Measuring Financial Cycle Time

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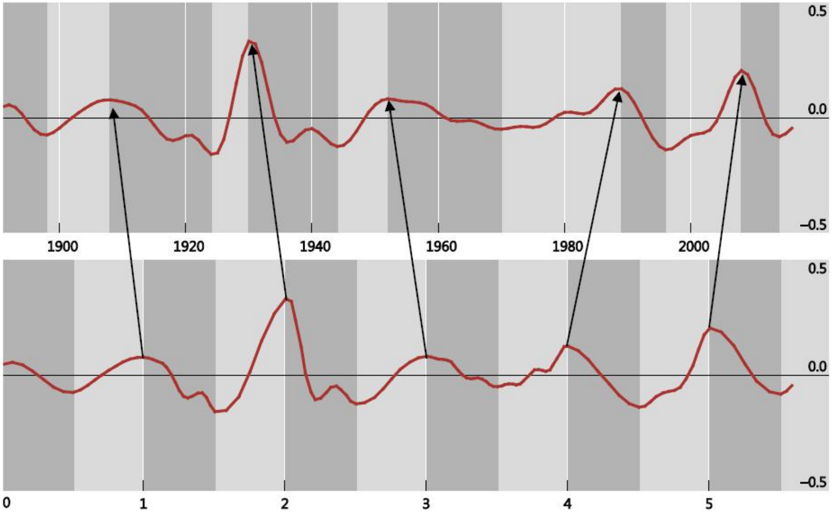
June 2020

Disclaimer

- The views expressed in the presentation are those of their authors and not necessarily the views of the Bank for International Settlements, Barclays, the Bank of England or any of its bodies.
- The paper was written when Marek was working for the Bank of England.

What is time deformation and why do we care?

Time deformation à la Burns and Mitchell (1946)



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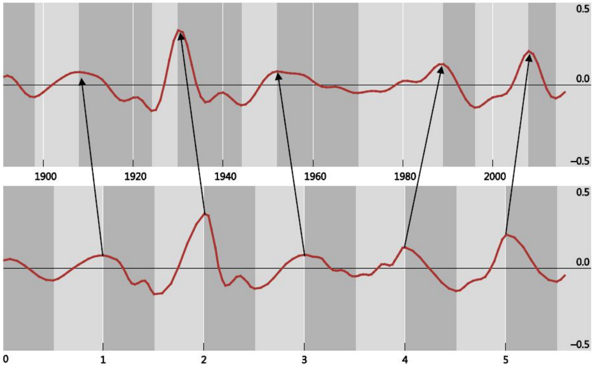

- Statistical accuracy - improvements in statistical testing/forecasting
- Policy insights - this time might NOT be different
- Economic insights into drawbacks when using benchmark macro-models

Contribution of this project

- Improve our understanding of financial cycle dynamics
- In particular, assess the extent of time deformation in the financial cycle i.e. shrinkage and dilation of cycles in calendar time?
- Try to identify variables that reflect the nature of the time deformation?

Modeling time deformation - Stock (1987)

- Financial cycle follows continuous-time autoregressive (CAR) process $f(s)$:

$$\frac{df(s)}{ds} = \lambda f(s) + d\eta(s) \qquad var(d\eta(s)) = Q$$

  - The operational time scale s differs from calendar time scale t
  - $f(s)$ has stable parameters: λ and Q in s
- The time scale is unknown so it is not possible to observe $f(s)$
- Observations Y_t are discretely sampled and equally-spaced in calendar time
  - This corresponds to sampling from the process $f(s)$ at an irregular frequency
- Denoting the mapping of s into t by s=g(t):
  - $Y_t = f(g(t)) \quad t = 1, ..., T$

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Mapping operational time into calendar time

- The g function has to be monotonically increasing, but can also depend on a set of z
  - While z can include (lags of) Y_t, it needs to be pre-determined at time t–1
- Following Stock(1988) we assume the following form of time deformation:

$$\Delta g = \frac{exp(\mathbf{c}'\mathbf{z_{t-1}})}{\sum_{i=2}^{N} exp(\mathbf{c}'\mathbf{z_{i-1}})/(N-1)} \Delta t$$

