Shock amplification and comovement generation of the production network in Poland

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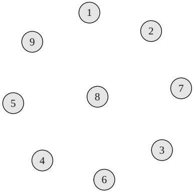
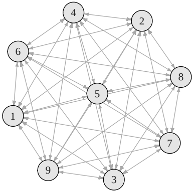
Basic network shapes - symmetry and density

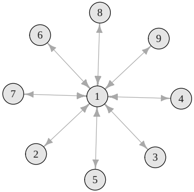
Empty network - symmetric and sparse



Interconnected network - symmetric and dense



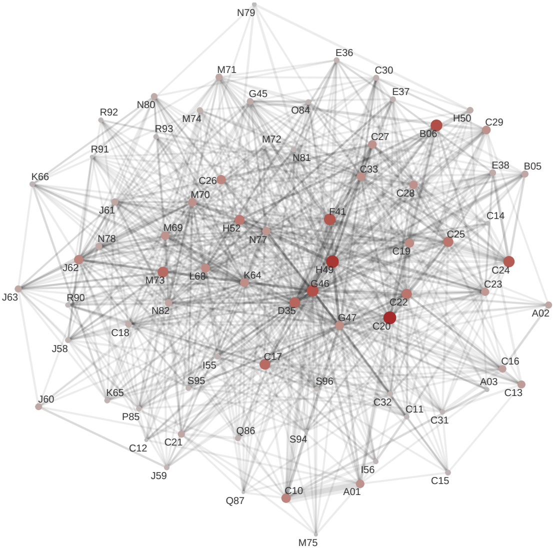
Star network - asymmetric



- What are the consequences of the shape of the network for the transmission and amplification of sectoral impulses into aggregates and prices?
- Lucas (1987) — as we disaggregate the economy into finer sectors, independent sectoral disturbances tend to average out, leaving aggregates unchanged (a weak propagation mechanism)

Production network of the Polish economy

- Graph of the network without small edges — below cost shares associated with symmetrically distributed costs
- Size and darkness of the nodes are proportional to Bonacich (1987) centrality measure
- Economy is a mixture of some important hubs plunged into almost-complete network



Literature overview

- Sources of macroeconomic fluctuations — micro-volatility (see e.g. Gabaix, 2011) or sectoral patterns
- The literature on production networks indicates non-neutrality of the network shape for the aggregates
  - Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) showed that a sufficient heterogeneity in sector's role as input supplier results in higher sectoral shock amplification
  - Baqaee and Farhi (2019) stressed the importance of non-linearities in shock propagation
  - In particular, Atalay (2017) showed that the extent of substitution is important — share of aggregate output volatility explained by sectoral shocks rises from 21% in CD case to 83% with $\epsilon - 0.1$ in the US
- Most of the literature concentrates on aggregates
  - Baqaee and Farhi (2018) is an important exception
  - Carvalho (2014) — cross-sectoral output correlation declines with network distance
- The research on prices is less developed:
  - mostly oriented on markups: Bigio and La'O (2020) or Baqaee and Farhi (2020)
  - or on input-price pass-through, like in Duprez and Magerman (2018)

Our contribution

- We concentrate on the ability of the network structure to:
  - amplify sectoral productivity shocks into the macroeconomic aggregates
  - generate comovement of sectoral variables
- We show the results not only for the real aggregates, but also, what is novel in the literature, for relative sectoral prices of output
- Most of the literature concentrates on US, we will presents results for Poland, a smaller, less developed but simultaneously interconnected economy
- Caveat: we do not use nonlinearities
- A sketch of results:
  - productivity shocks account for 24-30% of total economy GDP volatility and the network amplification is 3.1
  - closely connected sectors tend to comove
  - tfp shocks explain about 24%-29% of the observed dispersion in relative price variation, mostly through volatility, not covariance - the amplification factor is roughly 2.1
  - distance in network seems to be less pronounced in price comovement (than in case of output comovement)
  - covariance of tfp shocks is important in transmission into price comovement, but the shock explains a tiny fraction of its dispersion

