

Subjective Expectations and Uncertainty

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Outline

1 Main results

- Axiomatic theory of subjective expectations
- The Subjective Assessment Hypothesis (SAH)
- Uncertainty index

Outline

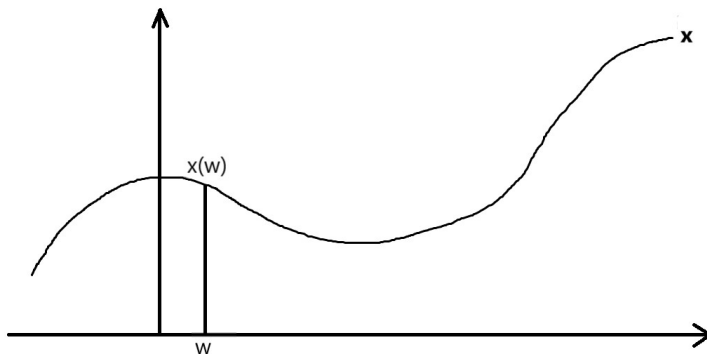
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Notation

- $\omega \in \Omega$ - state of the “world” understood as the context borne in mind by an individual when forming expectations at some particular moment of time
- $x := \{x(\omega) | \omega \in \Omega\}$ - unknown quantity (whole function, not its value at particular ω), and $\sup_{\omega \in \Omega} |x(\omega)| < \infty$
- From now on, unknown quantities in bold letter
- In particular $x(\omega) = 1$; for all $\omega \in \Omega$, denoted as **1**
- \mathcal{S} - linear (sub)space of all unknown quantities (feasible for given person to make expectations)

Unknown quantity



Axioms and uniqueness

Subjective expectation is a function $E : \mathcal{S} \rightarrow \mathbb{R}$ that obeys:

- ① $E(\mathbf{x} + \mathbf{y}) = E(\mathbf{x}) + E(\mathbf{y})$; for all $\mathbf{x}, \mathbf{y} \in \mathcal{S}$
- ② $\inf_{\omega \in \Omega} \mathbf{x}(\omega) \leq E(\mathbf{x}) \leq \sup_{\omega \in \Omega} \mathbf{x}(\omega)$; for all $\mathbf{x} \in \mathcal{S}$
- ③ $\mathbf{1}$ belongs to \mathcal{S}

Theorem

For each individual there is one and only one subjective expectation on \mathcal{S}

Subjective expectation and probability

Theorem

There exists one and only one finitely additive probability π , such that $E(\mathbf{x}) = \int_{\Omega} \mathbf{x}(\omega) \pi(d\omega)$, for all $\mathbf{x} \in \mathcal{S}$

but ... with further assumptions concerning

- measurability of unknown quantities
- “richness” of the linear space \mathcal{S} in relation to the states of the world (Ω)
- Hence we give up the idea to model subjective expectations as implication of dealing with fully specified probability

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Decision rule

We assume that the process of subjective assessment for each person may be approximated by the following decision rule:

$$\min_{\phi, \beta_1, \dots, \beta_n} E(\mathbf{y} - \hat{\mathbf{y}}) \cdot (\mathbf{y} - \hat{\mathbf{y}}) + \lambda E(\bar{\mathbf{y}}\mathbf{1} - \hat{\mathbf{y}}) \cdot (\bar{\mathbf{y}}\mathbf{1} - \hat{\mathbf{y}})$$

where $\mathbf{y} \cdot \mathbf{z} := y(\omega)z(\omega)$, and $\hat{\mathbf{y}} = \phi\mathbf{1} + \beta_1\mathbf{x}_1 + \dots + \beta_n\mathbf{x}_n$

- $\lambda \geq 0$, $\bar{y} \in \mathbb{R}$ are given
- **Note:** process of subjective assessment involves unknown quantities (whole functions) i.e. we look for $\hat{\mathbf{y}}$ and not $E(\hat{\mathbf{y}})$

The Subjective Assessment Hypothesis (SAH)

