

# INFLATION IN POLAND UNDER STATE-DEPENDENT PRICING\*

*Paweł BARANOWSKI – Mariusz GÓRAJSKI –  
Maciej MALACZEWSKI – Grzegorz SZAFRAŃSKI<sup>1</sup>*

*(University of Lodz, Faculty of Economics and Sociology,  
Department of Econometrics)*

## Abstract

We investigate the short-term dynamics of the Polish economy using a state-dependent pricing model with stochastic menu costs of Dotsey – King – Wolman (1999). We compare macroeconomic evidence of price rigidity in a small-scale DSGE framework with the state-dependent Phillips curve (SDPC) derived by Bakhshi – Khan – Rudolf (2007) to a benchmark model with the time-dependent pricing of Gali – Gertler (1999). To analyse the determinants of inflation in Poland under different price setting we estimate both models with Bayesian techniques. We focus on the comparison of microeconomic characteristics of price stickiness, the parameters of the aggregate Phillips curves, and impulse responses to macroeconomic shocks. The estimated state-dependent pricing model for Poland generates a median duration of prices about 4 quarters comparing to 8 quarters in the time-dependent model. The menu cost model is also able to identify higher variance of technology shocks, and higher persistence in preference shocks. Consequently, in the state-dependent pricing model in relation to the time-dependent counterpart the costs of monetary policy tightening are significantly lower in terms of output losses, while the impulse responses of inflation and interest rate in both models are similar in terms of scale and shape.

**Keywords:** inflation, state-dependent pricing, menu costs, Phillips curve, new-Keynesian DSGE.

**JEL classification indices:** C51, E31, E32, E52.

\* We thank the participants of the FindEcon'2012 conference and the seminars held in 2012 and 2013 at: the University of Lodz, National Bank of Poland, Wrocław University of Economics, Vilnius University and the Aboa Center for Economics in Turku for their valuable comments. A working version of this paper was published as ACE Discussion Papers no. 83/2013. We would like to thank Barbara Rudolf for providing us with the original replication codes from Bakhshi – Khan – Rudolf (2007). We acknowledge the financial support of Ministry of Science and Higher Education under grant no. 2094/B/H03/2010/39. Usual disclaimer applies.

---

<sup>1</sup> *Corresponding author:* Grzegorz Szafranski, Faculty of Economics and Sociology, Institute of Econometrics, Department of Econometrics, University of Lodz, Rewolucji St. 1905 r. 41/43, 90-214 Lodz, Poland.  
E-mail: gszafranski75@gmail.com

## 1. Introduction

New-Keynesian dynamic stochastic general equilibrium (DSGE) models are prominent tools for analysing short-term deviations of the economy from its steady state (see Woodford 2003; Gali 2008). The vast majority of DSGE models incorporate the Calvo (1983) time-dependent stochastic price stickiness, which yields real effects of monetary policy. In the Calvo pricing firms receive in every period an opportunity to reset a price with a constant probability. Under this assumption one can easily derive a New-Keynesian Phillips Curve describing a short-term relationship between inflation and output gap.

Despite its huge popularity, the Calvo price setting unrealistically assumes that the timing of the price decisions is exogenous. It results in a constant average frequency of price adjustment across firms and time, which is inconsistent with the microeconomic evidence worldwide (e.g. Dhyne et al. 2006; Klenow – Kryvtsov 2008; Midrigan 2010). Moreover, it is argued that the Calvo pricing alone is not able to create enough persistence in inflation observed in the data. Thus in empirical studies it is enhanced with ‘rule-of-thumb’ firms (Gali – Gertler 1999) or dynamic indexation (Christiano – Eichenbaum – Evans 2005). These exceptions to a purely forward-looking behaviour of firms in DSGE framework stand behind the specification of the New-Keynesian Hybrid Phillips Curve (NHPC) which is still very popular in empirical studies for Central European economies (Menyhert 2008; Basarac – Skrabac – Soric 2011; Vašíček 2011).

An alternative explanation of a price rigidity in a New-Keynesian paradigm is offered by introducing menu costs to price-setting decisions of firms. Under menu cost rigidity the frequency of price adjustments becomes state-dependent i.e. dependent on the shocks and current prices of all representative firms. The difference from the Calvo approach consists of both varying share of the firms adjusting the prices and their non-random selection. When the ‘selection effect’ is present (i.e. the propensity for price adjustment is bigger for firms with the prices farther away from their), it considerably complicates the derivation of the Phillips curve. In some of the state-dependent models the ‘selection effect’ may also have some consequences to the degree of money non-neutrality (e.g. Caplin – Spulber 1987; Golosov – Lucas 2007).

Only few papers take DSGE models with state-dependent pricing to the macroeconomic data. Usually authors calibrate the parameters to meet the microeconomic evidence (Golosov – Lucas 2007; Gertler – Leahy 2008) or they estimate hazard functions of price adjustment from macroeconomic data (Sheedy 2010). As the evidence from micro-level datasets in Poland is scarce, in the empirical part we focus on the papers that derive closed form solution for aggregate inflation equation. In this respect inflation-output relationship in Gertler – Leahy (2008) is a local

approximation of a state-dependent Phillips curve around a zero-inflation steady state that resembles a traditional forward-looking Phillips curve. On the other hand Bakhshi – Khan – Rudolf (2007) derive state-dependent Phillips curve around a positive steady-state inflation in Dotsey – King – Wolman (1999) model (henceforth: the DKW model). This approach implies additional terms in SDPC i.e. future marginal costs, as well as expected and lagged inflation. The authors after a series of exercises on a simulated data claim that SDPC offers empirical explanation of intrinsic persistence in inflation similar to NHPC. The paper of Bakhshi – Khan – Rudolf (2007), although it has not been challenged with any empirical data, is closely related to our paper being a theoretical background of our study. We build on SDPC specification of Bakhshi – Khan – Rudolf (2007) and we enhance it with external habit persistence.

In the empirical part of the paper we analyse short-term dynamics of Polish economy using small-scale New-Keynesian DSGE model with the DKW state-dependent pricing. We compare microeconomic and macroeconomic implications of the DSGE model with a state-dependent Phillips curve (SDPC) derived by Bakhshi – Khan – Rudolf (2007) to a benchmark model including hybrid New-Keynesian Phillips Curve (NHPC) of Gali – Gertler (1999). To replicate a short-term persistence in inflation and output both models include other sources of economic inertia (i.e. habit persistence in consumption, interest rate smoothing in a Taylor-type rule), which are routinely included in the empirical DSGE models.<sup>2</sup> To analyse monetary policy transmission mechanism we estimate both models with Bayesian techniques and focus on the comparison of distribution of price vintages, and a degree of price stickiness (expressed by median price duration) as well as parameters in Phillips curve equations, and impulse responses to macroeconomic shocks. We are interested whether the macroeconomic illustration of transmission mechanism and the assessment of microeconomic price rigidity depend on the choice of pricing mechanism.