A sketch of the theoretical model

- Consider a static economy where production takes place at n distinct nodes with

$$y_i = z_i \kappa_i l_i^{\alpha_i} \prod_{j=1}^{n} x_{ij}^{a_{ij}}, \tag{1}$$

- a_{ij} can be collected into the Leontief IO matrix $A = [a_{ij}]$, which defines a network with nodes, edges and weights
- With simple form of the utility of households, $u(c_1, \ldots, c_n) = \sum_{i=1}^{n} \beta_i \log\left(\frac{c_i}{\beta_i}\right)$, it can be shown that the prices that solve for competitive equilibrium are:

$$\hat{p} = A\hat{p} - \epsilon, \tag{2}$$

where $\hat{p}_i = \log(p_i/w)$ and $\epsilon_i = \log z_i$ measures tfp.
- GDP can be expressed as (for details see e.g. Carvalho and Tahbaz-Salehi, 2019):

$$\log GDP = \sum_{j=1}^{n} \lambda_j \epsilon_j, \tag{3}$$

where the Domar weights $\lambda_i = p_i y_i / GDP$ obey $\lambda = (I - A')^{-1}\beta$.

Volatility and comovement of real and nominal quantities

- Equation (3) implies that:

$$\text{std}(\log GDP) = \frac{\sigma/\alpha}{\sqrt{n}} \left(1 + (n\alpha)^2 \text{var}(\lambda_1, \lambda_2, \ldots, \lambda_n)\right)^{1/2}. \qquad (4)$$

  stating that volatility of GDP is a function of dispersion of size of the nodes in the network, which in turn with normalization of $\beta_i = 1/n$ becomes $\lambda_i = \varphi_i \, n$, where φ is a Bonacich (1987) measure of network centrality.
- Using definition of a Leontief matrix $L = (1 - A)^{-1}$ equation (2) implies that $\hat{p} = -L\epsilon$, so:

$$vcov(\hat{p}) = L\Sigma_\epsilon L' \qquad (5)$$

- We will use these equations, together with the three definitions of network to asses its ability to generate volatility and comovement
  - empirical (observed) network
  - symmetric dense network, with elements defined as: $\frac{1-\alpha}{n}$
  - symmetric sparse network with diagonal A and $a_{ii} = 1 - \alpha$

Data and the measurment of tfp and relative prices

- We use IO matrix for 2015 published by the Polish CSO to measure A and related structures and sectoral time series from Eurostat national accounts to measure volatilities
- We aggregated 77 sectors from original IO into 60 sectors consistent with Eurostat data.
- Log level of tfp is measured consistently with equation (1): it's defined for output, it uses materials, labor and additionally capital to account for measurable factors and nominal cost shares as factor shares (available for years 2000-2017)
- Phillis-Perron and ADF suggest I(1) for all sectors except for two sectors — tfp shocks ϵ_i are inferred form: $\Delta tfp_{it} = \rho_0 + \sum_j \rho_i \Delta tfp_{it-j} + \epsilon_t$ with three assumptions:
    - $\rho_j = 0$, assuming pure random walk shocks
    - $\rho_1 \neq 0, \rho_{j>1} = 0$, assuming estimated AR(1) coefficient
    - $\forall_j \rho_j = BIC$, estimated AR model with lags chosen using BIC criterion with max. lag = 3
- Relative prices $\hat{p}_{it}$ are measured as sectoral deflator of output relative to total economy value added deflator (1996-2019) — in 66% sectors real prices are I(0)

Outliers

Figure: Volatilities of tfp

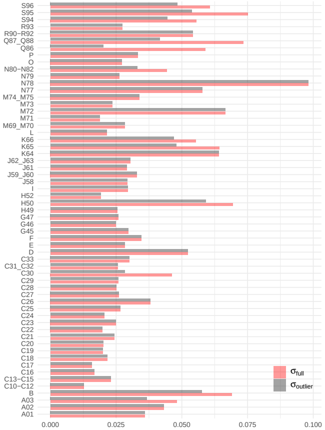
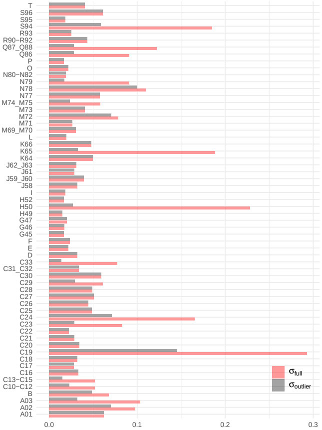


