Exchange rate predictability and dynamic Bayesian learning

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Overview

- We address short-term (one-month ahead) density forecasting of exchange rate returns
- We develop dynamic Bayesian methodology accounting for various sources of parameter and model uncertainty
- We work with a large number of differently specified VARs
- Dynamic model learning (DML) to switch between different VAR configurations in a data-based fashion
- Empirical application to forecasting exchange rates of G10 countries with a focus on economic evaluation criteria in a dynamic asset allocation exercise

Features of the Modelling Framework

- Multivariate setup
- Exogenous variables
- Minnesota-type prior with rich shrinkage patterns
- Time-varying coefficients
- Fast model switching motivated by "scapegoat" theory (Bacchetta & van Wincoop, 2004, AER)
- Decisions about specification choices (i.e. different predictors, different VARs, different degrees of model switching) are all made automatically in a time-varying fashion

Data

- Endogenous data: Monthly exchange rate returns of G10 countries vis-à-vis the US Dollar
  - Australian dollar (AUD), Canadian dollar (CAD), Deutsche mark/euro (EUR), Japanese yen (JPY), New Zealand dollar (NZD), Norwegian krone NOK), Swedish krona (SEK), Swiss franc (SWF), British pound (GBP)
- **Long sample** from $\mathbf{1973:01}$ to $\mathbf{2016:12}$ (without exogenous regressors)
- **Short sample** from $\mathbf{1986:01}$ to $\mathbf{2016:12}$ (with exogenous regressors)

Data

- Two types of exogenous regressors: "asset-specific" and "non asset-specific"
- **"Asset-specific"** exogenous data: country-specific information
  - Uncovered Interest parity (UIP)
  - Percentage change in aggregate stock prices over the last 12 months (STOCK_GROWTH)
  - Difference between long term and short term interest rates (INT_DIFF)
- **"Non-asset-specific"** exogenous data:
  - Oil price changes (OIL)
- We stationarize the data

(TVP)-VAR-(X) Model

State space representation

$$y_t = x_t \beta_t + \varepsilon_t, \;\; \varepsilon_t \sim N(0, \Sigma_t)$$
$$\beta_t = \beta_{t-1} + u_t, \;\; u_t \sim N(0, \Omega_t)$$

- y_t is an $M \times 1$ vector
- x_t is an $M \times k$ matrix defined so that each TVP-VAR equation contains an intercept, $p = 6$ lags of each of the M variables, N_x "asset-specific" variables and N_{xx} "non asset-specific" variables
- $k = \underbrace{M \cdot (1 + pM)}_{\text{standard VAR}} + \underbrace{M^2 \cdot N_x}_{\text{asset-specific exog.variables}} + \underbrace{M \cdot N_{xx}}_{\text{non-asset-specific exog.variables}}.$

Kalman Filtering and Discount Factors

Discount factors

$$\Omega_{t|t-1} = \frac{1}{\lambda}\Omega_{t-1|t-1} \qquad \lambda = 1 \text{ (for main results)}$$

$$\Sigma_{t|t-1} \sim IW(\delta n_{t-1}, S_{t-1}) \quad \delta = 0.97$$

The predictive density is multivariate t:

$$y_t|y^{t-1} \sim t_{\delta n_{t-1}}\left(\widehat{y}_{t|t-1}, F_t\Omega_{t|t-1}F_t^{'} + Q_{t|t-1}\right)$$

$$Q_{t|t-1} := E\left(\Sigma_{t|t-1}\right) = \frac{S_{t-1}}{\delta n_{t-1} + M - 1}$$

$$p\left(y_t|y^{t-1}\right) = \iint p\left(y_t|y^{t-1}, \beta_t, \Sigma_t\right) p\left(\beta_t|y^{t-1}, \Sigma_t\right) p\left(\Sigma_t|y^{t-1}\right) d\beta_t d\Sigma_t$$

Minnesota-type Prior

Prior distribution

$$\beta_0 \sim N\left(0_M, \Omega_0\right)$$

The Minnesota prior assumes Ω_0 to be diagonal. Let $\Omega_{0,i}$ denote the block of Ω_0 associated with the coefficients in equation i and $\Omega_{0,i,jj}$ its diagonal elements

$$\Omega_{0,i,jj} = \begin{cases} s_i^2 \gamma_1 & \gamma_1 \in \{0; 10\} \text{ for } \textbf{INTERCEPT} \\ \frac{\gamma_2}{r^2} & \gamma_2 \in \{0; 0.1; 0.5; 0.9\} \text{ for } \textbf{OWN LAGS} \text{ in the VAR, } r = 1, \ldots, 6 \\ \frac{\gamma_3 s_i^2}{r^2 s_j^2} & \gamma_3 \in \{0; 0.1; 0.5; 0.9\} \text{ for } \textbf{CROSS LAGS} \text{ in the VAR, } r = 1, \ldots, 6 \\ \gamma_4 s_i^2 & \gamma_4 \in \{0; 1\} \text{ for variable } \textbf{UIP} \\ \gamma_5 s_i^2 & \gamma_5 \in \{0; 1\} \text{ for variable } \textbf{STOCK_GROWTH} \\ \gamma_6 s_i^2 & \gamma_6 \in \{0; 1\} \text{ for variable } \textbf{INT_DIFF} \\ \gamma_7 s_i^2 & \gamma_7 \in \{0; 1\} \text{ for variable } \textbf{OIL} \end{cases}$$