The structure of the paper is as follows. In the Section 2 we describe the state-dependent pricing mechanism of Dotsey – King – Wolman (1999). Then, in the Section 3, we specify the DSGE model with DKW pricing. We also present the dataset as well as the assumptions on unobservables. Finally, in the Section 4 we discuss empirical results of the Bayesian estimation of the DSGE model. Furthermore, we compare the estimated DKW model with the benchmark Calvo models in terms of: distribution of price vintages, a degree of price stickiness, parameters in Phillips curve equations, and impulse responses to macroeconomic shocks.

---

<sup>2</sup> See e.g. Baranowski – Szafranski (2012), Torój – Konopczak (2012), Krajewski (2013), to mention just a few recent studies for Polish economy.

## 2. The model of the state-dependent pricing

We build on the DKW model, which introduces a stochastic menu cost as a source of price rigidity. As in standard DSGE models, continuum of firms indexed by  $i \in [0; 1]$  produce differentiated final goods and set their prices to maximize the profits under rational expectations. Each firm faces different stochastic menu costs, which are calculated in the units of labour costs and interpreted in terms of the amount of labour necessary to accomplish all activities connected with a price change. We treat those costs as if they were independent (across time and firms) realizations of a continuous random variable,  $\xi_{i,t}$ , distributed on  $[0, B]$  interval. Economically,  $B$  is an upper bound of menu cost distribution representing an opportunity cost of price adjustment. Menu costs discourage firms from changing the price in every period. A firm resets the price only if an expected marginal revenue from changing the price exceeds a realization of menu cost stochastic process. In a period  $t$  only a fraction of firms, drawing menu costs below a given threshold, sets a new price. If a firm decides to change the price in period  $t$ , it sets new optimal price,  $P_t^*$ , that maximizes its profits. The firms with relatively high menu costs leave their prices unaltered. In a consequence of these price-setting decisions, firms are assigned to different groups ('price vintages') that changed their price  $j$  periods ago ( $j = 1, 2, \dots, J$ ). Hence, the price set by firms from vintage  $j$  is  $P_{t-j}^*$ . In period  $t$  these firms solve the dynamic optimization problem, and change their price whenever:

$$v_{0,t} - v_{j,t} > \xi_{i,t} W_t, \quad (1)$$

where  $v_{0,t}$  and  $v_{j,t}$  are sums of discounted expected profits conditional on events of 'setting new price' ( $P_t^*$ ) and 'no price change' ( $P_{t-j}^*$ ), respectively, and  $W_t$  denotes economy-wide real wage rate in period  $t$  (cf. Appendix C).

Let, at the beginning of period  $t$ ,  $\omega_{j-1,t-1}$ ,  $j = 1, 2, \dots, J$  denote a fraction of all firms belonging to a price vintage  $j$ . In a period  $t$  a portion of firms,  $\alpha_{j,t}$ , from vintage  $j$  with relatively low menu costs, decides to reset its price to  $P_t^*$ . Next period they move to the first vintage ( $j = 1$ ). The rest of the firms from vintage  $j$  does not change the price, hence they migrate to a vintage  $j + 1$ . In the last vintage  $J$  the benefits from resetting the price are bigger than the upper bound  $B$  of menu cost, consequently, all of firms reset the price and migrate to the first vintage.<sup>3</sup> The dynamic relations between  $\omega_{j,t}$  and  $\alpha_{j,t}$  are described by identities (2) and (3):

---

<sup>3</sup> Due to strictly positive steady-state inflation  $\pi^{ss} > 0$  and bounded support of menu cost distribution, there exists a finite number of price vintages,  $J$ . The number of non-empty vintages  $J$  is a result of firms optimization decisions. It depends on the current shocks and such model parameters: steady-state inflation  $\Pi$ , price elasticity of demand and distribution of menu cost  $G$ .

$$\omega_{j,t} = (1 - \alpha_{j,t})\omega_{j-1,t-1}, \quad j = 1, 2, \dots, J-1 \quad (2)$$

$$\omega_{0,t} = \sum_{j=1}^J \alpha_{j,t}\omega_{j-1,t-1}. \quad (3)$$

Laws of motion given by equations (2) and (3) govern changes in a distribution of firms across price vintages (see Figure 1).

<Figure 1 over here>

As there are infinitely many firms and menu cost is drawn from a continuous distribution, in every price vintage  $j$  there exists a 'marginal' firm  $i^*(j)$  with profits from changing the price equal to menu cost:

$$v_{0,t} - v_{j,t} = \xi_{i^*(j),t} W_t \quad (4)$$

Identity (4) with a distribution function  $G$  (see Figure 2) determines a fraction of firms that in period  $t$  set common optimal price,  $P_t^*$ :

$$\alpha_{j,t} = G((v_{0,t} - v_{j,t})/W_t) = G(\xi_{i^*(j),t}) \quad (5)$$

<Figure 2 over here>

From the equations (2), (3) and (5) the probabilities of resetting the price  $\alpha_{1,t}, \alpha_{2,t}, \dots, \alpha_{J,t}$  are non-decreasing with  $j$ , and the distribution of firms across price vintages  $\omega_{0,t}, \omega_{1,t}, \dots, \omega_{J-1,t}$  is non-increasing. Due to the selection effect under positive steady-state inflation the later the firm resets its price the bigger the price change is. This phenomenon is not present in a pricing mechanism of Calvo (1983), in which the timing of price changes is exogenous and random. Moreover, in the Calvo pricing the number of firms in consecutive  $j = 1, 2, \dots$  fractions decline at the geometric rate ( $\omega_j = (1 - \theta)\theta^j$ ) and the adjustment probabilities are constant.

### 3. Estimation of the state-dependent Phillips curve

Here, we focus on the estimation of Phillips curve derived from DKW pricing mechanism around a non-zero steady-state inflation by Bakhshi – Khan – Rudolf (2007), state-dependent Phillips curve.