Figure: Volatilities of real prices



- Outliers detected as both: interquartile range (IQR) criterion at sectoral level AND 4% highest/lowest observations at economy level
- IQR criterion - outliers outside $[q_{0.25} - 1.5 \cdot IQR, q_{0.75} + 1.5 \cdot IQR]$

Aggregation of sectoral TFP volatility to output volatility

Table: Volatility of GDP implied by the Polish production network

shock	mean	σ_{GDP}	$\frac{1}{n}\sum(\sigma_i)$	σ_{IO}	$\sigma_{symmetric}$	$\frac{\sigma_{IO}}{\sigma_{GDP}}$	$\frac{\sigma_{symmetric}}{\sigma_{GDP}}$
$\rho_i = 0$	unweighted	0.047	0.035	0.014	0.004	0.293	0.094
$\rho_i = 0$	weighted	0.047	0.030	0.012	0.004	0.254	0.081
$\rho_1 \neq 0, \rho_{i>1} = 0$	unweighted	0.047	0.033	0.013	0.004	0.284	0.091
$\rho_1 \neq 0, \rho_{i>1} = 0$	weighted	0.047	0.029	0.012	0.004	0.243	0.078
$\rho_i = BIC$	unweighted	0.047	0.034	0.014	0.004	0.286	0.092
$\rho_i = BIC$	weighted	0.047	0.029	0.012	0.004	0.248	0.079

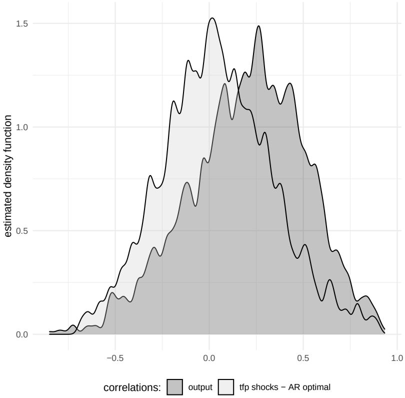
Notes: shock are defined as ϵ_t from model: $\Delta tfp_t = \sum_i \rho_i \Delta tfp_{t-i} + \epsilon_t$

- σ is measured as a a (weighted or unweighted) cross-sector average of σ_i, where σ_i is sectoral tfp volatility (standard deviation)
- The shock amplification of the network is 3.12
- Sectoral tfp volatility explains 24%-30% of total GDP volatility

Correlations across outputs and across tfp shocks

- TFP shock cross-correlations are centered at 0, but there exists both positive and negative industry pairs
- Cross-correlations between output log-changes are primarily positive, centered at 0.25, but there are also cases of negative cross-correlations

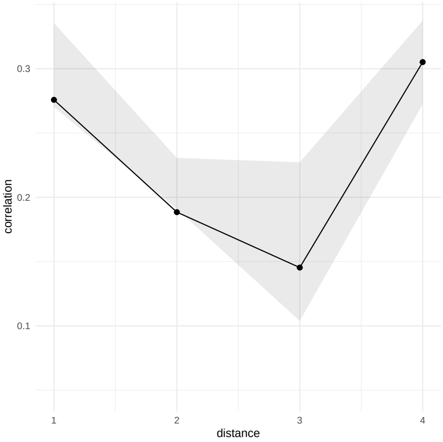
Figure: Cross-correlations between sectoral tfp shocks (optimal AR) and cross-correlations between sectoral outputs



Correlation of outputs and distance in network

- Network distance is defined as the shortest path between nodes
- Network connections were reduced to create more sparse network (density of original network is 0.92 vs 0.13 with threshold set at 3 times node size in symmetric dense network - 0.027) - see Appendix
- Confidence intervals calculated using a Fisher z-transformation to normal
- Sectors that are closer to each other tend to be more correlated, similar to Carvalho (2014)
- Results are robust to the threshold level - see Appendix

Figure: Correlation of outputs as a function of network distance



Volatility of observed and implied relative prices for empirical and symmetric sparse networks

Figure: Distributions of volatilities of relative prices - observed and implied by empirical network

