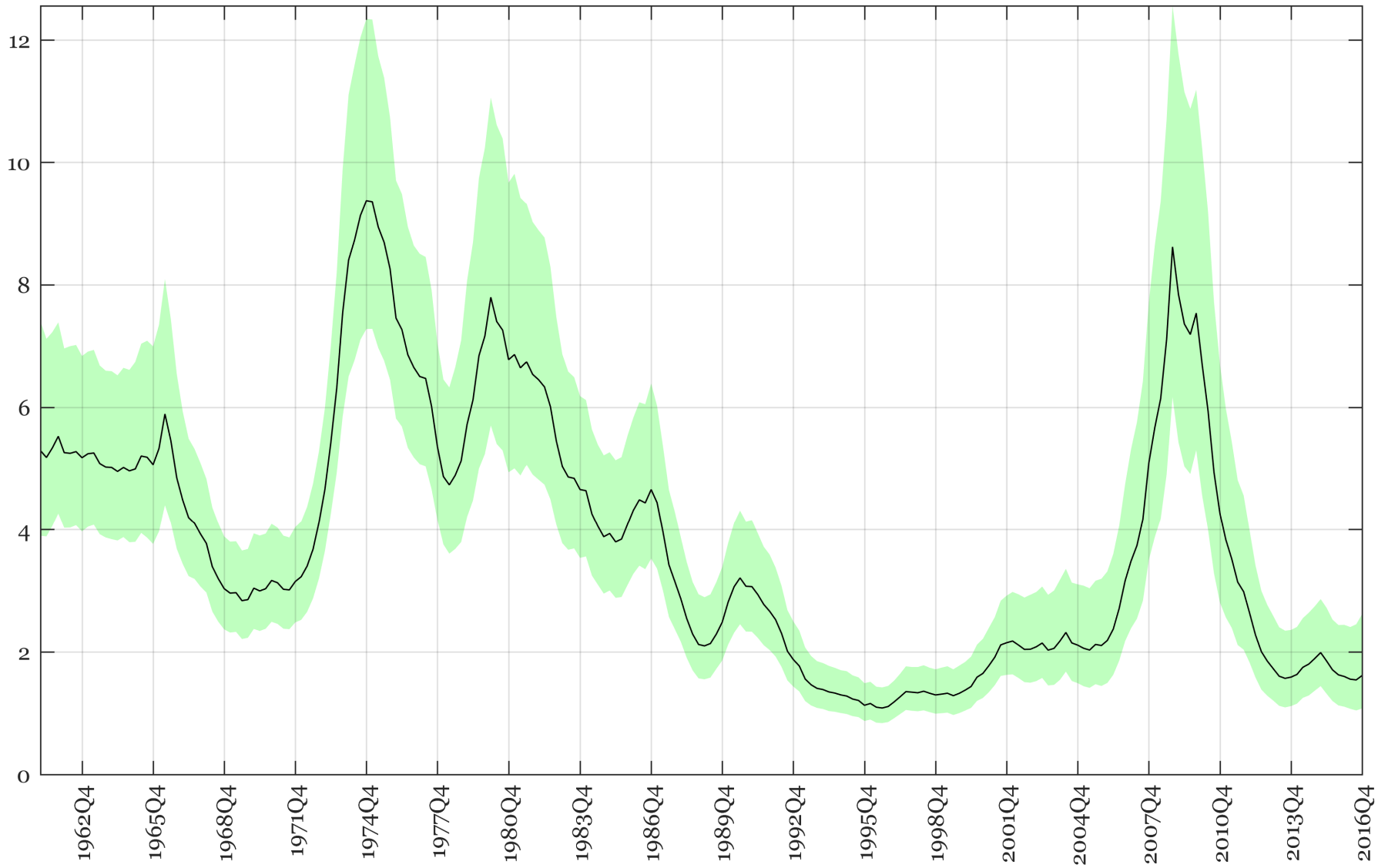


Residuals Volatility Decomposition, % shares

common (red), idio (blue)