We introduce one modification to original SDPC and incorporate external habit persistence in consumption (see Abel 1990), which results in a lagged term of output gap:<sup>4</sup>

$$\begin{aligned}\tilde{\pi}_t = & E_t \sum_{k=1}^{J-1} \delta'_k \tilde{\pi}_{t+k} + \sum_{l=1}^{\infty} \mu'_l \tilde{\pi}_{t-l} + E_t \sum_{j=0}^{J-1} \hat{\psi}'_j x_{t+j} + \vartheta x_{t-1} \\ & + \sum_{k=0}^{\infty} \eta_k \hat{\Omega}_{t-k} + E_t \sum_{j=0}^{J-1} \gamma_j (\tilde{\omega}_{j,t+j} - \tilde{\omega}_{0,t}) + v_t^\pi,\end{aligned}\quad (6)$$

where  $\tilde{\pi}_t$  is a deviation of inflation from its non-zero steady-state level,  $x_t$  is an output gap,  $\tilde{\omega}_{j,t} = \ln \omega_{j,t} - \ln \omega_j$ , and  $\hat{\Omega}_t = \sum_{j=0}^{J-1} \frac{1}{\omega_0} \Pi^{j(\varepsilon-1)} \omega_j \tilde{\omega}_{j,t}$  are, respectively, expected and past state-dependent unobserved terms. Parameters in equation (6) are given by the following formulas:

$$\delta'_j = \frac{1}{\mu_0} \sum_{k=j}^{J-1} (\varepsilon \rho_k - (1 - \varepsilon) \delta_k), \text{ for } j = 1, \dots, J-1, \rho_k = \frac{\beta^k \omega_k \Pi^{k\varepsilon}}{\sum_{j=0}^{J-1} \beta^j \omega_j \Pi^{j\varepsilon}}, \delta_k = \frac{\beta^k \omega_k \Pi^{k(\varepsilon-1)}}{\sum_{j=0}^{J-1} \beta^j \omega_j \Pi^{j(\varepsilon-1)}}$$

$$\mu_j = \frac{1}{\omega_0} \sum_{k=j+1}^{J-1} \omega_k \Pi^{k(\varepsilon-1)}, \hat{\psi}'_j = \frac{\hat{\psi}_j}{\mu_0}, \vartheta = \frac{\kappa_2 \rho_0}{\mu_0}, \text{ for } j = 0, 1, \dots, J-1$$

$$\hat{\psi}_j = \begin{cases} \kappa_1 \rho_j + \rho_j - \delta_j + \kappa_2 \rho_{j+1} & j = 0, 1, \dots, J-2 \\ \kappa_1 \rho_j + \rho_j - \delta_j & j = J-1 \end{cases}, \kappa_1 = \frac{\phi + \alpha + (1-\alpha)\sigma}{1-\alpha}, \kappa_2 = h(\sigma - 1),$$

and matrix formulas on  $\mu'_k, \eta_k$  for  $k = 1, 2, \dots$  are given in Appendix B to Bakhshi – Khan – Rudolf (2006).

To learn about the parameters of DSGE models with time- and state-dependent pricing mechanisms we perform a Bayesian estimation. We compare parameters of SDPC and NHPC and the distribution of firms across price vintages  $\omega_j$  in a steady-state. Then from the posterior distributions we generate impulse response functions, which describe a reaction of inflation, output gap and interest rate to a one-standard-deviation unanticipated shock. We perform the estimation in Dynare package (see Adjemian et al. 2011).

Both DSGE models are estimated on quarterly data for the Polish economy from the period 1997-2012. Inflation is measured by quarterly change of Consumption Price Index (CPI), interest rate is a short-term interest rate on 1 month interbank deposits (WIBOR 1m). Because a disinflation process is a dominating long-term component in the first 5 years of the sample, we perform the estimation

<sup>4</sup> For the general overview of the SDPC derivation and the structure of DSGE economy we refer the interested reader to the Appendix C.

on HP-detrended inflation and interest rates.<sup>5</sup> In the same fashion the output gap was calculated as percentage deviation of GDP from its HP-trend, which is a standard approach in determining steady-state level of production in the most of DSGE studies. The figures of the original data and their transformations are depicted in Appendix A.

We start from necessary modifications that facilitate the Bayesian estimation of SDPC presented in (6). Compared to the NHPC, it includes additional leads of inflation and output gap, and infinite number of lagged inflation terms. Moreover, there are unobserved characteristics of distributions of firms across price vintages that depend on the realisations of time-heterogeneous Markov process. Firstly, we examine the robustness of SDPC to changes in distribution of menu costs, steady-state markup and inflation. The exercise shows that a number of price vintages ( $J$ ) and the fractions of firms in consecutive price vintages are mostly sensitive to changes in markup and steady-state inflation<sup>6</sup>. Secondly, fractions of future price vintages  $\omega_{j,t+k}$ , vintages in steady state  $\omega_j$ , and their weighted absolute deviations  $\widehat{\Omega}_{t-k}$  are all unobserved. These components are solutions to dynamic optimization problem, cf. Appendix C, formula (15). For a technical and practical reasons we omit  $\omega_{0,t}, \omega_{1,t}, \dots, \omega_{J-1,t}$  from the estimated SDPC. Their numerical values are defined by a time-heterogeneous Markov chain and they depend on expected opportunity profits from resetting the price,  $v_{0,t} - v_{j,t}$ . To calculate these quantities in Bayesian estimation with Markov Chain Monte Carlo (MCMC) would be a matter of further numerical complication. Because  $\omega_{0,t}, \omega_{1,t}, \dots, \omega_{J-1,t}$  are not observable, their joint estimation would also introduce additional identification problems.

The other unobservables – steady-state fractions of firms in  $J$  price vintages  $\omega_0, \omega_1, \dots, \omega_{J-1}$  are realizations of a time-homogeneous Markov chain with transition matrix given by equation (17) in Appendix C. Moreover, they are directly related to structural parameters  $(\delta'_k, \widehat{\psi}'_j, \mu'_k)$ . To employ these values in posterior MCMC estimation we construct an exponential multinomial with interaction terms that interpolates  $\omega_j$  reasonably well. The grid is built on a joint domain of a markup  $m \equiv \frac{\varepsilon}{\varepsilon-1} \in [1.1; 1.35]$  and an upper bound of menu cost distribution function  $B \in [0.0075; 0.05]$  with a steady-state inflation  $\pi^{ss} = 4.6\%$  p.a. (average inflation rate in the sample) and number of price vintages  $J = 10$ .

In effect we have estimated a standard three-equation DSGE model with SDPC:

---

<sup>5</sup> We have also performed the estimation on non-demeaned inflation and interest rate (only successful for NHPC) with quantitatively different results (incredibly persistent technology shocks).

<sup>6</sup> Results of this exercises are available from the authors upon request.

$$\tilde{\pi}_t = E_t \sum_{j=1}^9 \delta_j' \tilde{\pi}_{t+j} + \vartheta x_{t-1} + E_t \sum_{j=0}^9 \hat{\psi}'_j x_{t+j} + \sum_{k=1}^4 \mu'_k \tilde{\pi}_{t-k} - v_t^\pi, \quad (7)$$

where error term,  $v_t^\pi$ , is an autoregressive process of order one for a negative technological shock,  $\epsilon_t^\pi \sim NID(0, \Sigma^\pi)$ :  $v_t^\pi = \rho_x v_{t-1}^\pi + \epsilon_t^\pi$ .