Dynamic Model Learning

- We have $\mathbf{J = 32}$ individual model configurations for the long sample and $\mathbf{J = 512}$ for the short sample
- In each period, we choose the predictive densities which would have maximized the **discounted joint predictive likelihood** until this point in time:

Evaluation criterion: Discounted joint predictive likelihood

$$DPL_{t|t-1,j} = \prod_{i=1}^{t-1} \left[p_j \left(y_{t-i} | y^{t-i-1} \right) \right]^{\alpha^i}$$

- We choose $\alpha \in \{0.20; 0.40; 0.50; 0.60; 0.70; 0.80; 0.90; 0.95; 0.99; 1\}$ in each period adaptively from the data

International FX Investment Strategy

- Consider a US investor who builds a portfolio by allocating her wealth between **10 bonds**: one domestic (US), and the nine foreign bonds
- The foreign bonds yield a **riskless return in local currency** and a **risky return in US dollars**
- The expectation of the **total** return at time $t-1$ is equal to

Expected total return

$$E_{t-1}(r_t) = i_{t-1} + \Delta s_{t|t-1}$$

- **Two steps** for the investor in each period: **select the currently best model** and **adjust portfolio weights**

Portfolio Optimization Problem

Mean-Variance Optimization

$$\max_{w_t} \left\{ \mu_{p,t|t-1} = w_t^{'} \mu_{t|t-1} + (1 - w_t^{'} \iota) r_f - \tau \left| w_t - w_{t-1} \circ \frac{1 + r_t}{1 + r_{p,t}} \right| \right\}$$

subject to

$$\left(\sigma_p^*\right)^2 = w_t^{'} \underbrace{\frac{\delta n_{t-1}}{\delta n_{t-1} - 2} \left(x_{t-1} \Omega_{t|t-1} x_{t-1}^{'} + Q_{t|t-1} \right)}_{\text{estimate of the conditional covariance matrix}} w_t$$

- $\sigma_p^* = 10\%$ (annualized)
- $\tau = 0.0008$

Economic Evaluation of Forecasting Performance

- Modified version of quadratic utility
- We measure how much a mean-variance investor is willing to pay for **conditioning on a particular VAR configuration with dynamic learning** (*) instead of basing portfolio decisions upon the simple **random walk (RW)** model