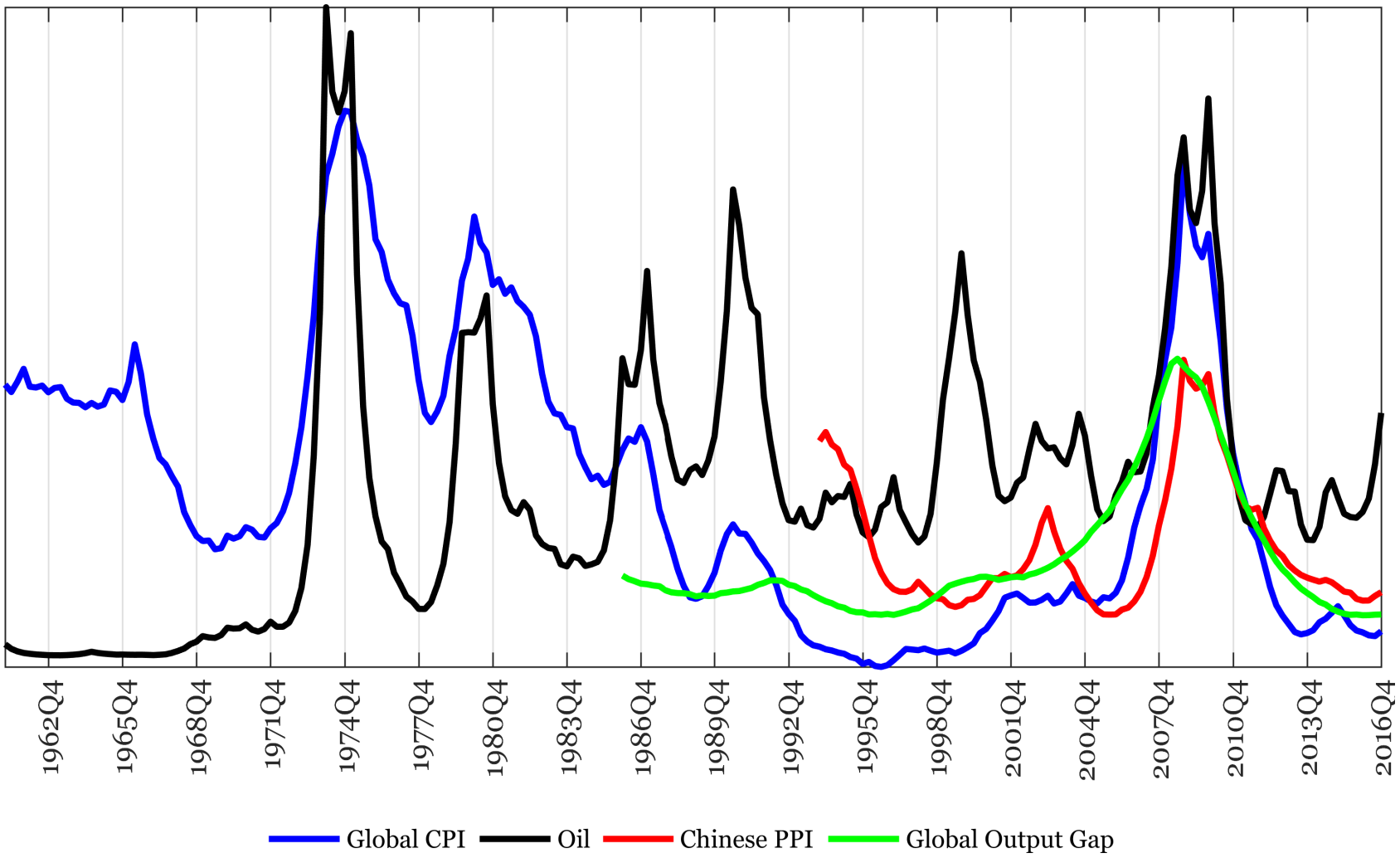


Global Inflation Volatility, $\mathbb{E}\left(\omega_t^2\right)$



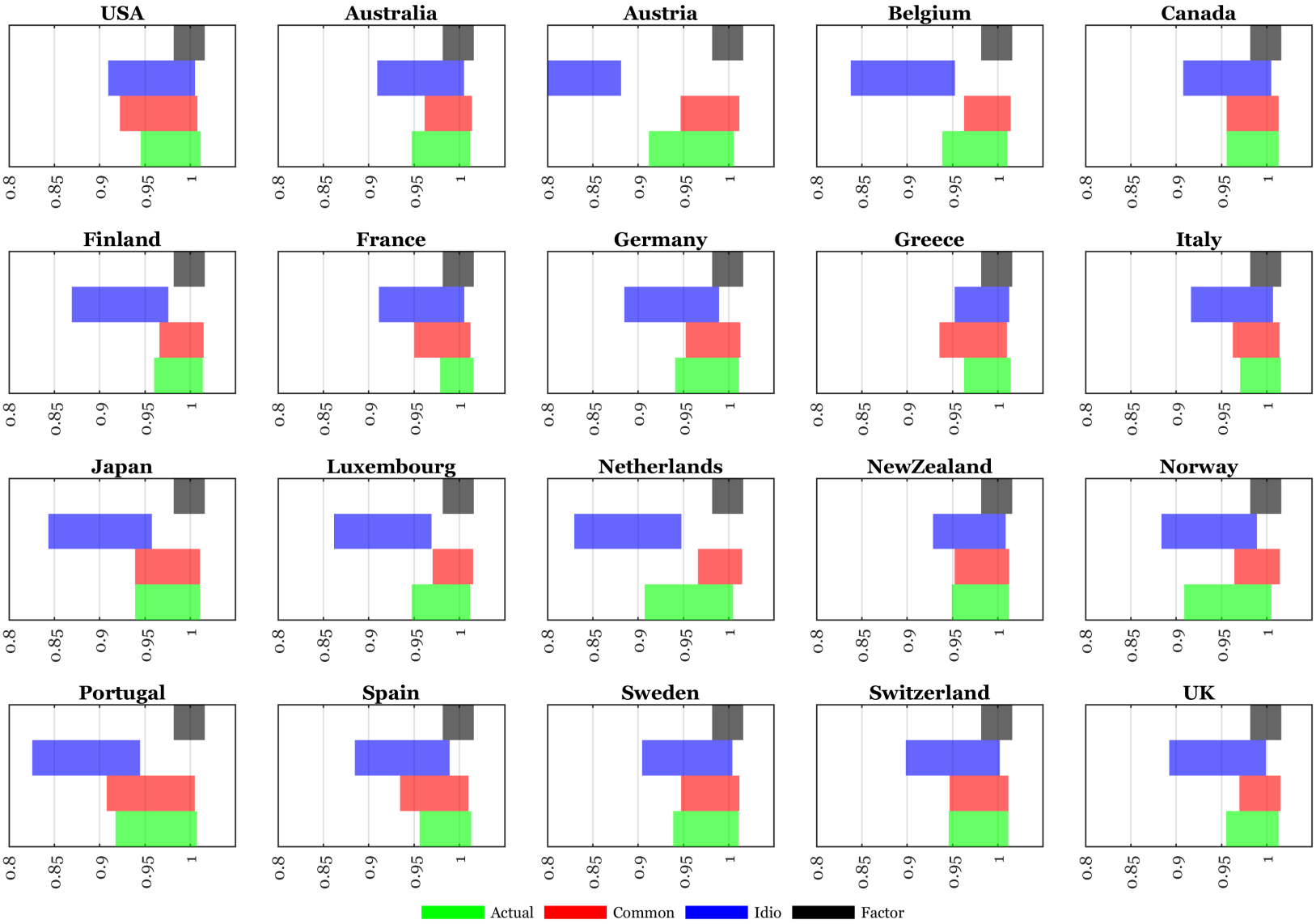
Global Inflation SV & Oil and Chinese PPI SVs

Global Inflation SV is correlated with: Oil SV (0.6), Chinese PPI SV (0.8) and Global Output Gap SV (0.6). Preliminary evidence of cointegration.