  - when $\mathbf{c} = \mathbf{0}$, calendar time is proportional to financial cycle time, hence there is no time deformation
  - $\mathbf{z_{t-1}}$ is set of macro-financial variables "slowing down" or "speeding up" of the financial cycle

The calendar time representation

- $f(s)$ can then be expressed in (discrete) calendar time a TVP-AR:

$$f(t) = a_t f(t-1) + v_t \qquad var(v_t) = Q_t$$

$$a_t = e^{\lambda \Delta g(t, cz_{t-1})}$$

$$Q_t = \int_0^{\Delta g(t, cz_{t-1})} e^{\lambda(\Delta g(t,cz_{t-1})-s)} Q e^{\lambda(\Delta g(t,cz_{t-1})-s)} ds$$

- The parameters vary systematically with the g function
  - A larger $\Delta g(t; z_{t-1})$ corresponds to a compression of t with respect to s
    - Calendar time accelerates
    - So the process has a larger variance
    - And a weaker dependence on its past values

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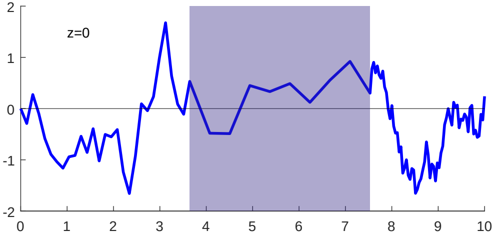
CAR(1) model - example of time deformation



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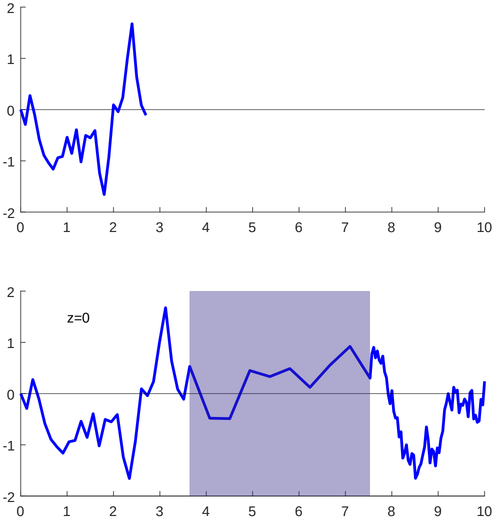

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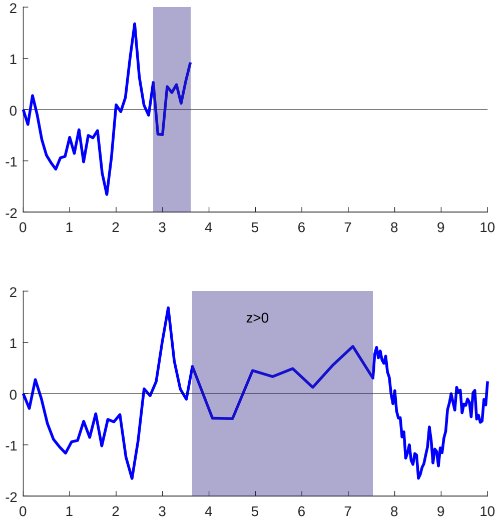

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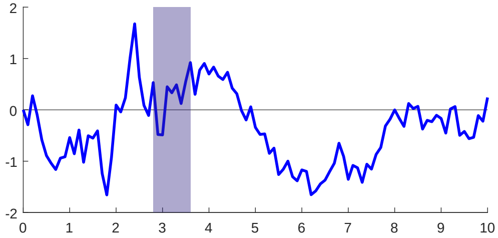

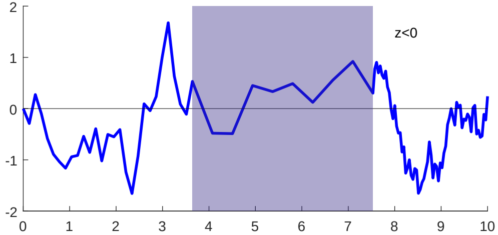

Our model

Observable variables driven by financial cycle:

$$y(t) = f(g(t, \boldsymbol{z}))$$

Financial cycle dynamics - CAR(1)

$$\frac{df(g(t,\boldsymbol{z}))}{dg(t,\boldsymbol{z})} = \lambda f(g(t,\boldsymbol{z})) + d\eta(g(t,\boldsymbol{z})) \qquad var(d\eta(g(t,\boldsymbol{z}))) = Q$$