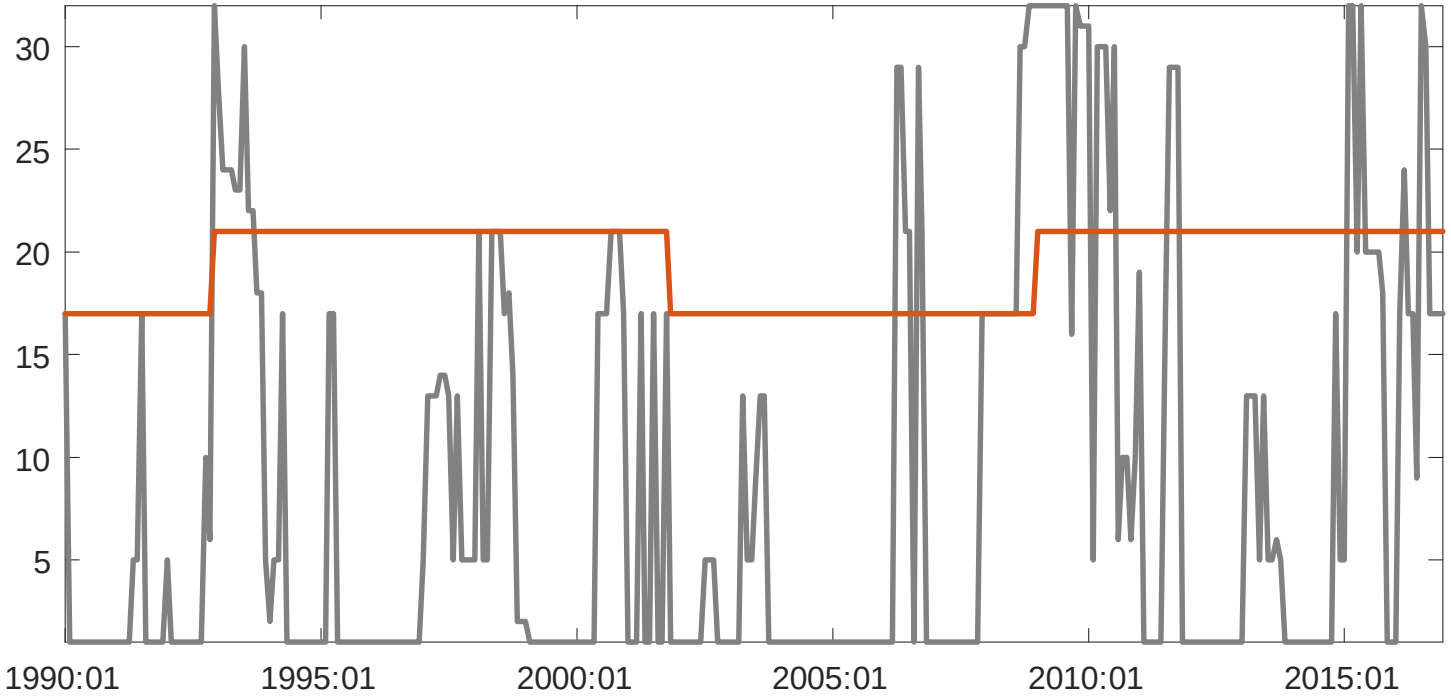
Calculation of Φ^{TC}

$$\underbrace{\sum_{t=0}^{T-1}\left\{\left(R_{p,t+1}^{*,TC}-\Phi^{TC}\right)-\frac{\theta}{2\left(1+\theta\right)}\left(R_{p,t+1}^{TC,*}-\Phi^{TC}\right)^{2}\right\}}_{\text{Average utility of flexible model}}=\underbrace{\sum_{t=0}^{T-1}\left\{R_{p,t+1}^{RW,TC}-\frac{\theta}{2\left(1+\theta\right)}\left(R_{p,t+1}^{RW,TC}\right)^{2}\right\}}_{\text{Average utility of RW model}}$$

- $R_{p,t+1}^{TC,*}$ is the gross portfolio return of the flexible model
- Relative risk aversion: $\theta = 2$

Fast Model Switching

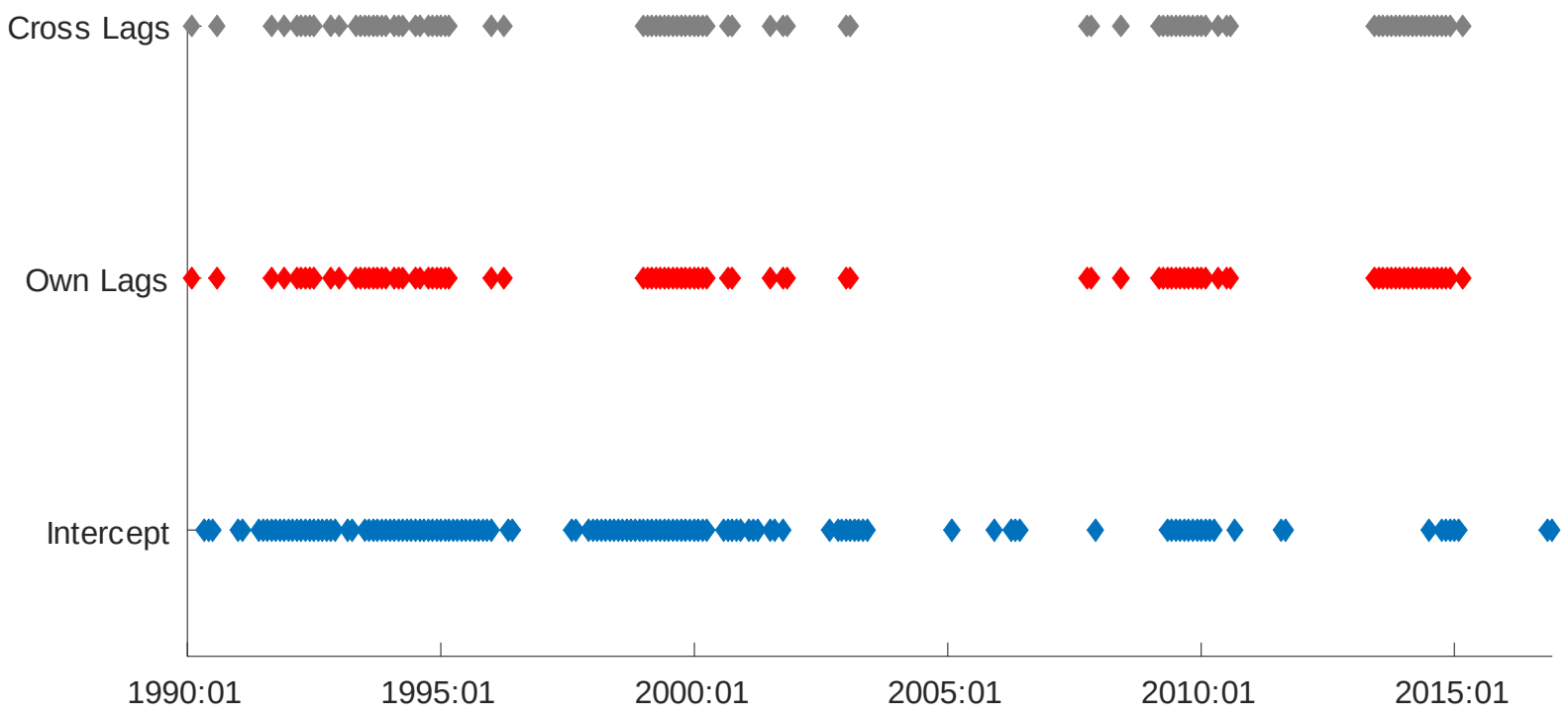
Long Sample



- red line: $\alpha = 1$
- grey line:
  $\alpha \in \{0.20; 0.40; 0.50; 0.60; 0.70; 0.80; 0.90; 0.95; 0.99; 1\}$