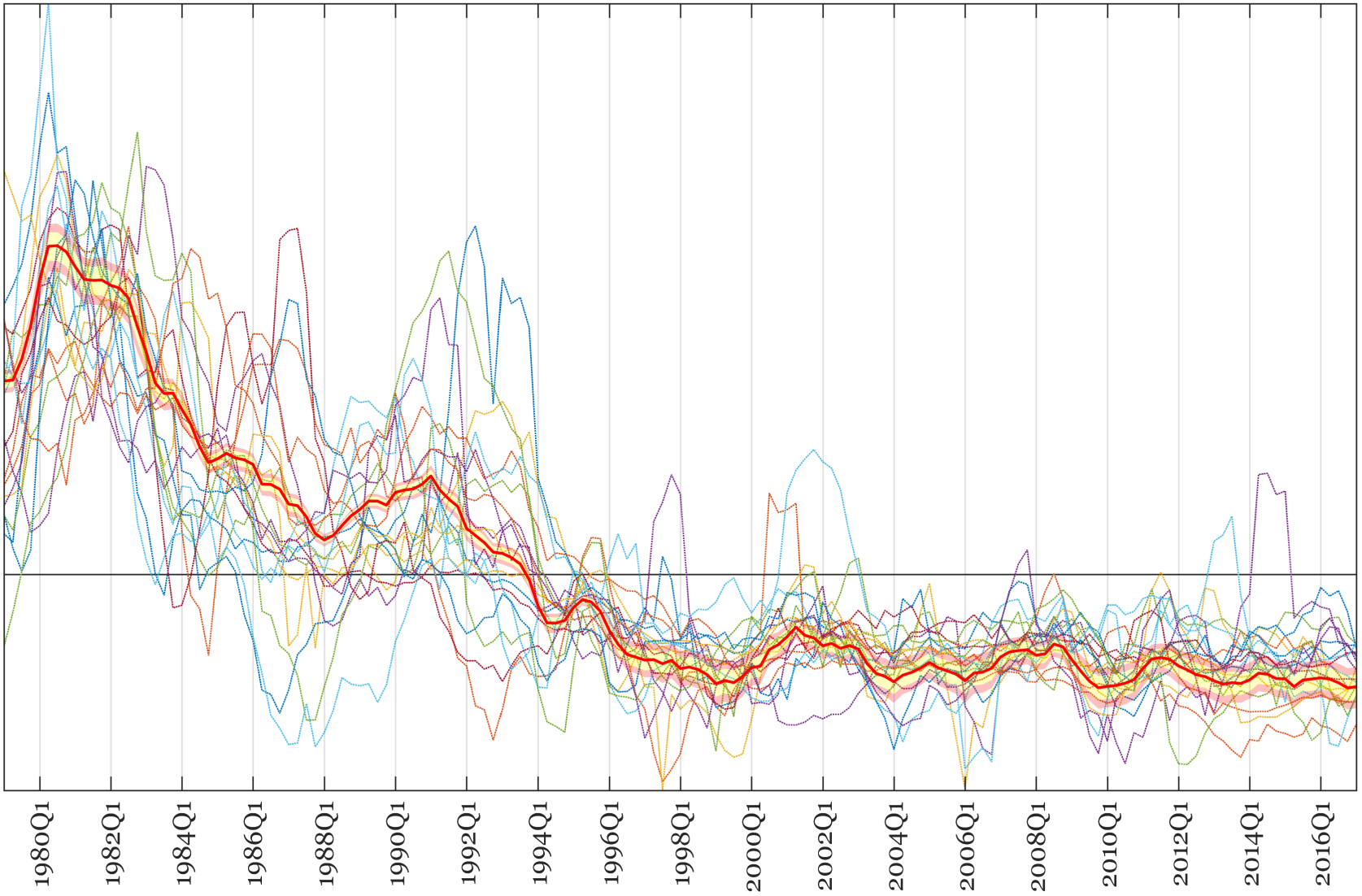
Persistence analysis

90% confidence interval for the largest AR root, Stock (1991)