The other structural equations are:

- dynamic IS curve with habit persistence:

$$x_t = \gamma E_t(x_{t+1}) + (1 - \gamma)x_{t-1} - \sigma_1(\tilde{i}_t - E_t \tilde{\pi}_{t+1}) + v_t^x, \quad (8)$$

where the parameters depend on  $\sigma$  (cf. Appendix C), and  $h$  ( $\gamma = \frac{\sigma}{\sigma+h(\sigma-1)}$ ,  $\sigma_1 = \frac{1}{\sigma+h(\sigma-1)}$ ), and the error term,  $v_t^x$ , is an autoregressive process of order one for a preference shock,  $\epsilon_t^x \sim NID(0, \Sigma^x)$ :  $v_t^x = \rho_x v_{t-1}^x + \epsilon_t^x$ .

- Taylor rule with interest rate smoothing:

$$\tilde{i}_t = \lambda \tilde{i}_{t-1} + (1 - \lambda)(\phi_\pi \tilde{\pi}_t + \phi_x x_t) + \epsilon_t^i, \quad (9)$$

where,  $\epsilon_t^i$ , is a white noise monetary policy shock:  $\epsilon_t^i \sim NID(0, \sigma^i)$ .

In the next section we compare the dynamics of the system to a benchmark time-dependent pricing framework with the hybrid Phillips curve (NHPC) of Gali – Gertler (1999):

$$\tilde{\pi}_t = \beta_f E_t(\pi_{t+1}) + \beta_b \pi_{t-1} + \chi_0 x_t + \chi_1 x_{t-1} + v_t^\pi, \quad (10)$$

where  $\beta_f = \frac{\beta\theta}{\theta+\tau(1-\theta(1-\beta))}$ ,  $\beta_b = \frac{\tau}{\theta+\tau(1-\theta(1-\beta))}$  and  $\chi_0 = \frac{\varphi+\alpha+\sigma(1-\alpha)(1-\tau)(1-\theta)(1-\beta\theta)}{1-\alpha}$ ,  $\chi_1 = \frac{h(1-\sigma)(1-\tau)(1-\theta)(1-\beta\theta)}{\theta+\tau(1-\theta(1-\beta))}$ .

## 4. Results

The estimation results of the state-dependent pricing model are included in Table 1 and depicted in Figure 3 and Figure 4. They come from a simulation of the two Markov chains (1 million each with 25% burn-in initial cycles) following Metropolis random-walk algorithm implemented in Dynare. We have started the simulations with fairly diffuse priors. The parameter  $B$  describing an upper bound on a menu cost has been initiated with a flat prior on an interval close to a grid domain. We have restricted the prior distribution of gross markup  $m$  above one with an adjusted gamma distribution. The mean of its prior distribution (1.25) gives an average elasticity of substitution between products equal to 5, which is close to the baseline parametrisation in Dotsey – King – Wolman (1999). The mean of a risk aversion parameter ( $\sigma$ ) is about 3 which leads to a reasonable curvature of consumer preferences (e.g. Mehra – Prescott 1985; Szpiro 1986). The values of  $h, \rho_\pi, \rho_x$  was bounded in  $[0; 1]$

interval by beta distribution, and standard deviations of shocks come from inverse gamma distribution, which are standard assumptions in many DSGE studies. The interest rate smoothing parameter ( $\lambda$ ) has been given considerably diffuse prior distribution. The other parameters of a Taylor rule ( $\phi_\pi, \phi_x$ ) priors were more concentrated to meet the Blanchard-Kahn conditions in the simulation exercise.

<Table 1 over here>

<Figure 3 over here>

<Figure 4 over here>

The results of Bayesian estimation of all of the parameters, except for a markup ( $m$ ) and coefficients at inflation in the Taylor rule ( $\phi_\pi$ ) are considerably updated by the data (see Figure 3). From a negative asymmetry of posterior histogram of  $B$  we conclude that to allow for a degree of price stickiness observed in the data one needs the maximal menu costs to be rather above mean value of its prior density. A mean value of an upper bound of menu costs 3.1% in terms of real wages reads circa 2.5% in terms of real output. This extreme (and possibly only potential) value of menu costs is a sufficient barrier for price rigidity observed in the Polish data. Also a mean of the posterior distribution of  $\sigma$ , which explains consumption smoothing preferences, is shifted to the right compared to a mean of prior distribution and takes a relatively big value (6.4). There is also a strong evidence of the inertial behaviour in a monetary policy reaction function (with  $\lambda$  about 0.8) and habit formation (with  $h$  about 0.9). All of the three parameters ( $\sigma, h, \lambda$ ) posterior distributions are very close to the time-dependent pricing case (see results in Appendix B). The only significant difference can be found in the shock characteristics. In comparison to the time-dependent pricing model the variances of technological ( $\sigma^\pi$ ) and preference shocks ( $\sigma^x$ ) are bigger in the state-dependent pricing model. It is however partly compensated by a lower persistence of a preference shock (measured with  $\rho_x$ ) in the state-dependent pricing model.

In the next step we analyse the median parameters<sup>7</sup> from both estimated Phillips curves as a potential explanation of the differences in the determination of inflation by its persistence as well as by expectations on inflation and output. Compared to NHPC (which is mostly forward-looking) one-period inflation expectation parameter in SDPC is of much lower magnitude (see Figure 5). The impact of inflation expectations in SDPC is more prolonged in time and it decreases with a time

---

<sup>7</sup> As the posterior histograms of the structural parameters (being the functions of posterior distributions of 'deep' DSGE parameters) are of a complex shape we analyse their medians only.

horizon. The sum of inflation expectation terms in SDPC is about 71% of one-period expectation parameter from NHPC which means that the estimated SDPC is less forward-looking than the estimated NHPC. On the other hand, SDPC gives an appealing economic (menu costs) explanation of intrinsic inflation persistence. The sum of the estimated parameters at lagged inflation in SDPC is more than 0.5 which is 3.5 times bigger than the impact in NHPC. Summing up, from the perspective of SDPC intrinsic persistence is a prevailing force of inflation determination in Polish economy contrary to the NHPC results which is mainly forward looking.