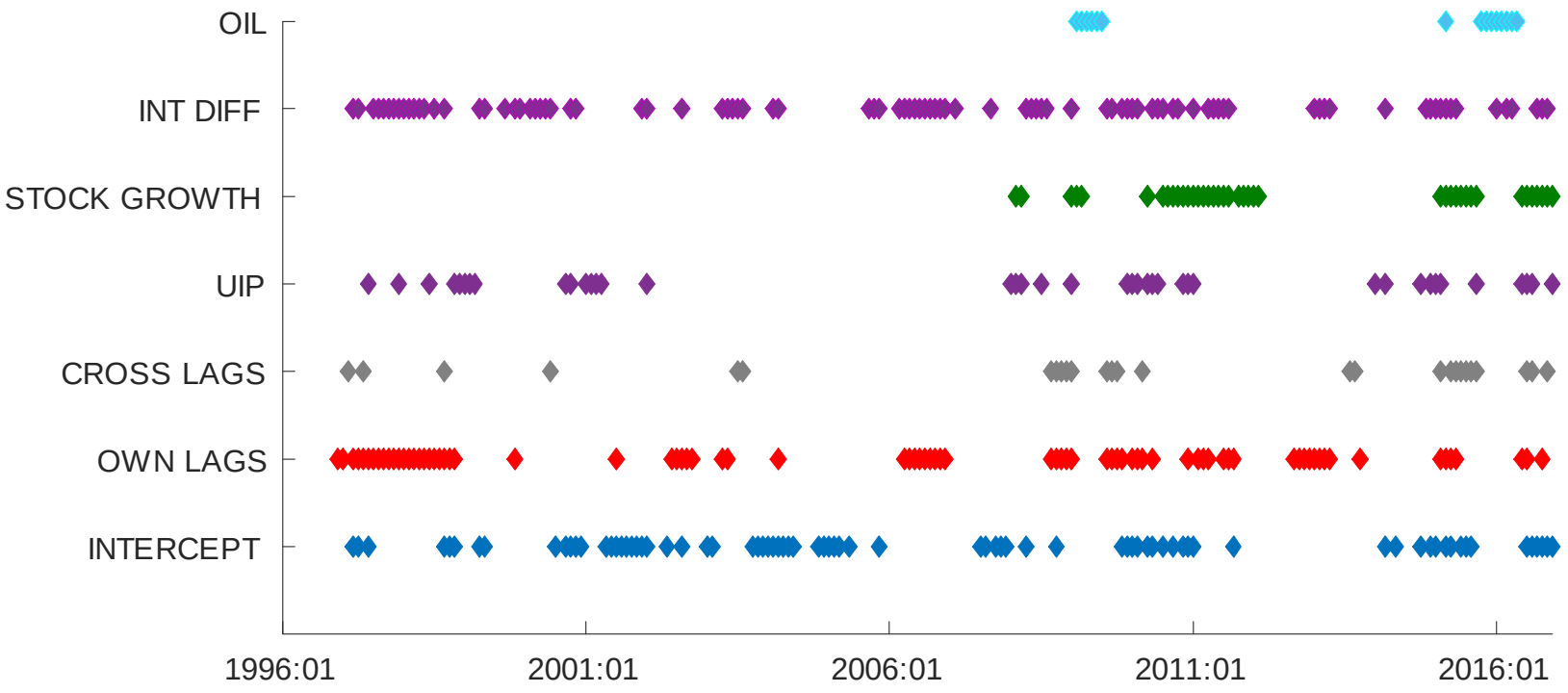
Dynamic Sparsity

Long Sample



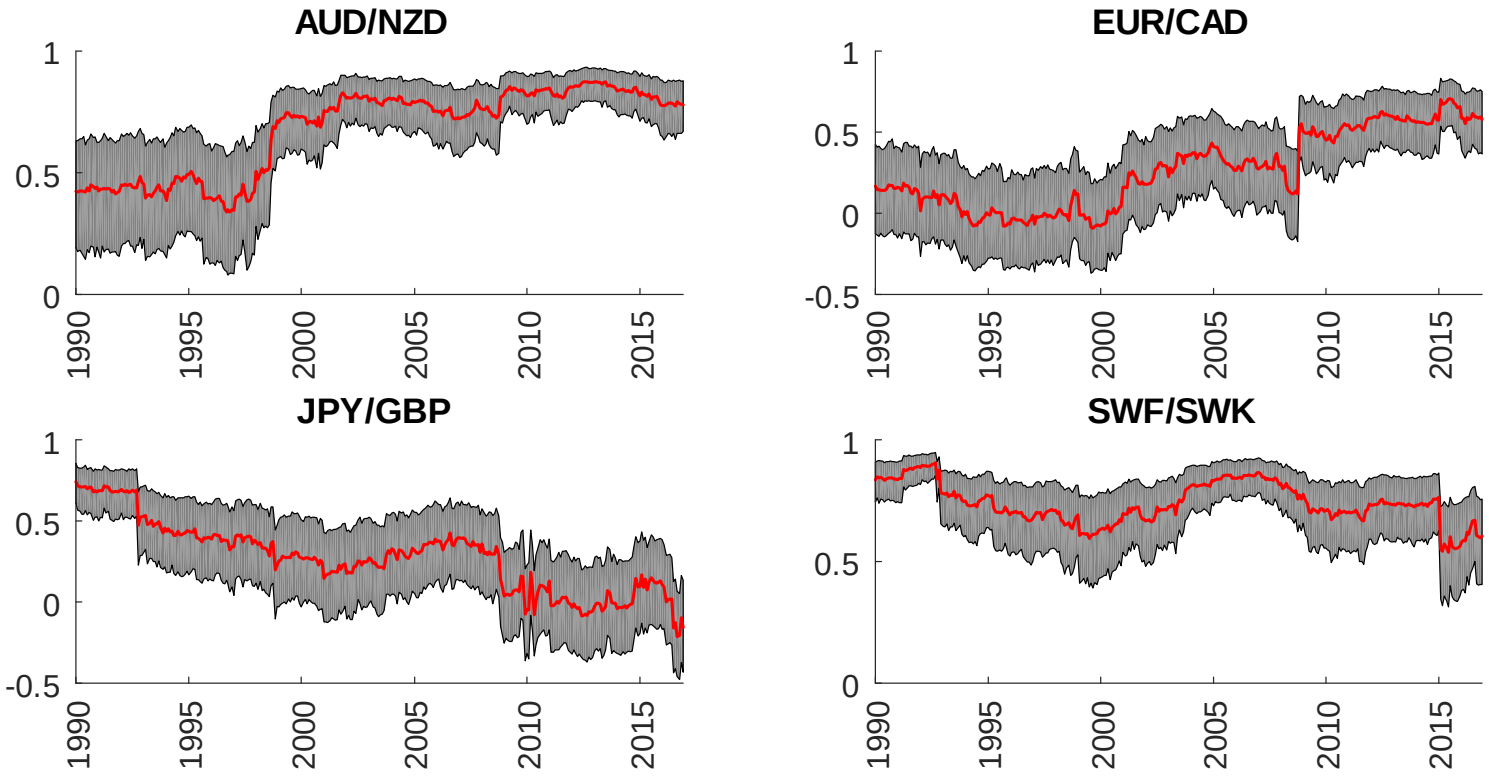
Dynamic Sparsity

Short Sample



Correlation Forecasts and 90% Credibility Intervals

Long Sample



Results: Long Sample

Evaluation period:1990-2016

	Φ^{TC}	SR	SR^{TC}	PLL
Flexible VAR	485**	1.08**	0.92**	22.05
Type of restrictions: VAR lags				
Flexible VAR without own lags ($\gamma_2 = 0$)	365*	0.82*	0.72*	21.86
Flexible VAR without cross lags ($\gamma_3 = 0$)	278	0.80	0.66	21.78
Type of restrictions: Random walk				
Random walk (without drift) with time-varying (co-)variance	17	0.47	0.46	21.65
Type of restrictions: Model selection dynamics				
$\alpha = 1$	−255	0.35	0.19	21.65
$\alpha = 0.99$	−194	0.40	0.24	21.65
$\alpha = 0.95$	60	0.59	0.45	21.68
$\alpha = 0.90$	238	0.78	0.65	21.88
$\alpha = 0.80$	485**	1.08**	0.92**	22.05
$\alpha = 0.70$	478*	1.07**	0.90*	22.06
$\alpha = 0.60$	486*	1.12**	0.93**	22.06
$\alpha = 0.50$	409*	1.04**	0.84**	22.05
$\alpha = 0.40$	276	0.93*	0.70	22.03
$\alpha = 0.20$	181	0.85	0.60	21.98