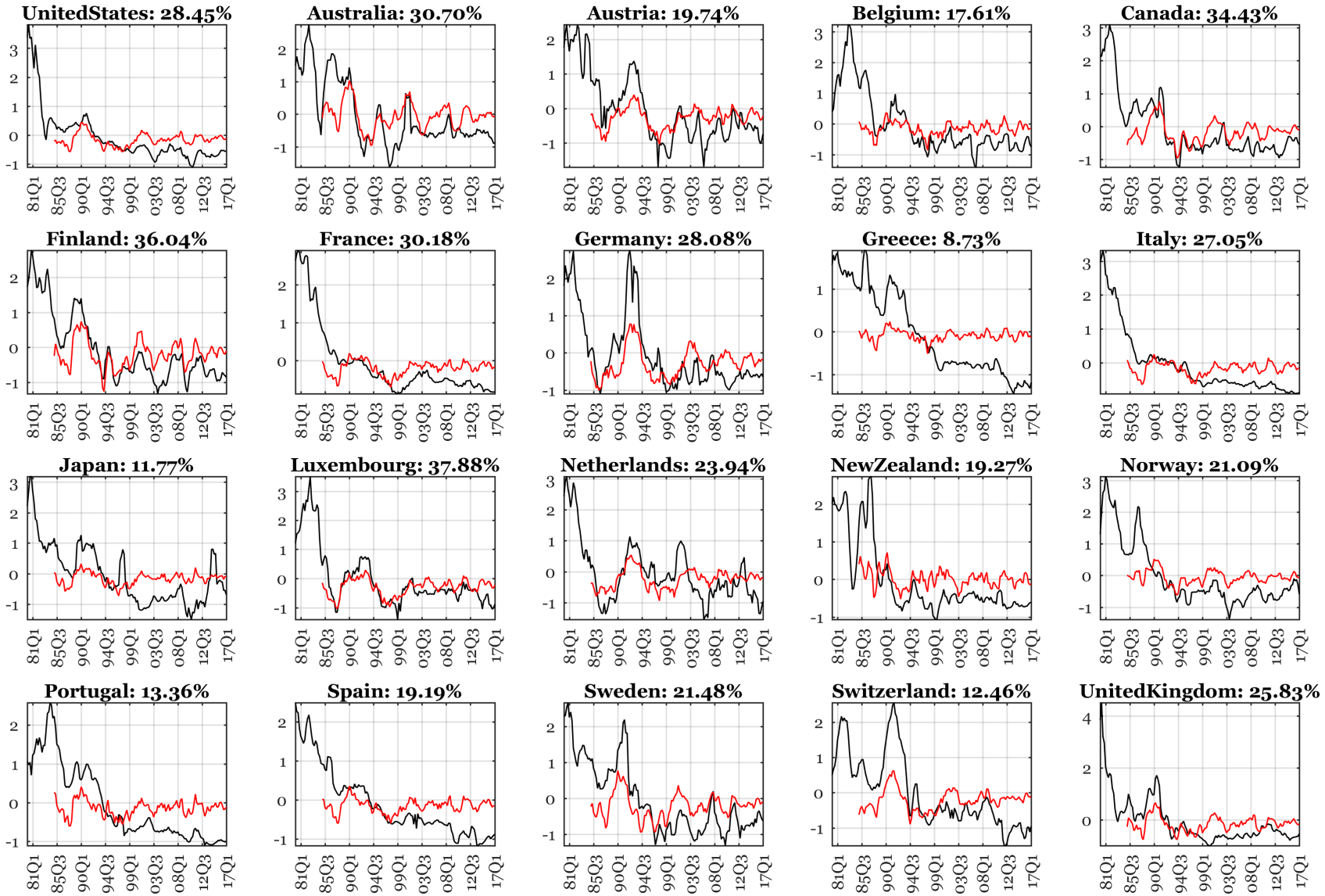
Data Vs Global Factor, Posterior bands

Core inflation



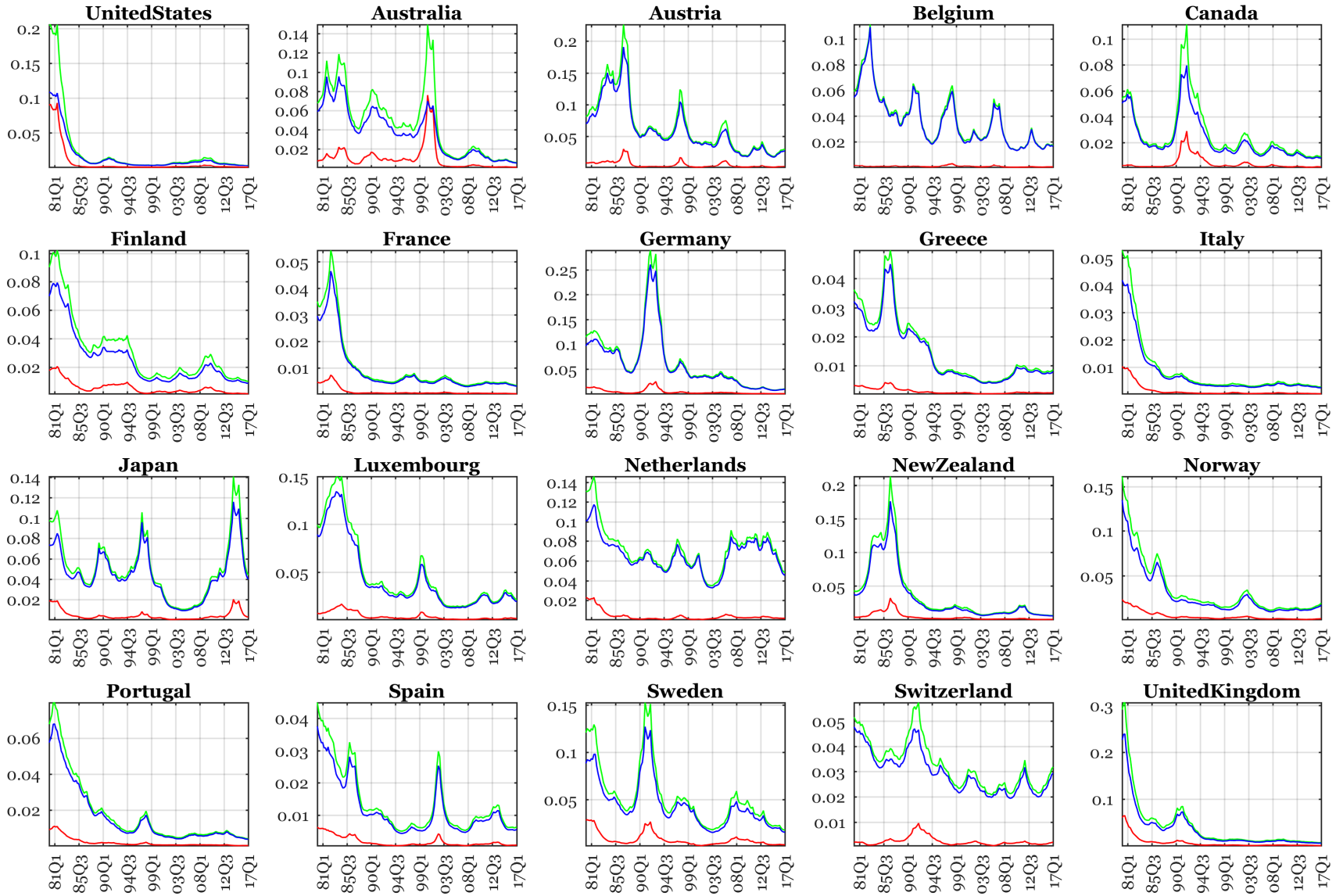
Actual series and Common Components

Core inflation



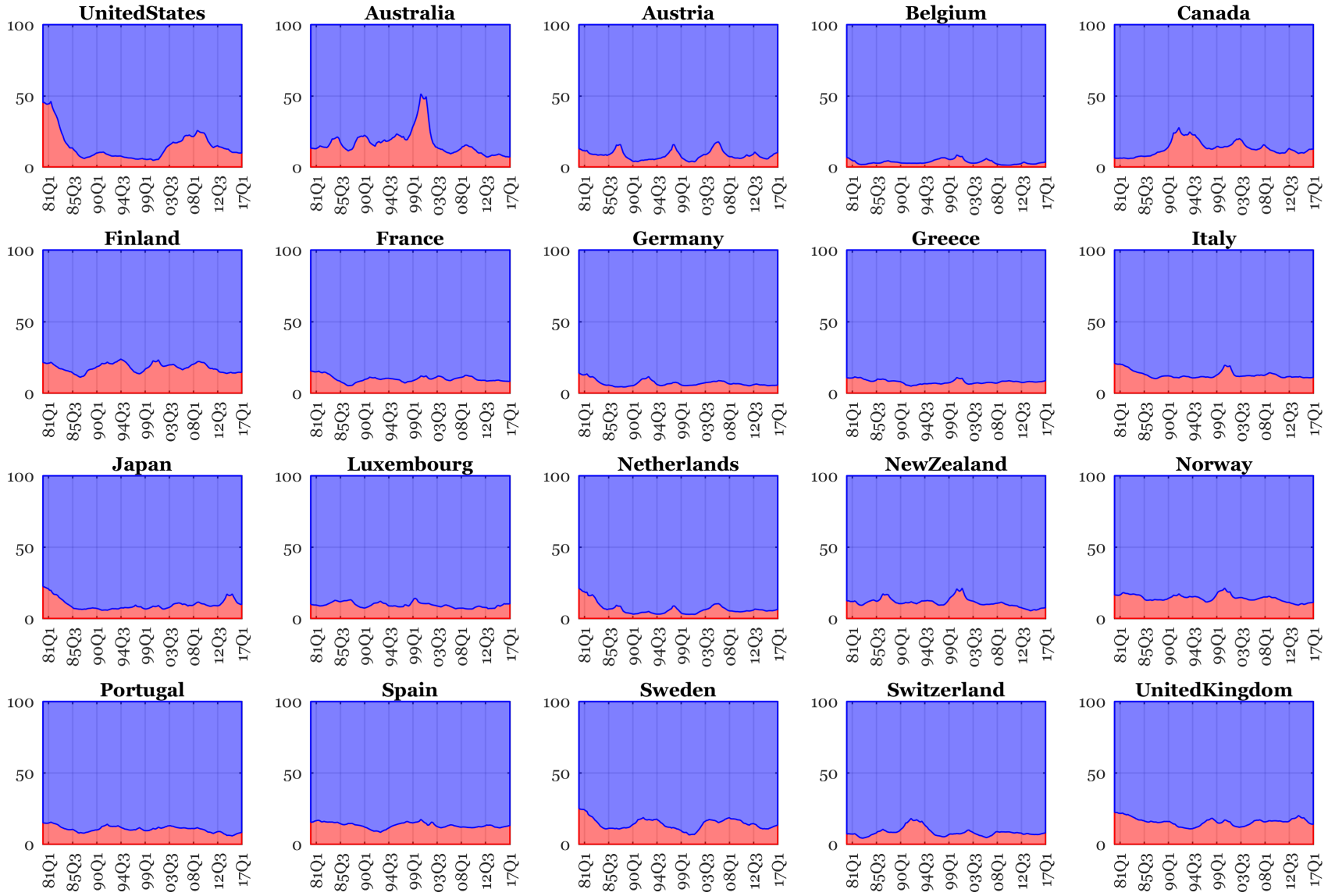
Residuals Volatility Decomposition

Core inflation. total (green), common (red), idio (blue)