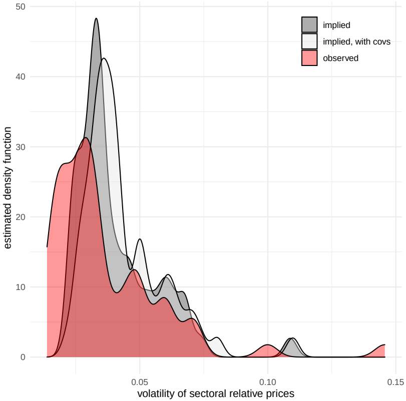
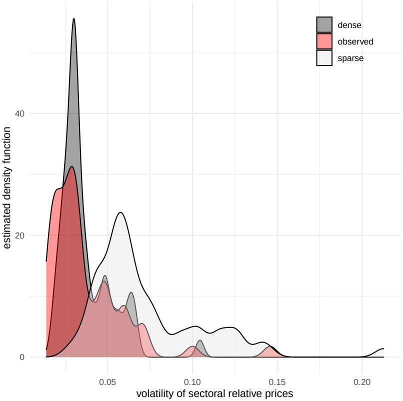


Figure: Distributions of volatilities of relative prices - observed and implied by symmetric networks



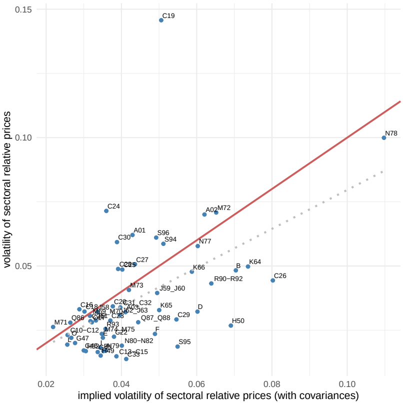
Volatility of observed and implied relative prices

- Consider an equation:

$$\sigma_i^p = \beta_0 + \beta \sigma_i^{implied} + \eta_i \qquad (6)$$

- Ideally, $\beta_0 = 0, \beta = 1$ and $R^2 = 1$
- $\beta_0 \neq 0$ implies bias, on average
- $\beta \neq 1$ informs about the average scaling factor
- R^2 informs about fraction of variance explained

Figure: Implied and observed volatilities of outputs



Transmission of shock volatility into price volatility

shock	network	β	t_β	R^2_β	γ	t_γ	R^2_γ
$\rho_i = 0$	observed	0.781	4.822	0.290	-0.842	-1.118	0.021
$\rho_1 \neq 0, \rho_{i>1} = 0$	observed	0.707	4.235	0.239	-1.350	-1.513	0.039
$\rho_i = BIC$	observed	0.758	4.702	0.279	-0.952	-1.218	0.025
$\rho_i = BIC$	dense	0.551	3.050	0.140	-0.18	-0.127	0
$\rho_i = BIC$	sparse	0.253	3.029	0.139	—	—	—

Notes: shock are defined as ϵ_t from model: $\Delta tfp_t = \sum_i \rho_i \Delta tfp_{t-i} + \epsilon_t$

- In all estimations β_0 was statistically 0 — no bias
- Implied variance is too high, on average it needs to be rescaled down by a factor of 0.75
- Implied variance explain about 24%-29% of observed variation in σ^p
- The symmetric networks explains 14% of volatility dispersion - the amplification factor is roughly 2.1
- The variance (diagonal terms) are the most important in variance propagation — last 3 columns show the estimates of:

$$\sigma_i^p = \gamma_0 + \gamma(\sigma_i^{implied} - \sigma_i^{diag}) + \eta_i \tag{7}$$

- It explains why volatility is the highest in sparse network - diagonal network streamlines directly tfp volatility into prices

Cross-correlations in sectoral prices

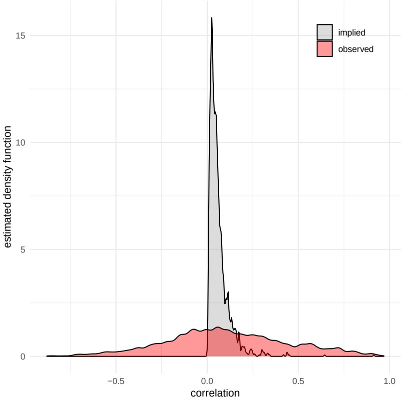


Figure: Distributions of cross-correlations in sectoral prices - variance of tfp (AR optimal) shock only