- The solution to the decision rule is called the SAH:

$$\hat{y} = (1 - \alpha)[E(y)\mathbf{1} + \hat{\beta}_1(\mathbf{x}_1 - E(\mathbf{x}_1)\mathbf{1}) + \cdots + \hat{\beta}_n(\mathbf{x}_n - E(\mathbf{x}_n)\mathbf{1})] + \alpha \bar{y}\mathbf{1}$$

$$\text{where } 0 \leq \alpha = \frac{\lambda}{1+\lambda} < 1$$

Theorem

\hat{y} is unique

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Construction

- Using the SAH we decomposed the revision expectations as

$$\begin{aligned} & E_{t+1}(\hat{y}_{t+1}) - E_t(\hat{y}_{t+1}) \\ &= \beta_{1,t}(x_{1,t} - E_t(\hat{x}_{1,t})) + \dots + \beta_{n,t}(x_{n,t} - E_t(\hat{x}_{n,t})) \\ & - \alpha\beta_{1,t}(x_{1,t} - \bar{x}_{1,t}) - \dots - \alpha\beta_{n,t}(x_{n,t} - \bar{x}_{n,t}) + \varepsilon_{t+1} \end{aligned}$$

- $E_h(\cdot)$ - subjective expectation made at time "h"
- Parameters $\beta_{1,t}, \dots, \beta_{n,t}$ must change in time (it follows from the SAH)
- ε_{t+1} is the carrier of uncertainty tj. part of the revision that can't be explained by the SAH and knowing $x_{1,t}, \dots, x_{n,t}$

The empirical model

- Uncertainty index is volatility of the uncertainty carrier ε_t

$$E_{t+1}(\hat{y}) - E_t(\hat{y}) = c_t + (x_t - E_t(\hat{x}))'\beta_t - \alpha(x_t - \bar{x}_t)'\beta_t + \varepsilon_t$$

$$\varepsilon_t \sim N(0, e^{h_t}) \Rightarrow \{e^{\frac{1}{2}h_t}\}_{t=1}^T = \text{uncertainty index}$$

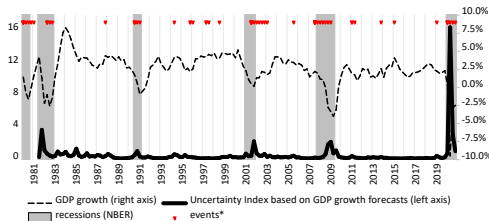
$$h_t = \mu + \psi(h_{t-1} - \mu) + u_t; \quad u_t \sim N(0, \sigma^2)$$

$$\begin{bmatrix} c_t \\ \beta_t \end{bmatrix} = v + F \left(\begin{bmatrix} c_{t-1} \\ \beta_{t-1} \end{bmatrix} - v \right) + \omega_t; \quad \omega_t \sim N(0, \Sigma)$$

Data

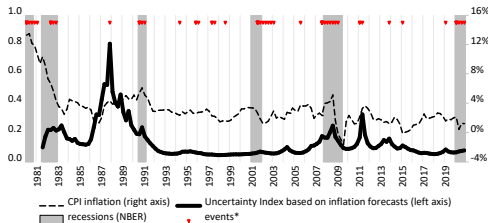
- Survey of Professional Forecasters, 1981:Q3 - 2020:Q4
- Two indices: $E_t(\hat{y})$ is expectation of GDP growth or inflation
- $E_t(\hat{x})$ are expectations of GDP growth, inflation and interest rates
- x_t - realizations of GDP growth, inflation and interest rates
- \bar{x}_t - consensus from previous survey