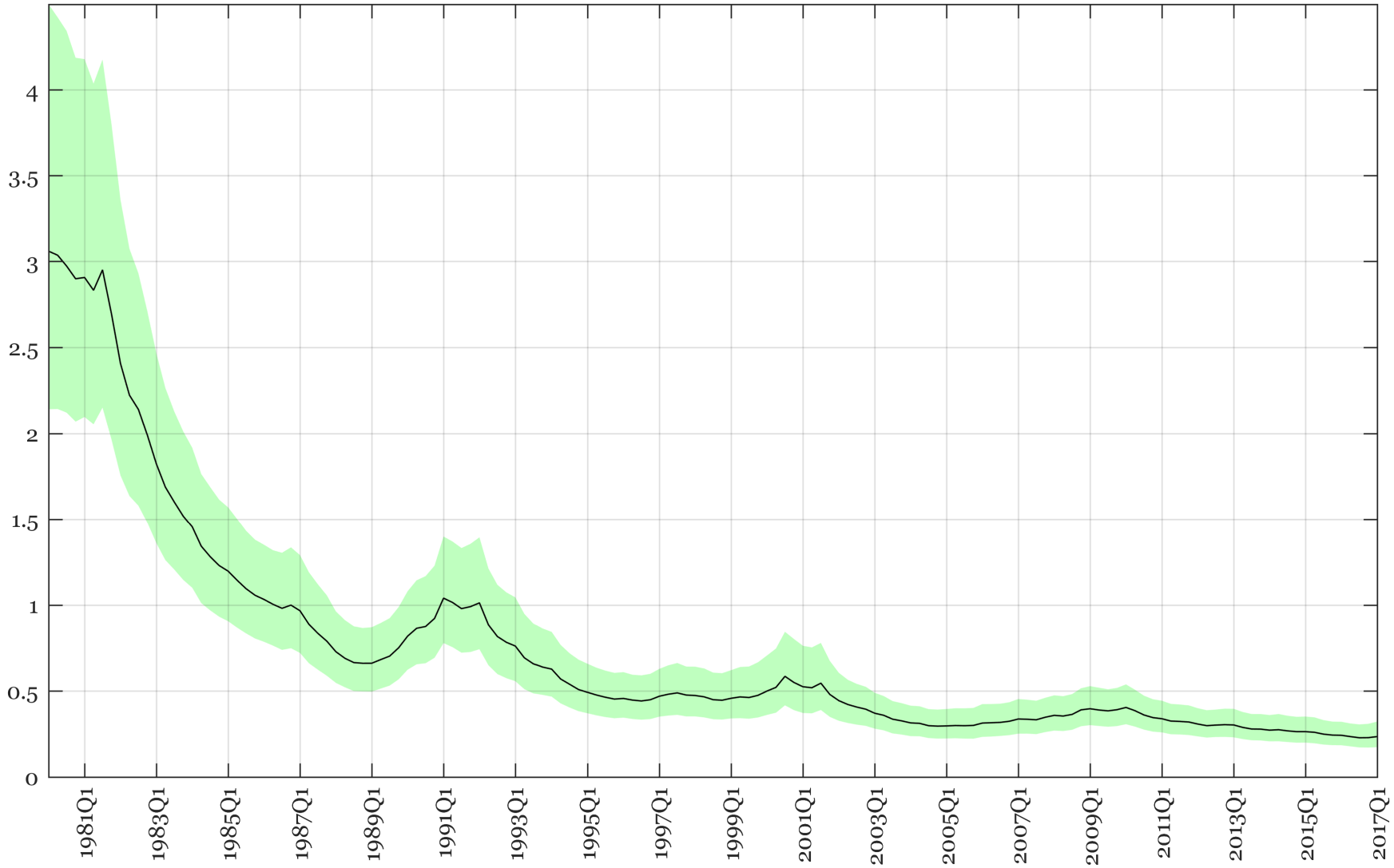
Residuals Volatility Decomposition, % shares

Core inflation. common (red), idio (blue)



Global Inflation Volatility, $\mathbb{E}\left(\omega_t^2\right)$

Core inflation



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CPI Inflation Pseudo Out of Sample Forecasting

- Recursive Estimation and Out of Sample Forecasting
- **101 Quarterly Vintages**
  (estimation window endpoint spanning from 1989Q4 to 2014Q4)
- **From 1 quarter to 2 years ahead**: $h \in \{1, \ldots 8\}$
- Specifications with 4 lags
- **Six Models Evaluated**:
  MAI-AR-SV (benchmark), MAI-AR, Univariate AR, Univariate AR-SV, VAR, VAR-SV
- Prior distributions calibrated as Univariate Random Walks across models
- **Extensive usage of parallelization** to perform MCMC estimation of several vintages simultaneously

Forecasting Point and Density Diagnostics

- Forecasting diagnostics framework of Clark and Ravazzolo (2015)
- For each model $m \in \{1,\ldots,M\}$, variable $j \in \{1,\ldots,n\}$ and horizon $h \in \{1,\ldots,H\}$
  - **Root Mean Squared Forecast Error** (RMSFE):

$$RMSE_{j,h}^{m} = \sqrt{\frac{1}{T^*}\sum_{t=T+1}^{T+T^*}\left(y_{j,t+h} - \widehat{y}_{j,t+h}^{m}\right)^2}$$

  - **Log Predictive Scores** obtained via non-parametric kernel smoothing density estimators:

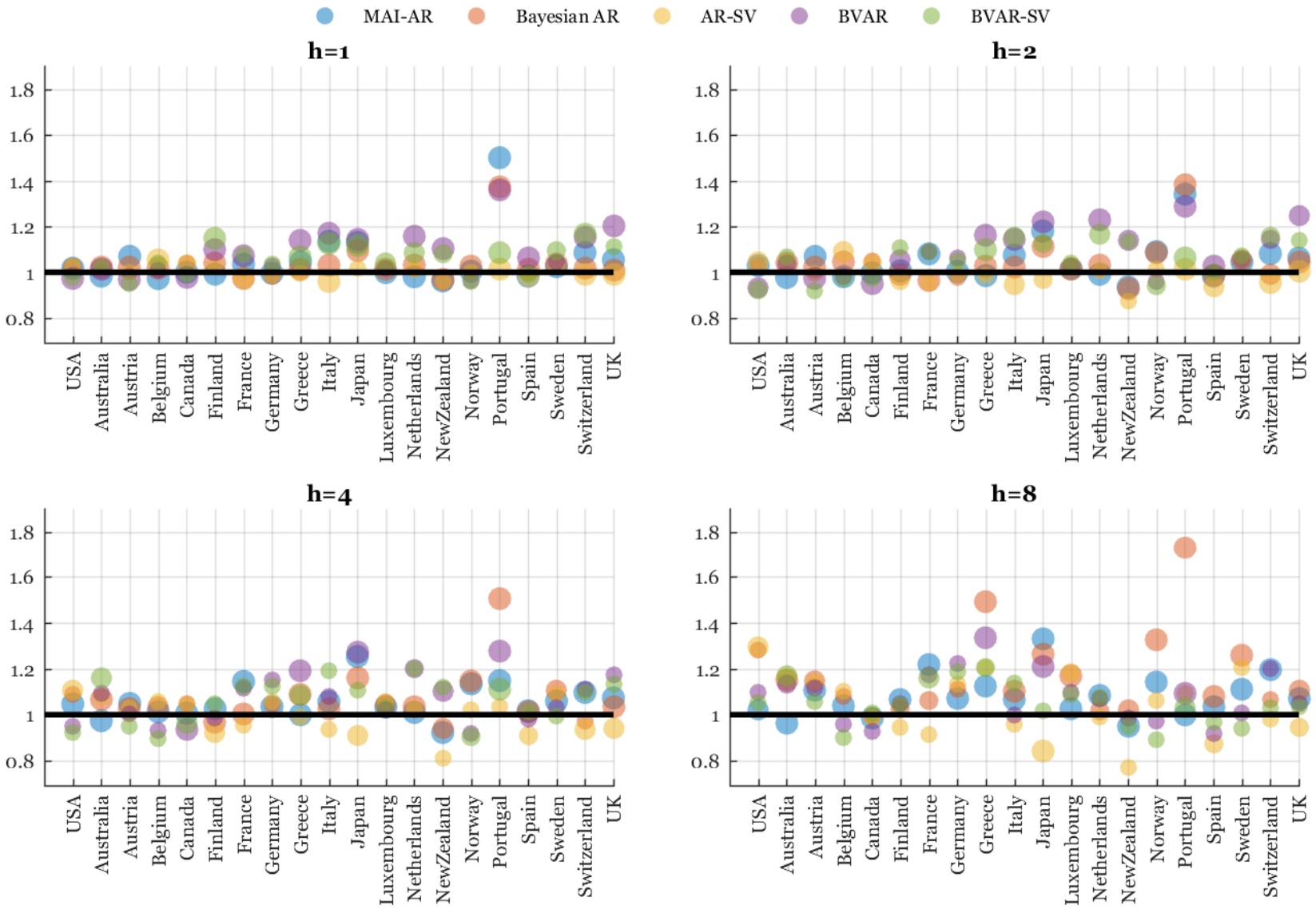
$$\overline{logScore}_{j,h}^{m} = \frac{1}{T^*}\sum_{t=T+1}^{T+T^*}\log\left(\frac{1}{\widehat{\mathcal{H}}\cdot L_c}\sum_{i=1}^{L_c}\mathcal{K}_{\mathcal{N}}\left(\frac{y_{j,t+h} - \widehat{y}_{j,t+h}^{m,i}}{\widehat{\mathcal{H}}}\right)\right)$$