The effect of current output gap on inflation is weaker in the estimated SDPC for the Polish economy compared to the analogous results in NHPC. In the SDPC there is however an additional influence of output gap expectations on inflation, which by its construction is absent from NHPC (see Figure 6). This impact of output gap expectations on inflation is a medium-term phenomenon lasting up to several quarters. It is also stronger than the contemporaneous impact up to 3 quarters ahead. A maximum effect of output gap expectations is located at the two-quarter, and then it slowly decays. In result of those medium-term output gap expectations and despite similar strength of habit persistence the lagged effect of output gap in SDPC is also stronger than in NHPC. In summary, firms from the perspective of state-dependent pricing are on average more forward looking in determining aggregate inflation than their counterparts from the model of time-dependent pricing of Gali – Gertler (1999).

<Figure 5 over here>

<Figure 6 over here>

The next point in a Bayesian analysis of both pricing mechanisms is a comparison of steady-state distribution of firms across price vintages, which are described as  $\omega_j$  in SDPC (see Figure 7). The posterior histograms of  $\omega_j$  (black line) are depicted together with *a priori* beliefs of the parameters (grey line) to realize the extent of information update after looking at the data. In NHPC where the number of price vintages is infinite we regain their values from the posterior histograms of  $\theta$ . In SDPC estimation we have truncated the number of price vintages at 10, but for a convenience of the presentation we further restrict it to 9.

<Figure 7 over here>

<Figure 8 over here>

From Figure 7 it is important to learn that the mode of the posterior distribution is decaying faster in SDPC than in NHPC. In the state-dependent pricing model it is an outcome of an increase in the fraction of firms resetting the price ( $\alpha_{j,t}$ ) for the consecutive price vintages. Contrarily, in the time-dependent pricing model the probability of price adjustment ( $\theta$ ) is constant. It results in an estimated median duration of price at about 4 quarters in DKW economy compared to about 8 quarters in NHPC. The clear difference in price rigidity observed on the aggregate level is surprising and it may also be of some importance for the results of micro-price studies. However, these results for state-dependent economy should be treated with a caution, as it seems that posterior distributions of  $\omega_j$  in SDPC are to some extent (at least in shape) determined by their prior distribution generated from prior distribution of  $m$  and  $B$ . On the contrary in NHPC the effect of data clearly increases with the number of a price vintage.

The last part of the Bayesian analysis is about comparing impulse response functions (IRF) in both DSGE models using their posterior distributions. In the Figure 8 we put a dashed line for an average IRF path of responses in the time-dependent pricing model, and a shaded area and a black line are, respectively, 90% HPD interval and a median of state-dependent pricing model.

The results of impulse response functions in both time- and state-dependent model are economically plausible. They also exhibit similar a hump-shaped pattern of reaction, widely observed in other studies. In the first row of Figure 8 there are the effects of technology shocks on three observed variables. The unanticipated negative shock to technology lowers firms productivity, which implies that the re-optimizing firms set higher prices (and consequently lower production that opens the negative output gap), both in state- and time-dependent pricing model. We can also observe central bank response to a higher inflation resulting in higher interest rate, what generates further decrease of the production. In the second row there are the effects of monetary policy shock. An 'extra' raise of central bank interest rate (above the interest rate consistent with the Taylor rule) tend households to postpone a part of its current consumption. This generates negative output gap, which results in lower inflation. Third row describes the effects of a preference shock. The shock raises the weight of current utility in the lifetime utility path. This makes current consumption more valuable, which results in a positive output gap and consequently higher inflation. At last central bank raises interest rate in response to higher economic activity. The estimated DSGE model with state-dependent pricing identifies less persistent preference shocks than in a comparable time-dependent model. To sum up it is important to note that the time-dependent and state-dependent IRFs estimated for the Polish economy are generally hard to distinguish. The differences being negligible in economic terms, are statistically significant only for reaction after monetary policy

shock (negligibly slower responses of inflation, and quicker adjustments in output gap and interest rates in the model with SDPC), and the responses of output gap to technological shocks (faster adjustment).

## **5. Conclusions**

The estimated state-dependent pricing model generates a median duration of prices above 4 quarters compared to 8 quarters in the time-dependent model. On the other hand in the state-dependent pricing model, inflation after a monetary policy shock approaches a steady-state path in a similar (or little prolonged) pace relative to the time-dependent pricing model. DKW mechanism identifies higher variance of technology shocks, and higher persistence in preference shocks. Despite those significant differences the dynamics of impulse responses in the time- and state-dependent pricing models are hard to distinguish. We conclude that it may have important consequences in interpreting the results of future microeconomic surveys of price stickiness.

The results obtained in the state dependent pricing model should be treated with caution. Firstly, we have only analysed small-scale DSGE models without many important nominal and real frictions. Secondly, the estimation sample contains a 5-year period of considerable disinflation process. We have used HP-filtered series to cope with the issue in a proper manner. The other limits of our study come from the state-dependent terms which have been omitted to facilitate the Bayesian estimation of SDPC. Particularly, the estimated short-term response of output and inflation to the monetary policy shock in the state-dependent pricing model may be stronger if one fully accounts for the short-term dynamics in fractions of firms across price vintages. Nevertheless, the conclusions from the IRF analysis should be valid if small shocks are considered and steady-state inflation is close to 4% p. a.