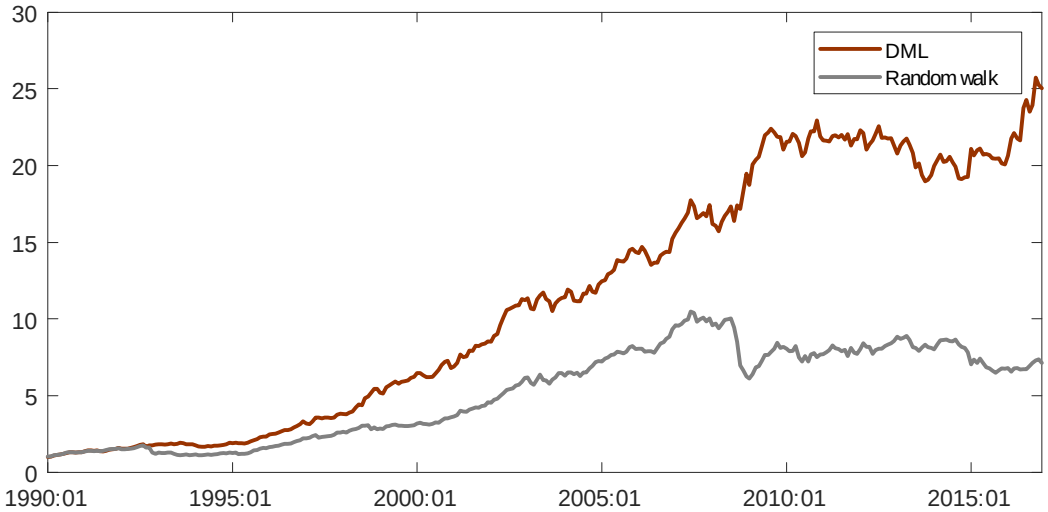
Results: Short Sample

Evaluation period:1996-2016

	Φ^{TC}	SR	SR^{TC}	PLL
Flexible VAR	327	1.01*	0.82*	22.02*
With Regressors				
Flexible VAR with OIL	199	0.89	0.70	22.03*
Flexible VAR with UIP	464*	1.12**	0.93**	22.01*
Flexible VAR with INT_DIFF	388*	1.06*	0.88*	22.02*
Flexible VAR with STOCK_GROWTH	368*	1.06*	0.88*	22.06*
Flexible VAR with ALL REGRESSORS	397*	1.02*	0.87*	22.04*
Type of restrictions: VAR lags				
Flexible VAR without own lags ($\gamma_2 = 0$)	98	0.72	0.60	21.97
Flexible VAR without cross lags ($\gamma_3 = 0$)	200	0.86	0.79	21.78**
Type of restrictions: Random walk				
RW without drift and time-varying (co-)variance	5	0.54	0.53	21.72*
Type of restrictions: Model selection dynamics				
$\alpha = 1$	−427	0.34	0.11	21.69
$\alpha = 0.99$	−464	0.28	0.08	21.66
$\alpha = 0.95$	−167	0.51	0.34	21.79*
$\alpha = 0.90$	98	0.77	0.60	21.96*
$\alpha = 0.80$	266	0.94	0.76	22.02*
$\alpha = 0.70$	327	1.01*	0.82*	22.02*
$\alpha = 0.60$	251	0.97	0.75	22.02*
$\alpha = 0.50$	84	0.82	0.60	21.98
$\alpha = 0.40$	−31	0.71	0.48	21.96
$\alpha = 0.20$	11	0.75	0.52	21.94

Statistics of the Portfolio Returns and Evolution of Wealth

	Long Sample	Short Sample
Mean return annualized (in %)	13.92	12.48
Volatility annualized (in %)	10.36	10.25
Skewness	0.07	−0.23
Kurtosis	3.37	3.16
Positive returns (>0 in %)	64	65
First-order autocorrelation of returns	0.08	0.10
First-order autocorrelation of squared returns	−0.03	−0.04
Correlation to S&P 500 returns	−0.04	−0.01



Summary

- New multivariate approach to exchange rate forecasting with large utility gains for an FX investor
- **Main conceptual advantages:**
  - Approach nests many simpler models as special cases
  - Flexible shrinkage prior
  - Transparency
  - Computational feasibility
- **Main empirical findings:**
  - Sparsity
  - Fast model switching
  - Important for prediction: VAR lags and UIP
  - Findings align with implications of the theoretical and empirical literature

Koop Korobilis (2013) Minnesota-Prior

$$\Omega_{0,i,jj} = \begin{cases} a \cdot s_i^2 & a = 100 \text{ for INTERCEPTS} \\ \frac{\gamma}{r^2} & \gamma \in \left\{10^{-5}; 0.001; 0.005; 0.01; 0.05; 0.1\right\} \text{ for } r = 1, ..., 6 \end{cases} .$$

- Single shrinkage parameter γ
- All variables are treated as engonenous
- This results in large VARs (+27 additional variables in short sample)

Results for Koop Korobilis (2013) Minnesota-Prior

Short Sample

	Φ^{TC}	SR	SR^{TC}	PLL
DML with ALL VARIABLES	-130	0.42	0.36	21.84
Type of restrictions: Model selection dynamics				
DML $(\alpha = 1)$	-201	0.33	0.30	21.63
DML $(\alpha = 0.99)$	-201	0.33	0.30	21.63
DML $(\alpha = 0.95)$	-253	0.32	0.26	21.69
DML $(\alpha = 0.90)$	-287	0.29	0.23	21.79
DML $(\alpha = 0.80)$	-210	0.35	0.30	21.84
DML $(\alpha = 0.70)$	-116	0.43	0.38	21.86
DML $(\alpha = 0.60)$	-94	0.45	0.40	21.86
DML $(\alpha = 0.50)$	-21	0.51	0.45	21.87
DML $(\alpha = 0.40)$	-14	0.53	0.47	21.86
DML $(\alpha = 0.20)$	-21	0.53	0.46	21.84