  - **Continuous Ranked Probability Score** (CRPS):

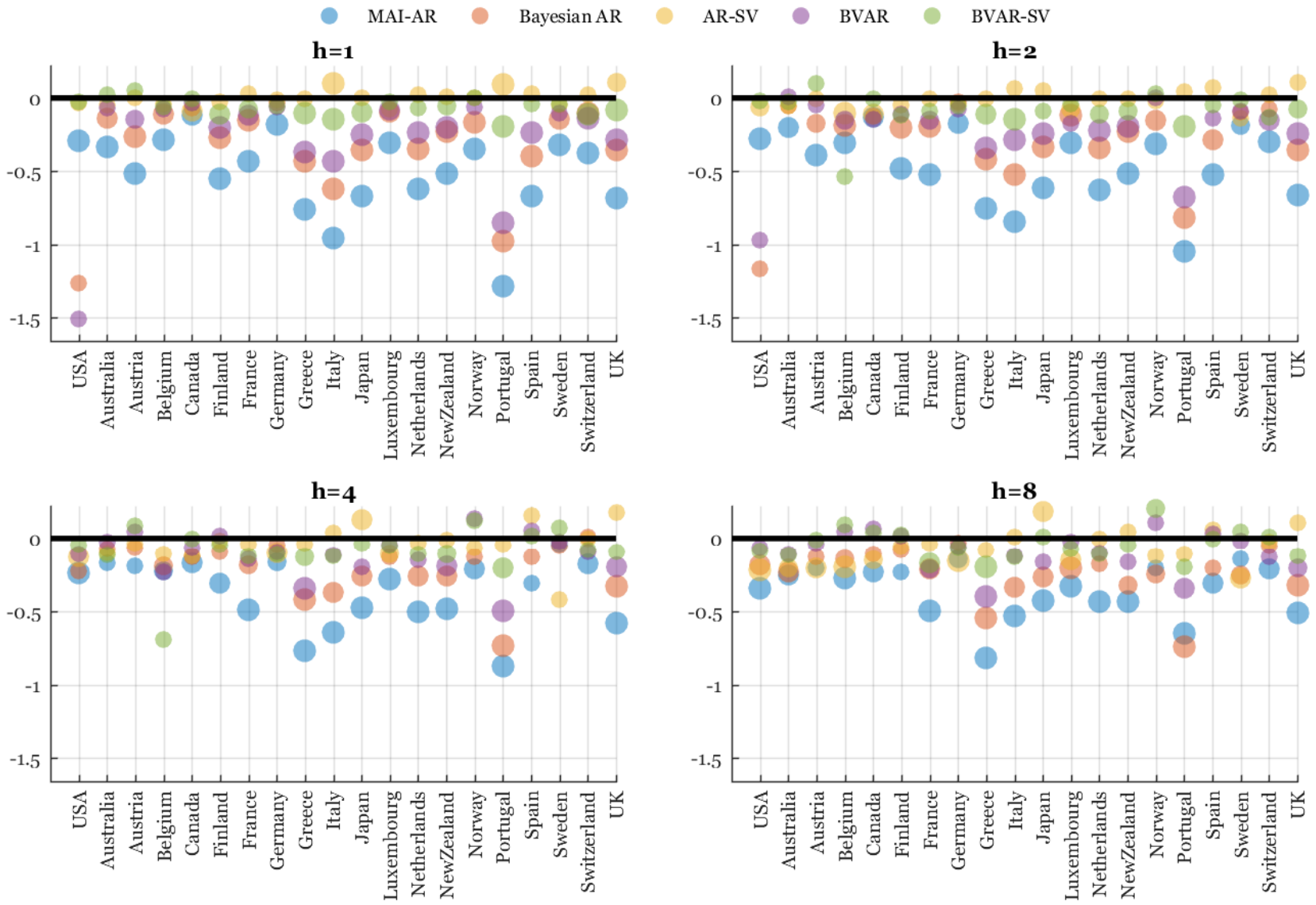
$$\overline{CRPS}_{j,h}^{m} = \frac{1}{T^*}\sum_{t=T+1}^{T+T^*}\left(\frac{1}{L_c}\sum_{i=1}^{L_c}\left|\widehat{y}_{j,t+h}^{m,i} - y_{j,t+h}\right| - \frac{1}{2\cdot L_c}\sum_{i=1}^{L_c}\left|\widehat{y}_{j,t+h}^{m,i} - \widehat{y}_{j,t+h}^{m,i'(i)}\right|\right)$$

- To test for significantly different performances: Diebold and Mariano (1995) t-tests for equality are computed for all diagnostics.