## References

1. Abel, A. (1990): Asset Prices under Habit Formation and Catching Up with the Joneses, *American Economic Review*, 80 (2): 38 - 42.
2. Adjemian, S. – Bastani, H. – Juillard, M. – Mihoubi, F. – Perendia, G. – Ratto, M. – Villemot, S. (2011): *Dynare: Reference Manual, Version 4*, Dynare Working Papers, nr 1.
3. Bakhshi, H. – Khan, H. – Rudolf, B. (2006): The Phillips Curve under State-Dependent Pricing, *CEPR Discussion Papers*, no. 5945.
4. Bakhshi, H. – Khan, H. – Rudolf, B. (2007): The Phillips Curve under State-Dependent Pricing, *Journal of Monetary Economics*, 54 (8): 2321-2345.
5. Baranowski, P. – Szafranski, G. (2012): Effects of Monetary Policy in a Small-Scale DSGE Model for Poland – how much the Estimation Method Determine the Results? (in Polish), *Bank i Kredyt*, 43(4): 119–144.
6. Basarac, M. – Skrabic, B. – Soric, P. (2011): The Hybrid Phillips Curve: Empirical Evidence from Transition Economies, *Finance a Uver – Czech Journal of Economics and Finance*, 61(4), 367-383.
7. Calvo, G. (1983): Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics*, 12 (3): 383-398.
8. Cambell, J. (1999): Asset Prices, Consumption, and the Business Cycle. In: Taylor J.B. – Woodford M. (eds.): *Handbook of Macroeconomics*, vol. 1. Amsterdam: Elsevier, 1231-1303.
9. Caplin, A. – Spulber, D. (1987): Menu Costs and the Neutrality of Money, *Quarterly Journal of Economics*, 102 (4): 703-725.
10. Christiano, L. – Eichenbaum, M. – Evans, C.L. (2005): Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, University of Chicago Press, 113(1): 1-45.
11. Dhyne, E. – Alvarez, L. - Le Bihan, H. – Veronese, G. – Dias, D. – Hoffmann, J. – Jonker, N. – Lünemann, P. – Rumler, F. – Vilmunen, J. (2006): Price Changes in the Euro Area and the United States: Some Facts from Individual Consumer Price Data, *Journal of Economic Perspectives*, 20 (2): 171-192.
12. Dixit, A. – Stiglitz, J. (1977): Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67 (3): 297-308.
13. Dotsey, M. – King, R. – Wolman, A. (1999): State-Dependent Pricing and the General Equilibrium Dynamics of money and output, *Quarterly Journal of Economics*, 114 (2): 655-690.
14. Gali, J. (2008): *Monetary policy, inflation, and the business cycle: An introduction to the New Keynesian Framework*, Princeton University Press.
15. Gali, J. – Gertler, M. (1999): Inflation Dynamics: A Structural Econometric Analysis, *Journal of Monetary Economics*, 44 (2): 195-222.
16. Gertler, M. – Leahy, J. (2008): A Phillips Curve with an Ss Foundation, *Journal of Political Economy*, 116 (3): 533-572.
17. Golosov, M. – Lucas, R. (2007): Menu Costs and Phillips Curves, *Journal of Political Economy*, 115 (2): 171-199.
18. Krajewski, P. (2013): The Impact of Government Spending on Labor Market (in Polish), *Wiadomości Statystyczne*, 1: 1-13.
19. Klenow, P.J. – Kryvtsov, O. (2008): State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?, *Quarterly Journal of Economics*, 123 (3): 863-904.
20. Mehra, R. – Prescott, E. C. (1985): The Equity Premium: A Puzzle, *Journal of Monetary Economics*, 15(2): 145-161.
21. Menyhert, B. (2008): Estimating the Hungarian Phillips Curve, *Acta Oeconomica*, 58(3), 295-318.
22. Midrigan, V. (2010): Is Firm Pricing State or Time Dependent? Evidence from US Manufacturing, *The Review of Economics and Statistics*, 92(3): 643-656.

23. Sheedy, K.D. (2010): Intrinsic Inflation Persistence, *Journal of Monetary Economics*, 57(8): 1049-1061.
24. Smets, F. – Wouters, R. (2003): An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association*, 1 (5): 1123-1175.
25. Szpiro, G.G. (1986): Relative Risk Aversion Around the World, *Economics Letters*, 20(1): 19-21.
26. Taylor, J.B. (1993): Discretion Versus Policy Rules in Practice, *Carnegie-Rochester Series on Public Policy*, 39.
27. Torój, A. – Konopczak, K. (2012): Crisis Resistance versus Monetary Regime: A Polish-Slovak Counterfactual Exercise, *Central European Journal of Economic Modelling and Econometrics*, 4: 1-22.
28. Vašíček, B. (2011): Inflation Dynamics and the New Keynesian Phillips Curve in Four Central European Countries, *Emerging Markets Finance and Trade*, 47(5), 71-100.
29. Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, New York : Princeton University Press, Princeton.

## Tables and Figures

Table 1. Characteristics of prior and posterior distributions in the state-dependent pricing model.

Parameter symbols	prior distribution			posterior distribution	
	Type of distribution	Mean	Standard deviation	Mean	HPD interval 90%
$m$	Gamma (1; $\infty$ )	1.25	0.15	1.240	[1.064; 1.477]
$B$	Uniform [0.0075; 0.05]	0.029	0.012	0.031	[0.016; 0.050]
$\sigma$	Gamma (0; $\infty$ )	3.00	1.50	6.327	[4.127; 8.330]
$h$	Beta	0.50	0.25	0.930	[0.855; 0.999]
$\phi_\pi$	Normal	2.00	0.05	2.019	[1.843; 2.187]
$\phi_x$	Normal	0.00	0.05	0.039	[-0.044; 0.122]
$\lambda$	Beta	0.75	0.15	0.820	[0.785; 0.855]
$\rho_\pi$	Beta	0.50	0.15	0.323	[0.145; 0.470]
$\rho_x$	Beta	0.50	0.15	0.404	[0.253; 0.519]
$\sigma^\pi$	Gamma <sup>-1</sup>	1.0	$\infty$	0.552	[0.460; 0.643]
$\sigma^x$	Gamma <sup>-1</sup>	1.0	$\infty$	0.203	[0.157; 0.248]
$\sigma^i$	Gamma <sup>-1</sup>	1.0	$\infty$	0.229	[0.195; 0.262]

Source: Own calculations with Dynare 4.3 (Adjemian et al. 2011). Note: HPD stands for highest posterior density.

Figure 1. DKW mechanism of migration between price vintages.