Results for Koop Korobilis (2013) Minnesota-Prior

Long Sample

	Φ^{TC}	SR	SR^{TC}	PLL
DML with ALL VARIABLES	−70	0.50	0.36	21.88
Type of restrictions: Model selection dynamics				
DML ($\alpha = 1$)	−123	0.45	0.30	21.74
DML ($\alpha = 0.99$)	−19	0.57	0.44	21.73
DML ($\alpha = 0.95$)	43	0.61	0.45	21.83
DML ($\alpha = 0.90$)	25	0.58	0.44	21.91
DML ($\alpha = 0.80$)	36	0.60	0.46	21.93
DML ($\alpha = 0.70$)	−44	0.53	0.39	21.93
DML ($\alpha = 0.60$)	−20	0.56	0.41	21.95
DML ($\alpha = 0.50$)	115	0.68	0.55	21.95
DML ($\alpha = 0.40$)	70	0.65	0.49	21.94
DML ($\alpha = 0.20$)	−69	0.53	0.36	21.90

"Dense" Prior

$$\Omega_{0,i,jj} = \begin{cases} \gamma_1 s_i^2 & \gamma_1 \in \{0; 10\} \text{ for INTERCEPTS} \\ \frac{\gamma_2}{r^2} & \gamma_2 \in \{0.0001; 0.001; 0.1\} \text{ for OWN LAGS} \\ \frac{\gamma_3 s_i^2}{r^2 s_j^2} & \gamma_3 \in \{0.0001; 0.001; 0.1\} \text{ for CROSS LAGS} \\ \gamma_4 s_i^2 & \gamma_4 \in \{0.0001; 0.001; 0.1\} \text{ for variable UIP} \\ \gamma_5 s_i^2 & \gamma_5 \in \{0.0001; 0.001; 0.1\} \text{ for variable STOCK_GROWTH} \\ \gamma_6 s_i^2 & \gamma_6 \in \{0.0001; 0.001; 0.1\} \text{ for variable INT_DIFF} \\ \gamma_7 s_i^2 & \gamma_7 \in \{0.0001; 0.001; 0.1\} \text{ for variable OIL} \end{cases}$$

Results for "Dense" Prior

Long Sample

	Φ^{TC}	SR	SR^{TC}	PLL
DML with ALL POTENTIAL REGRESSORS	343	0.98	0.75	22.01
Type of restrictions: Model selection dynamics				
DML ($\alpha = 1$)	−141	0.69	0.29	21.78
DML ($\alpha = 0.99$)	−230	0.45	0.21	21.70
DML ($\alpha = 0.95$)	379*	1.00*	0.79*	21.77
DML ($\alpha = 0.90$)	409*	0.97	0.80	21.90
DML ($\alpha = 0.80$)	337	0.92	0.73	21.94
DML ($\alpha = 0.70$)	497*	1.08	0.88	22.03
DML ($\alpha = 0.60$)	343	0.98	0.75	22.01
DML ($\alpha = 0.50$)	245	0.89	0.65	21.99
DML ($\alpha = 0.40$)	214	0.87	0.63	21.98
DML ($\alpha = 0.20$)	131	0.79	0.54	21.95

Results for "Dense" Prior

Short Sample

	Φ^{TC}	SR	SR^{TC}	PLL
DML with ALL REGRESSORS	206	0.96	0.72	22.04*
Type of restrictions: Model selection dynamics				
DML ($\alpha = 1$)	−321	0.29	0.21	21.71*
DML ($\alpha = 0.99$)	−163	0.53	0.34	21.72*
DML ($\alpha = 0.95$)	−10	0.65	0.47	21.85*
DML ($\alpha = 0.90$)	83	0.74	0.57	21.95*
DML ($\alpha = 0.80$)	189	0.92	0.71	22.00*
DML ($\alpha = 0.70$)	278	1.03	0.81	22.05*
DML ($\alpha = 0.60$)	243	1.01	0.78	22.05*
DML ($\alpha = 0.50$)	231	0.96	0.74	22.03*
DML ($\alpha = 0.40$)	206	0.96	0.72	22.04*
DML ($\alpha = 0.20$)	61	0.83	0.57	21.97*

Fama Regression/UIP

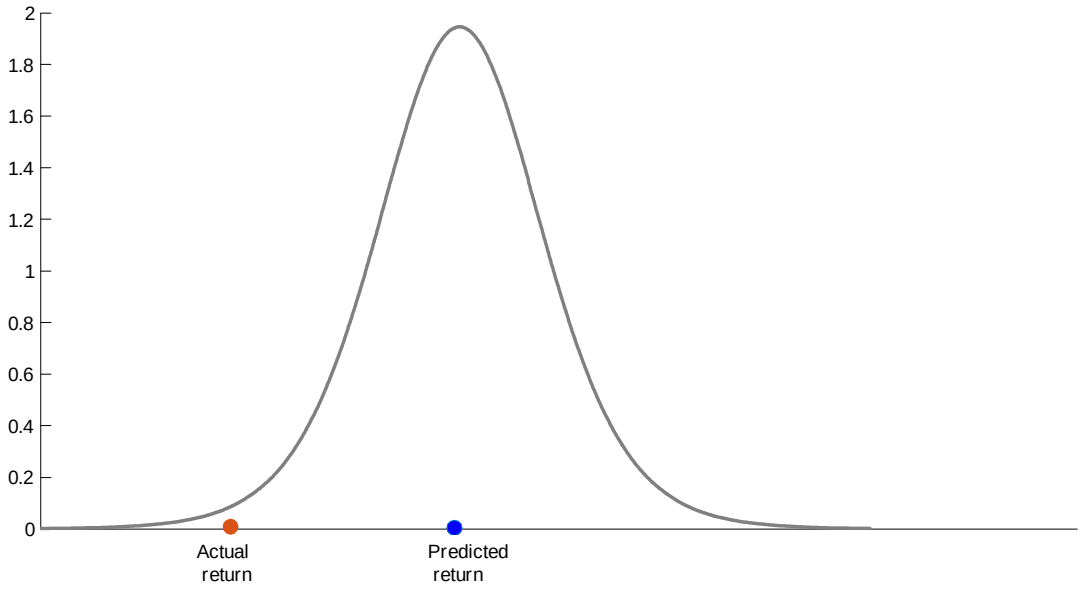
The UIP condition postulates that the difference in interest rates between two countries should equal the expected change in exchange rates between the countries' currencies (Engel 2013):