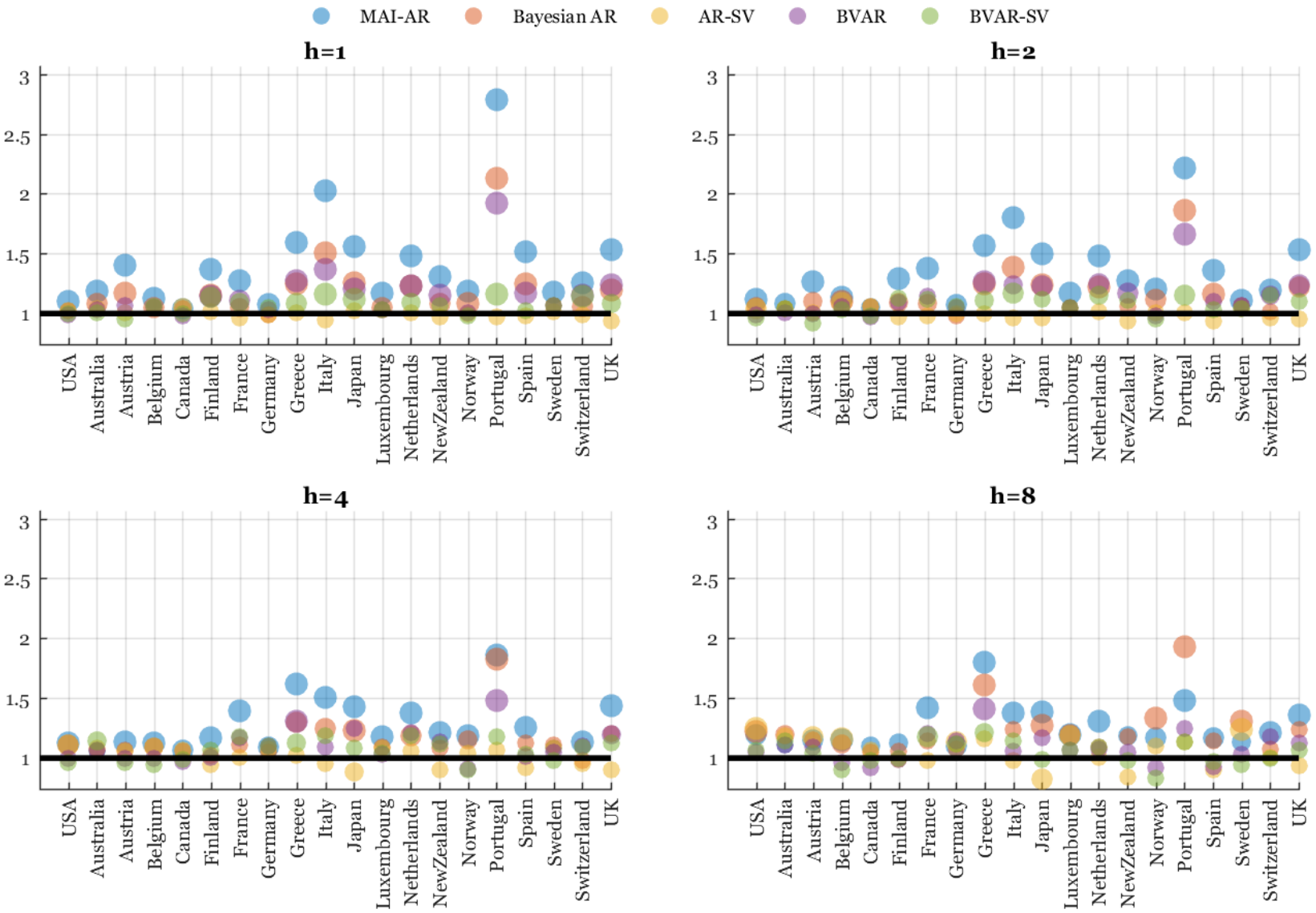
Relative RMSFE (ratios with mai-ar-sv)



Relative Log-Scores (differences with mai-ar-sv)

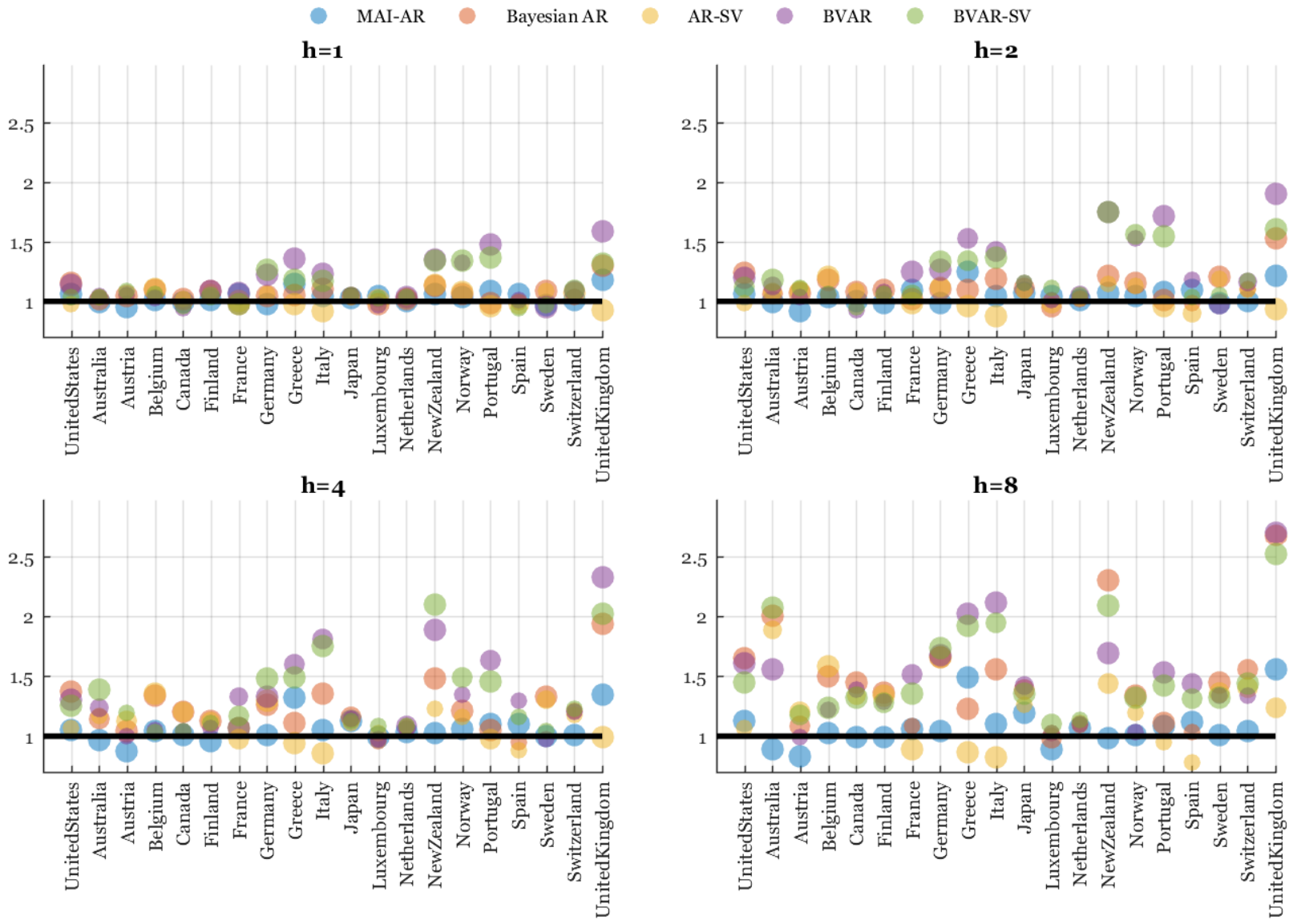


Relative CRPS (ratios with mai-ar-sv)