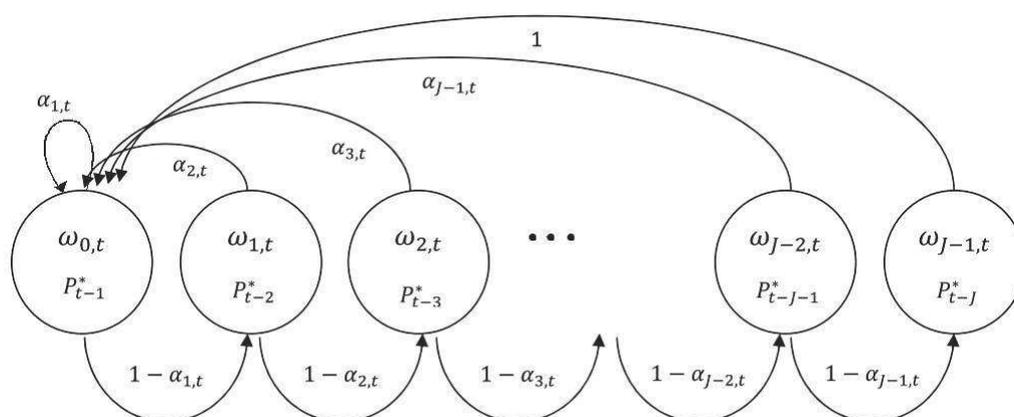
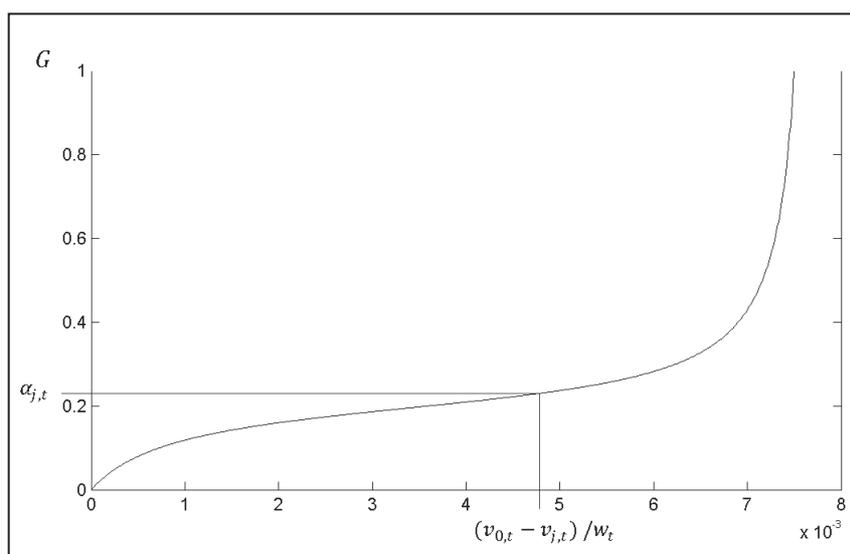
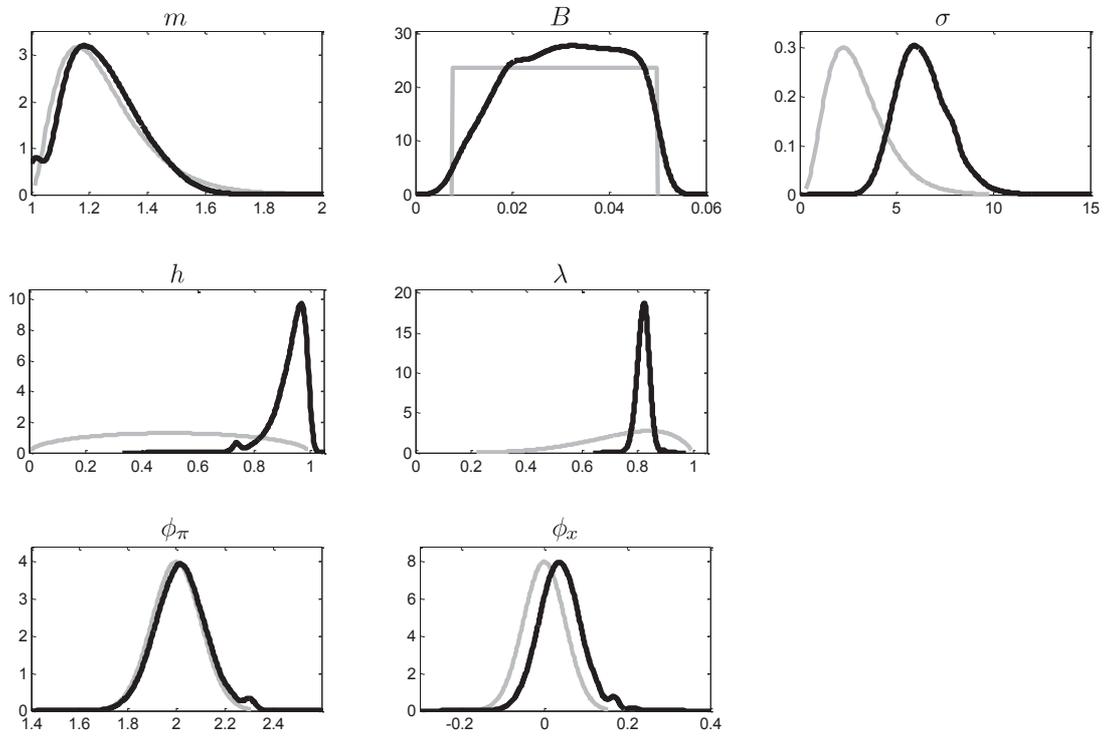


Figure 2. Menu cost distribution function  $G$ , and the fraction of firms  $\alpha_{j,t}$  resetting the price in period  $t$ .



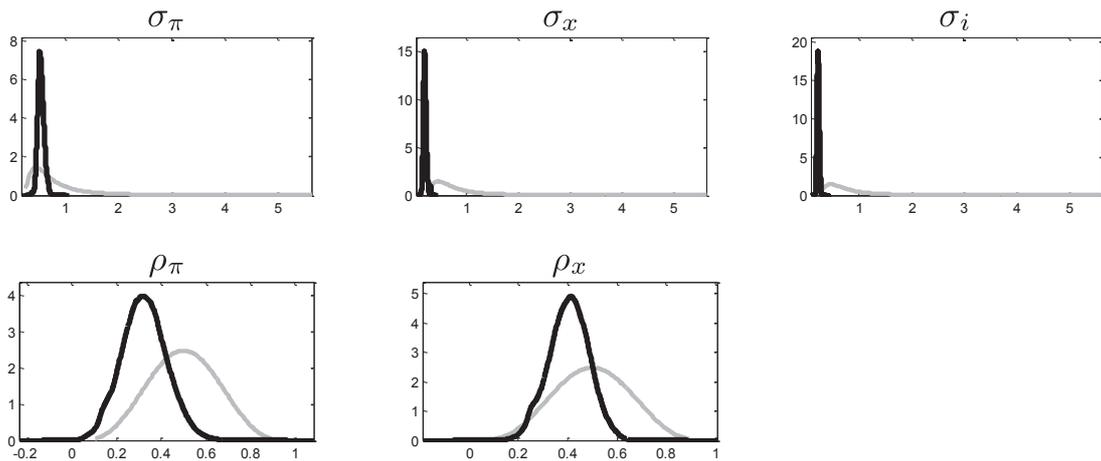
Note: The function  $G$  is of the form:  $G(x) = c_1 + c_2 \cdot \tan(c_3 \cdot x + c_4)$ , where  $c_1 = 0,1964$ ,  $c_2 = 0,0625$ , and  $c_3 = 2.7558/B$ ,  $c_4 = 1.2626$ ,  $B = 0.0075$ , (cf. Dotsey – King – Wolman 1999)

Figure 3. Prior (grey lines) and posterior (black lines) distributions of parameters of DSGE with DKW mechanism