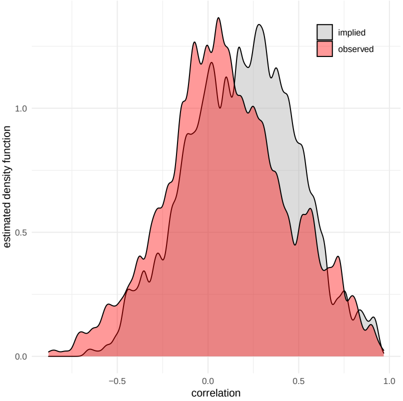


Figure: Distributions of cross-correlations in sectoral prices - whole vcov matrix of tfp (AR optimal) shock

Transmission of shock volatility into price volatility

shock	cov	network	CD	p-val	Steiger	p-val
$\rho_i = BIC$	diag	empirical	10.60	0	2012.65	0
$\rho_i = BIC$	full	empirical	32.37	0	3033.21	0
$\rho_i = BIC$	full	dense	37.00	0	2891.35	0
$\rho_i = BIC$	full	sparse	7.71	0	2798.47	0
$\rho_1 \neq 0, \rho_{i>1} = 0$	diag	empirical	10.06	0	2009.87	0
$\rho_1 \neq 0, \rho_{i>1} = 0$	full	empirical	27.46	0	2806.43	0
$\rho_i = 0$	diag	empirical	10.42	0	2013.73	0
$\rho_i = 0$	full	empirical	30.10	0	2921.34	0

Notes: shock are defined as ϵ_t from model: $\Delta tfp_t = \sum_i \rho_i \Delta tfp_{t-i} + \epsilon_t$

- Pesaran (2021) CD tests of cross-sectional dependence show in all cases that the implied covariance matrices are different from zero (with the smallest statistics in symmetric sparse network)
- All Steiger (1980) tests reject the null hypothesis all the correlations are equal to the observed one
- So, vcov matrices of sectoral prices generated by tfp shocks are not zero but simultaneously do not equal to the observed one

Transmission of shock volatility into price volatility

shock	cov	network	β	p-val	R^2	$\lvert\rho\rvert$	SMI	RV_{adj}	PSI
$\rho_i = BIC$	diag	empirical	0.44	0	0.01	0.06	0.13	0.04	0.44
$\rho_i = BIC$	full	empirical	0.15	0	0.02	0.29	0.17	0.10	0.41
$\rho_i = BIC$	full	dense	0.17	0	0.02	0.29	0.13	0.10	0.40
$\rho_i = BIC$	full	sparse	0.18	0	0.02	0.24	0.13	0.14	0.43
$\rho_1 \neq 0, \rho_{i>1} = 0$	diag	empirical	0.50	0	0.01	0.06	0.13	0.05	0.44
$\rho_1 \neq 0, \rho_{i>1} = 0$	full	empirical	0.18	0	0.03	0.27	0.18	0.12	0.44
$\rho_i = 0$	diag	empirical	0.43	0	0.01	0.06	0.15	0.05	0.44
$\rho_i = 0$	full	empirical	0.17	0	0.02	0.28	0.17	0.13	0.44

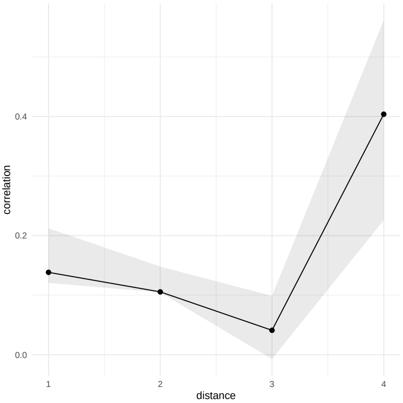
Notes: shock are defined as ϵ_t from model: $\Delta tfp_t = \sum_i \rho_i \Delta tfp_{t-i} + \epsilon_t$