Time deformation function:

$$\Delta g = \frac{exp(\boldsymbol{c}'\boldsymbol{z_{t-1}})}{\sum_{i=2}^{N} exp(\boldsymbol{c}'\boldsymbol{z_{i-1}})/(N-1)} \Delta t$$

Data

- Y: Credit to households scaled by (potential) GDP
- z: variables likely to accelerate/slow down the financial cycle
- We hypothesize that variables related to attitudes towards risk could play a role:
  - Real long-term interest rate
  - Volatility of inflation
  - NVIX
  - Corporate spreads

Baseline Results

	Model 1	Model 2	Model 3	Model 4	Model 5	**Model 6**
λ	-0.83	-0.65	-0.84	-0.83	-0.82	**-0.67**
	[-3.13]	[-3.07]	[-3.38]	[-3.29]	[-3.33]	**[-3.01]**
Q	0.01	0.01	0.01	0.01	0.01	**0.01**
Real LT rate		0.53				**0.33**
		[6.92]				**[3.16]**
Inflation vol.			0.26			**0.30**
			[2.50]			**[2.17]**
Corp. spread				0.87		**0.56**
				[9.36]		**[4.76]**
NVIX					0.54	**0.22**
					[5.93]	**[2.00]**
LogL	-722	-747	-730	-761	-741	**-771**
LR-test		50	16	78	38	98
p-value		0.00	0.00	0.00	0.00	0.00

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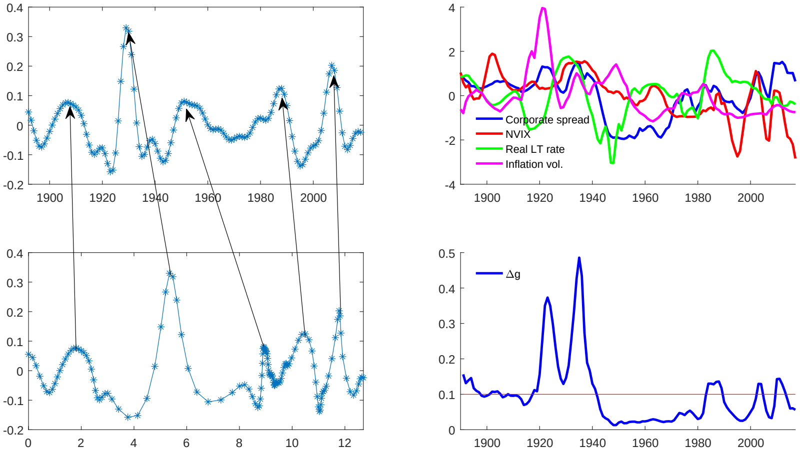
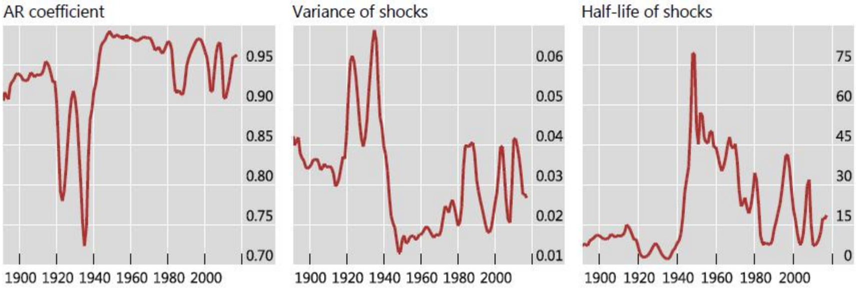