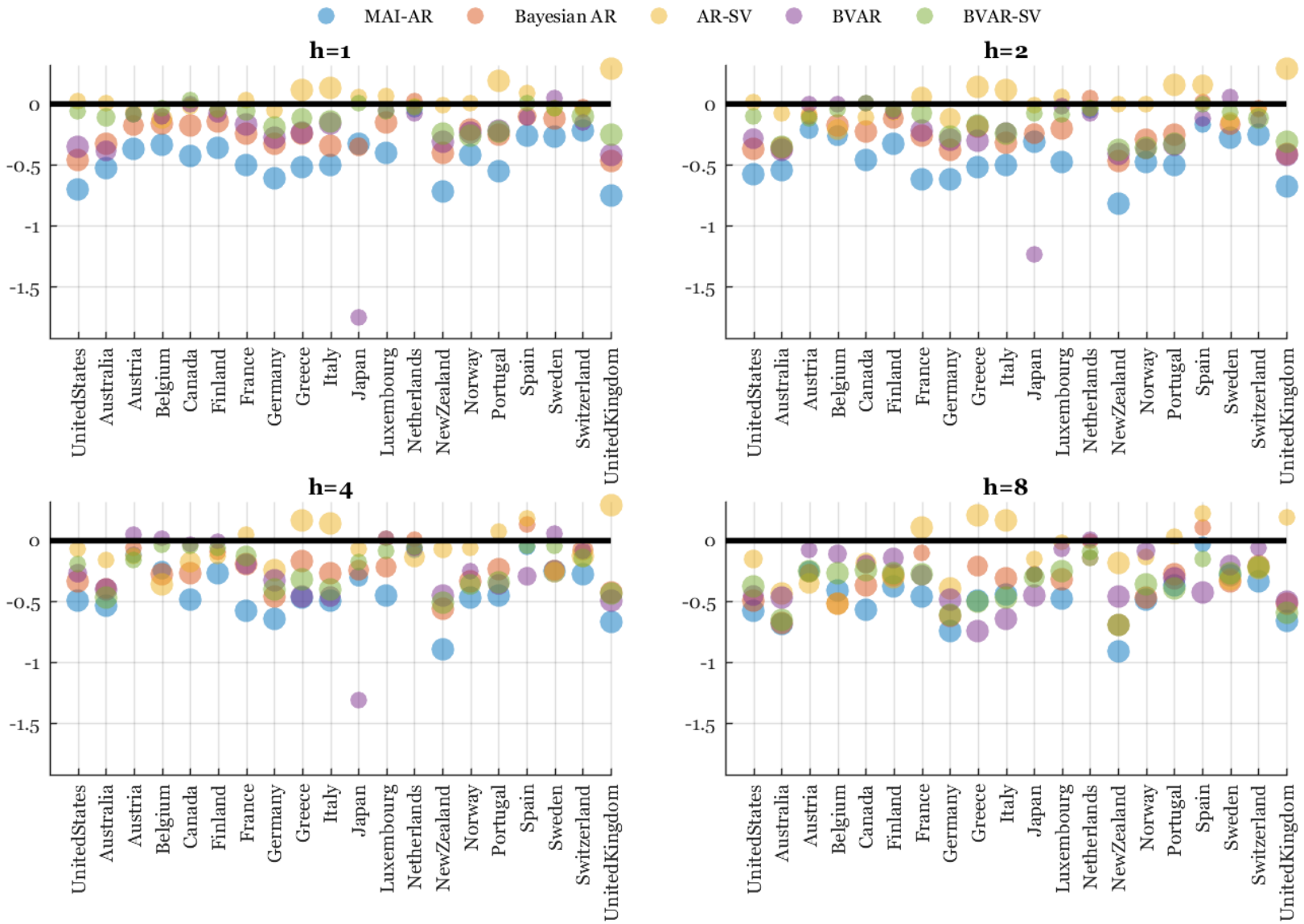
Relative RMSFE (ratios with mai-ar-sv)

core inflation



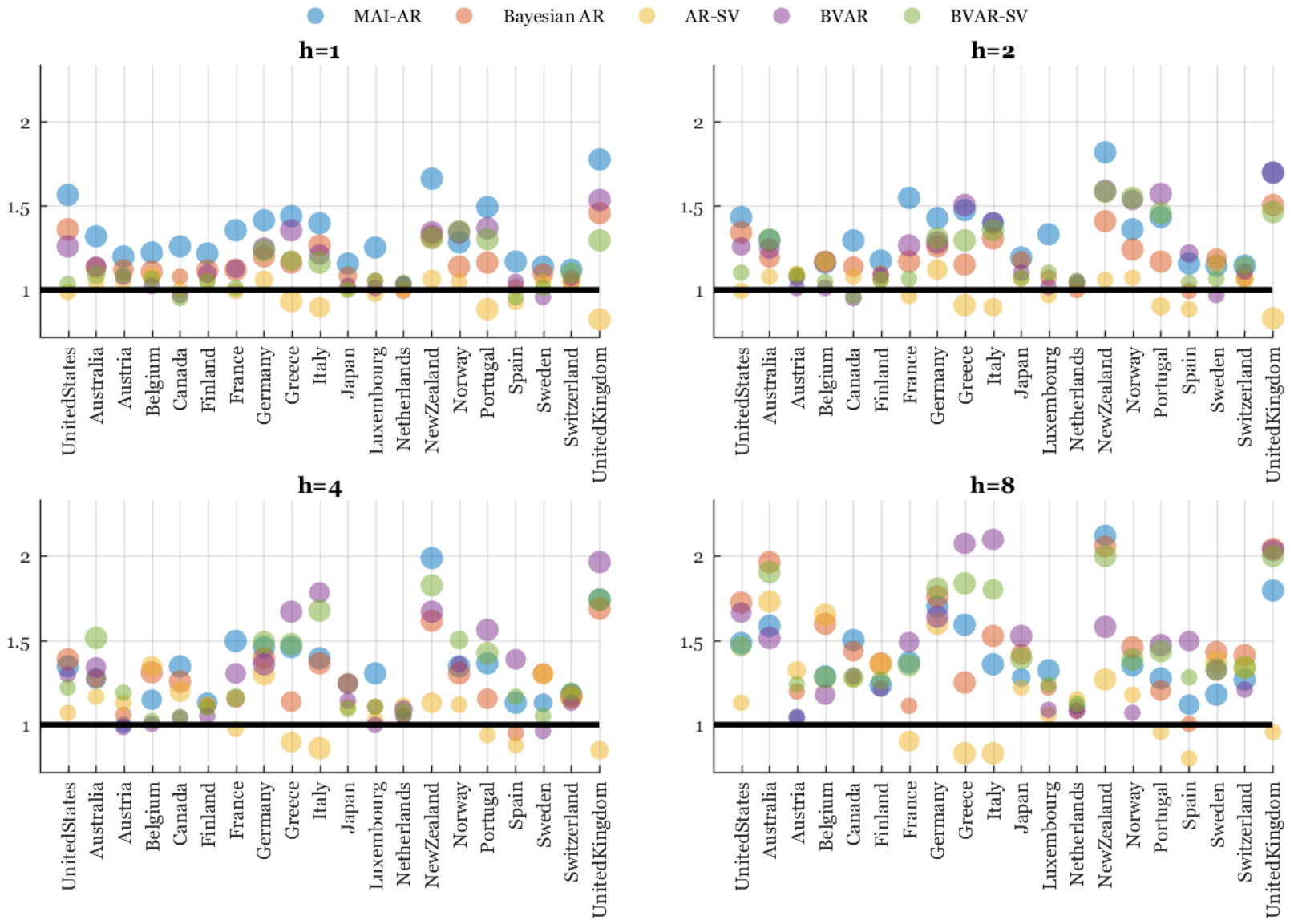
Relative Log-Scores (differences with mai-ar-sv)

core inflation



Relative CRPS (ratios with mai-ar-sv)

core inflation



1 Introduction

2 The Model

3 Estimation & Data

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6 Conclusions

Concluding remarks

- Using a novel model (MAI-AR-SV) that allows to disentangle between common and country specific components, we find that:
  - **a large fraction of headline CPI inflation volatility can be attributed to a global factor**, which is also driving the inflation levels
  - global inflation dynamics since the early ′90s are **related with the Chinese PPI and Oil inflation**
  - core inflation rates show a smaller and less time-varying degree of commonality
- **Proper modeling of inflation volatility should be considered by policy makers:**
  - to better evaluate inflation shocks in periods with different degrees of volatility
  - to disentangle whether the volatility originates locally or globally
- **The MAI-AR-SV shows also very good out of sample performance.**

THANK YOU