- β from regression $\rho^p_{ij} = \beta_0 + \beta\rho^{implied}_{ij} + \eta_{ij}$ (for $j < i$) show that covariance of tfp shocks is important in transmission into price comovement, but it explains up to 3% of its dispersion
- Different matrix similarity measures — *SMI* of Indahl, Næs, and Liland (2018), matrix correlation RV_{adj} (Mayer, Lorent, and Horgan, 2011) and Procrustes Similarity Index (Sibson, 1978) show:
  - lower similarity when only variances of tfp shocks are considered
  - not much differences between tfp identification schemes and, surprisingly, between empirical and symmetric network
- $\lvert\rho\rvert$ measures mean absolute correlation and is 0.28 in the data

Comovement of relative prices in the network

- Theoretical model generates the general shape of comovement found in the data (density and mean-abs correlation) but is weak at predicting individual cross-correlations
- It may be due to the different shocks affecting prices...
- but also the sectoral dimension of the price cross-correlations is not very strong

Figure: Cross-correlation of relative prices as a function of network distance



Conclusions

- Productivity shocks account for 24-30% of total economy GDP volatility and the network amplification is 3.1
- The output of closely connected sectors tend to comove
- TFP shocks explain about 24%-29% of the observed dispersion in relative price variation
    - mostly through shock volatility - the role of covariance of tfp shock is negligible
    - the amplification factor is roughly 2.1 (fraction of dispersion explained of observed network versus the fraction of hypothetical symmetric network)
- Theoretical model generates the general shape of comovement found in the data (density and mean-abs correlation) but is weak at predicting individual cross-correlations
- A distance in network seems to be less important in price comovement (than in case of output comovement)
- Covariance of tfp shocks is important in transmission into price comovement, but the shock explains a tiny fraction of its dispersion

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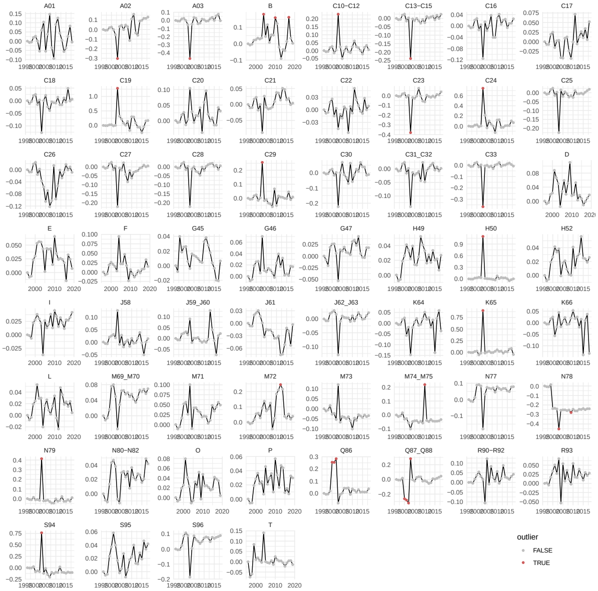
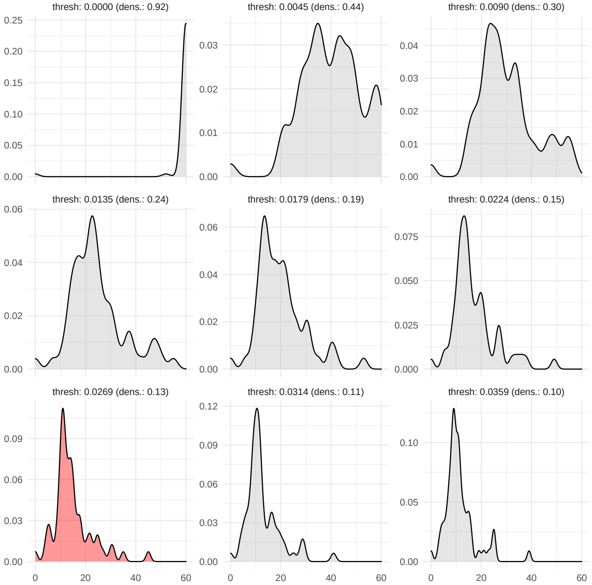
Section 9

Appendix

Outliers in real prices



Network degree as a function of cut-off threshold



The importance of threshold in distance in network

Figure: Correlation as a function of network distance for different thresholds cutting small connections