Baseline results - different perspective



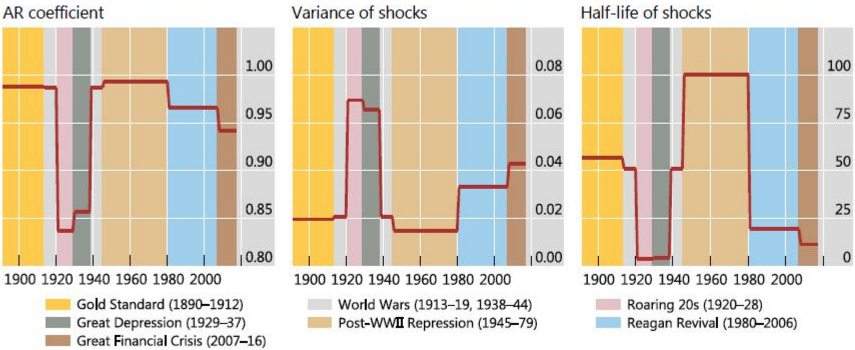
Additional variables

	Model 6	Model 7	Model 8	Model 9	Model 10
λ	[-3.01]	[-2.84]	[-3.01]	[-3.04]	[-2.91]
Real LT rate	[3.16]	[1.84]	[3.63]	[1.88]	[3.52]
Inflation vol.	[2.17]	[1.06]	[-0.45]	[3.51]	[1.62]
Corp. spread	[4.76]	[5.07]	[6.71]	[4.62]	[5.07]
NVIX	[2.00]	[1.22]	[-0.85]	[2.10]	[2.15]
Rates slope		[-1.78]			
GDP vol.			[3.93]		
NBER recession				[5.18]	
FC contraction					[2.47]
LogL	-771.41	-773.00	-779.17	-788.69	-774.48
LR		3.17	15.52	34.56	6.14
p-value		0.05	0.00	0.00	0.01

Different regimes

	Gold standard	Post-WWII Repression	Great Recession
Gold standard 1890-1913	x	0.57	-1.6
	x	[2.11]	[-4.04]
World Wars 1914-20, 1939-45	0.1	0.68	-1.49
	[0.3]	[2.12]	[-3.48]
Roaring Twenties 1921-28	2.68	3.25	1.08
	[6.72]	[8.58]	[2.28]
Great Depression 1929-38	2.54	3.11	0.94
	6.08	[7.8]	[1.92]
Post-WWII Repression 1946-79	-0.57	x	-2.17
	[-2.11]	x	[-5.79]
Regean Rivival 1980-2006	1.06	1.63	-0.54
	[3.67]	[6.26]	[-1.39]
Great Recession 2007-16	1.6	2.17	x
	[4.04]	[5.79]	x

Different regimes



What drives time-deformation a bit of theory

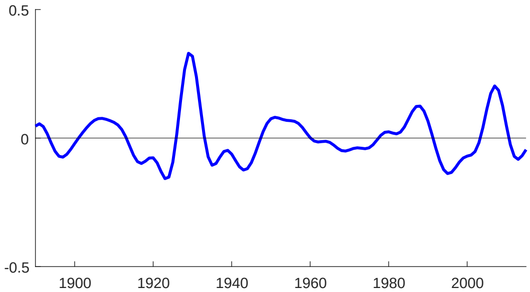
- State-contingent decisions
  - Rational inattention - Sims (2010); Bacheta and van Wincoop (2010)
  - Behavioural inattention - Gabaix (2017)
- Psychological time interest rates (Forgetfulness and discounting) - Allais (1972), Allais and Barthalon (2014)
  - Time preference shocks - Albuquerque, Eichenbaum, Rebelo (2016)
  - Diagnostic expectations - Bordalo, Gennaioli and Shleifer (2017)

Conclusions

- We jointly estimate the time series behaviour of financial conditions and the nature of the time deformation
  - We found statistical evidence of significant time deformation in the US financial cycle
  - Its extent appears to be associated with measures of subjective risk perceptions
- To understand the wider implications of these findings theoretical models are needed
  - Models should be able to match the stylised facts about time deformation
  - Having identified variables associated with time deformation provides a road map

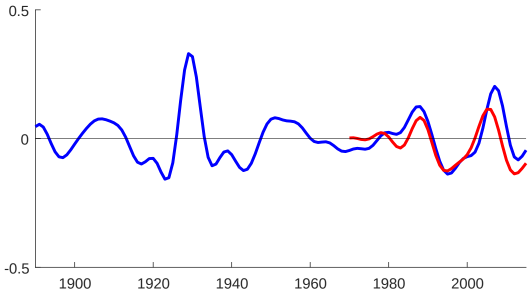
Thank you!