GDP growth-based uncertainty index



Notes: * 1987Q4 – Black Monday (19th October), 1990Q3-1991Q1 – Gulf War I, 1994Q2 – surge in commodity prices, 1995Q4-1996Q1 – federal government shutdown, 1997Q2-1997Q3 – Asian Crisis, 1998Q3 – Russian LTCM Default, 2001Q3 – 9/11 terrorist attack, 2001Q4-2002Q3 – Worldcom and Enron, 2002Q4-2003Q1 – Gulf War II, 2005Q3 – hurricane Katrina, 2007Q3-2009Q1 – credit crunch & sub-prime crisis, 2011Q1-2011Q2 – debt ceiling dispute, 2012Q3 – fiscal cliff, 2013Q4 – federal government shutdown, 2015Q1 – coalition against ISIL, 2019Q1 – federal government shutdown, since 2020Q1 –

Inflation-based uncertainty index



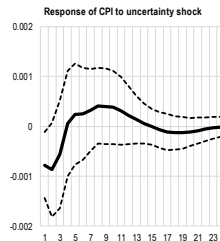
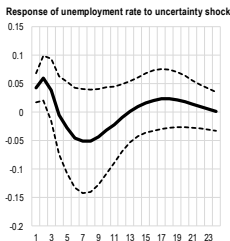
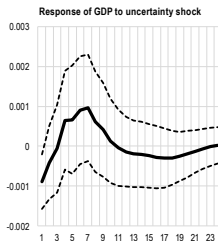
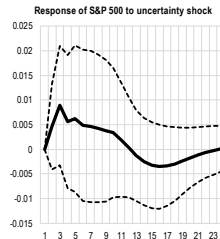
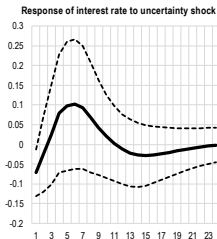
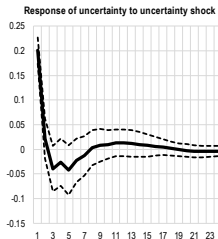
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Correlations with other indices

Uncertainty measure	GDP-growth-forecast based	inflation-forecasts based
VXO	0.29***	0.27***
EPU	0.65***	0.12
EPU (Monetary Policy)	0.25***	0.24***
JLN	0.55***	0.15
forecast disagreement (GDP growth)	0.96***	0.10
forecast disagreement (inflation)	0.21***	0.24***

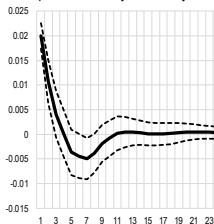
Notes: Symbols *** / ** / * denote that correlation is statistically different from zero at the 0.01 / 0.05 / 0.10 levels. Due to data availability correlations are calculated on different time ranges – VXO: 1986q3 – 2020q4; EPU: 1985q1 – 2020q4; JLN: 1981q4 – 2011q4; forecast disagreement: 1981q4 – 2020q4. JLN uncertainty measure refers to 3-month-ahead uncertainty. Forecast disagreement is measured by interquartile range of one-quarter-ahead forecasts. In parentheses are shown correlations on sample ending before the Covid-19 pandemic.

IRFs for GDP-forecast based index

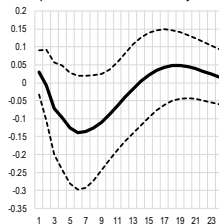


IRFs for inflation-forecast based index

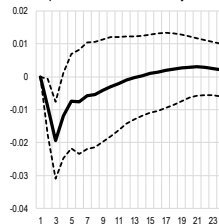
Response of uncertainty to uncertainty shock



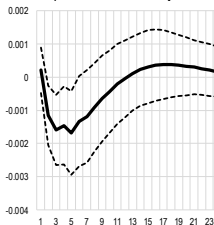
Response of interest rate to uncertainty shock



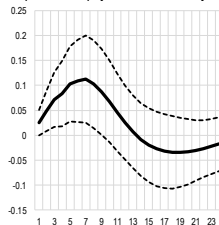
Response of NASDAQ to uncertainty shock



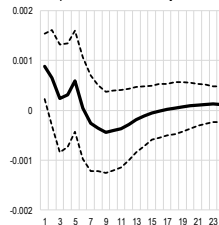
Response of GDP to uncertainty shock



Response of unemployment rate to uncertainty shock



Response of CPI to uncertainty shock



Thank you for attention