UIP

$$E_t \Delta s_{t+1} = int_t - int_t^*,$$

where $\Delta s_{t+1} \equiv s_{t+1} - s_t$. $E_t \Delta s_{t+1}$ denotes the expected change (at time t for $t+1$) of log exchange rates, denominated as US dollar per foreign currency. int_t (int_t^*) is the one-period nominal interest rate US (foreign) securities. We use $z_{1,t} = int_t - int_t^*$ as a predictor.

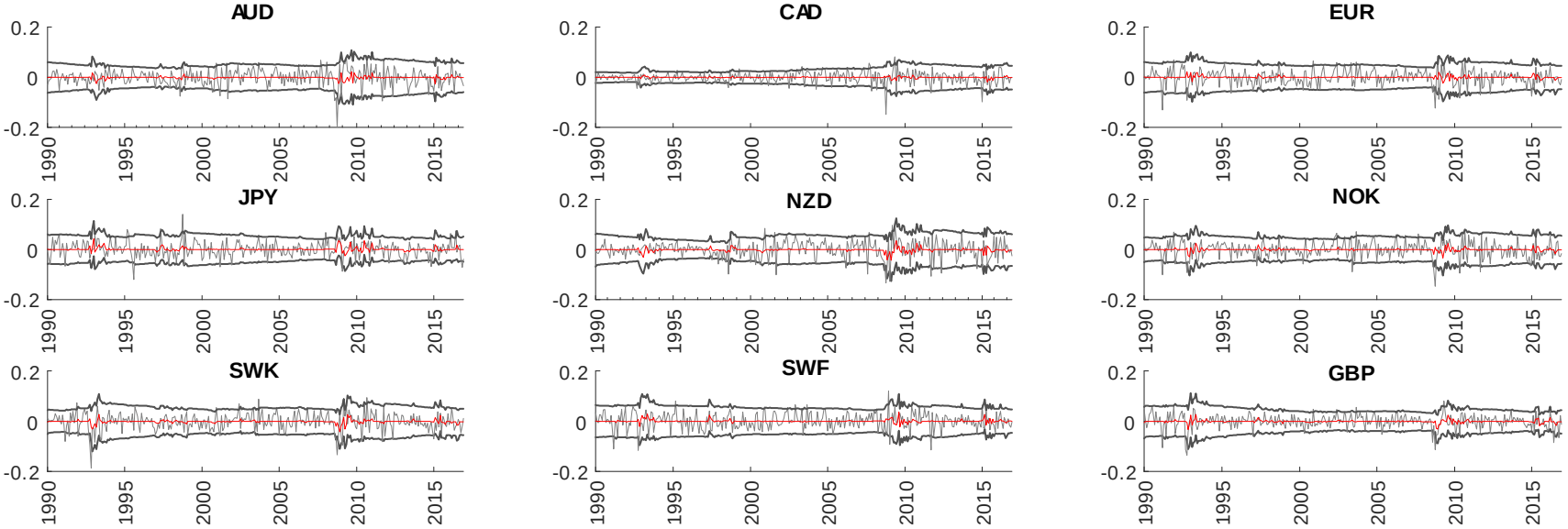
Predictive Likelihoods



- Exponential downweighting of past forecasting performance:
- Example for $\alpha = 0.95$: forecast performance three years ago receives $\sim 15\%$ as much weight as the forecast performance last period
- Example for $\alpha = 0.90$: forecast performance three years ago receives $\sim 2\%$ as much weight as the forecast performance last period

Interval Forecasts

Long Sample



Related Work on Exchange Rate Forecasting

- Abbate & Marcellino (2018, *Royal Statistical Society Series A*)
- Ackermann, Pohl and Schmedders (*2017, Management Science*)
- Kouwenberg, Markiewicz, Verhoeks and Zwinkels (*2017, Journal of Financial and Quantitative Analysis*)
- Li, Tsiakas and Wang (*2015, Journal of Financial Econometrics*)
- Berge (*2014*, *Journal of Applied Econometrics*)
- Carriero, Kapetanios and Marcellino (*2009, International Journal of Forecasting*)
- Della Corte, Sarno and Tsiakas (*2009, Review of Financial Studies*)
- Wright (*2008, Journal of Econometrics*)