Our measure of the financial cycle?

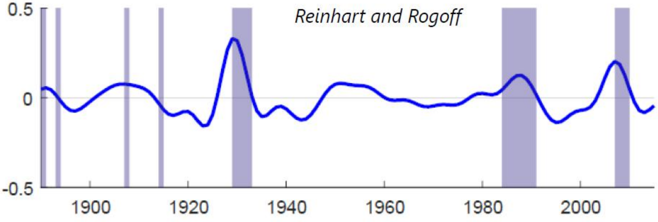


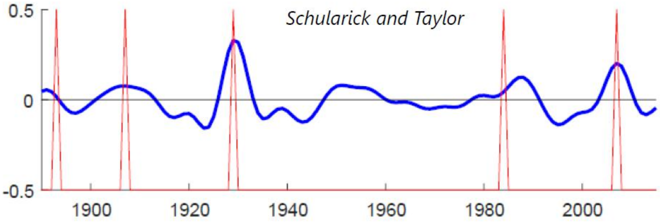
- Credit to private non-financial sector scaled by potential GDP;
- Smoothed using band pass filter - (7,30)

… vs Drehmann et al (2012)



The financial cycle and financial crises

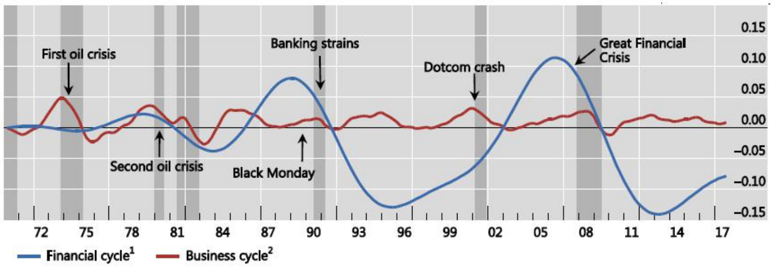




- Financial cycle vs business cycle
- Financial cycle measured using credit and house-price data

LINK1
LINK2

The financial and business cycles in the United States



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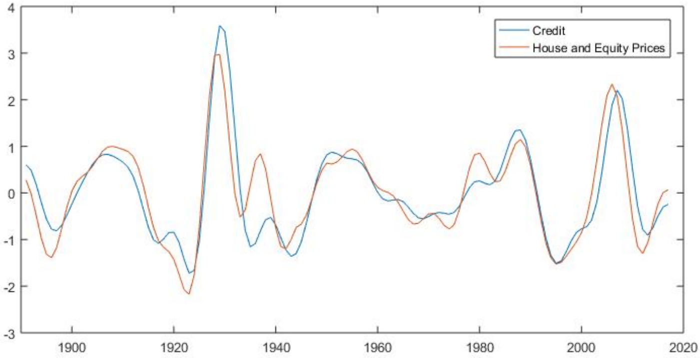
Financial cycle measured using credit and house-price data



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Robustness - credit and property prices

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λ	-1.10	-1.00	-1.09	-1.04	-1.15	**-0.92**
	[-7.58]	[-3.56]	[-3.66]	[-3.57]	[-3.75]	[-3.39]
Q	1.96	0.43	0.44	0.46	0.46	**0.64**
Real LT rate		0.44				**0.36**
		[4.04]				[2.68]
Inflation vol.			0.20			**0.21**
			[1.98]			[1.58]
Corp. spread				0.76		**0.53**
				[6.01]		[3.69]
NVIX					0.33	**0.06**
					[2.75]	[0.49]
LogL	-117	-126	-120	-135	-122	**-140**

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To-do list

- Increase AR order
  - Improvement in the statistical fit
- Go multivariate
  - Joint modeling of different financial indicators (e.g. credit and property prices)
- Consider long memory
  - Does time deformation induce spurious long memory in the observed series