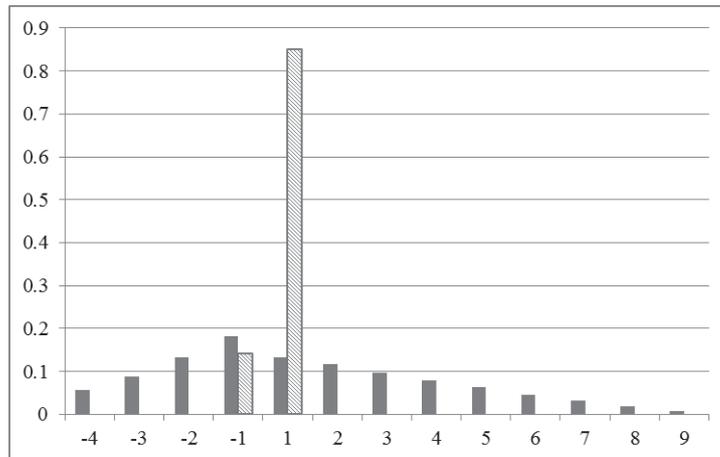
Source: own calculations with Dynare 4.3 (Adjemian et al. 2011)

Figure 4. Prior (grey lines) and posterior (black lines) of parameters of DSGE with DKW mechanism



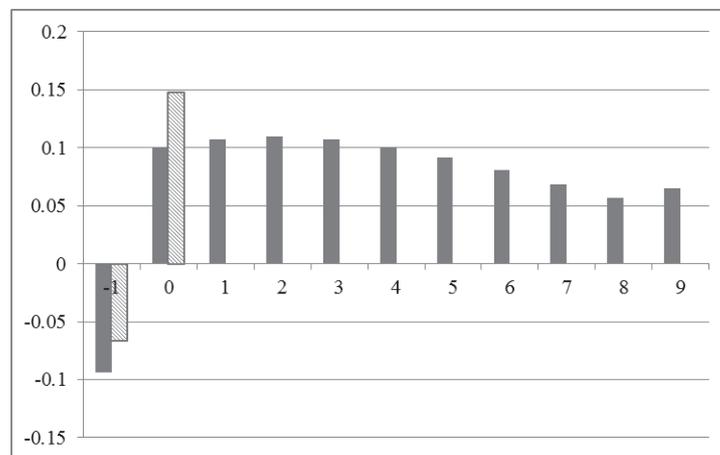
Source: own calculations with Dynare 4.3 (Adjemian et al. 2011).

Figure 5. Median of posterior distributions of the parameters at lagged (-) and expected (+) inflation in SDPC (in black) and NHPC (shaded)



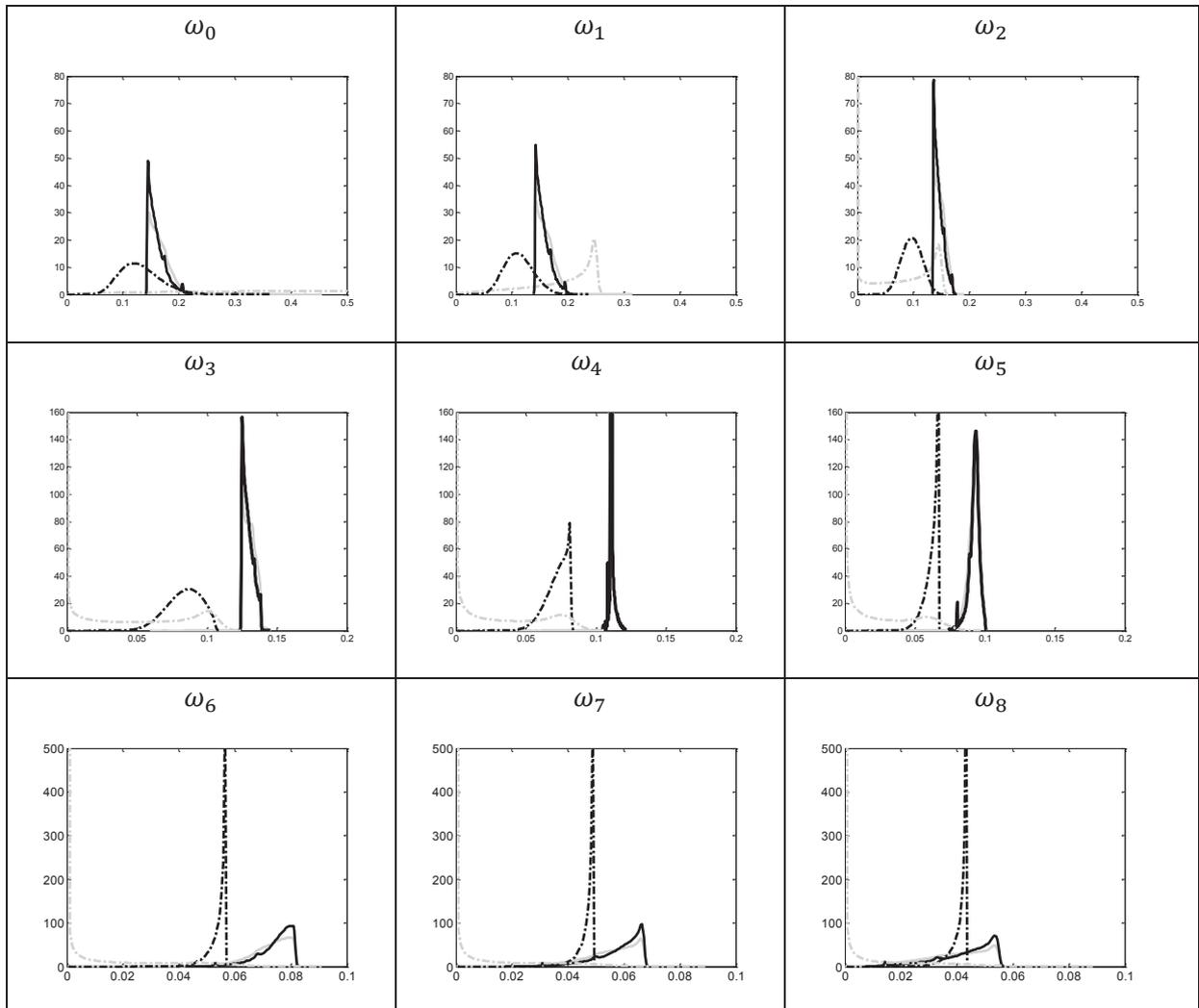
Source: Own calculations. Note: in SDPC the parameters at lagged terms ( $k = -1, \dots, -4$ ) are denoted by  $\mu'_k$ , and at lead terms by  $\delta'_k$  ( $k = 1 \dots 9$ ). In NHPC they are denoted by  $\beta_b$  ( $k = -1$ ), and  $\beta_f$  ( $k = 1$ ).

Figure 6. Median of posterior distribution of parameters at lagged (-), current ( $l = 0$ ) and expected (+) output gap in SDPC (black) and NHPC (shaded).



Source: Own calculations. In SDPC:  $\vartheta$  for  $l = -1$ ,  $\hat{\psi}'_l$  for  $l = 0, \dots, 9$ , and in NHPC:  $\chi_1$  i  $\chi_0$ .

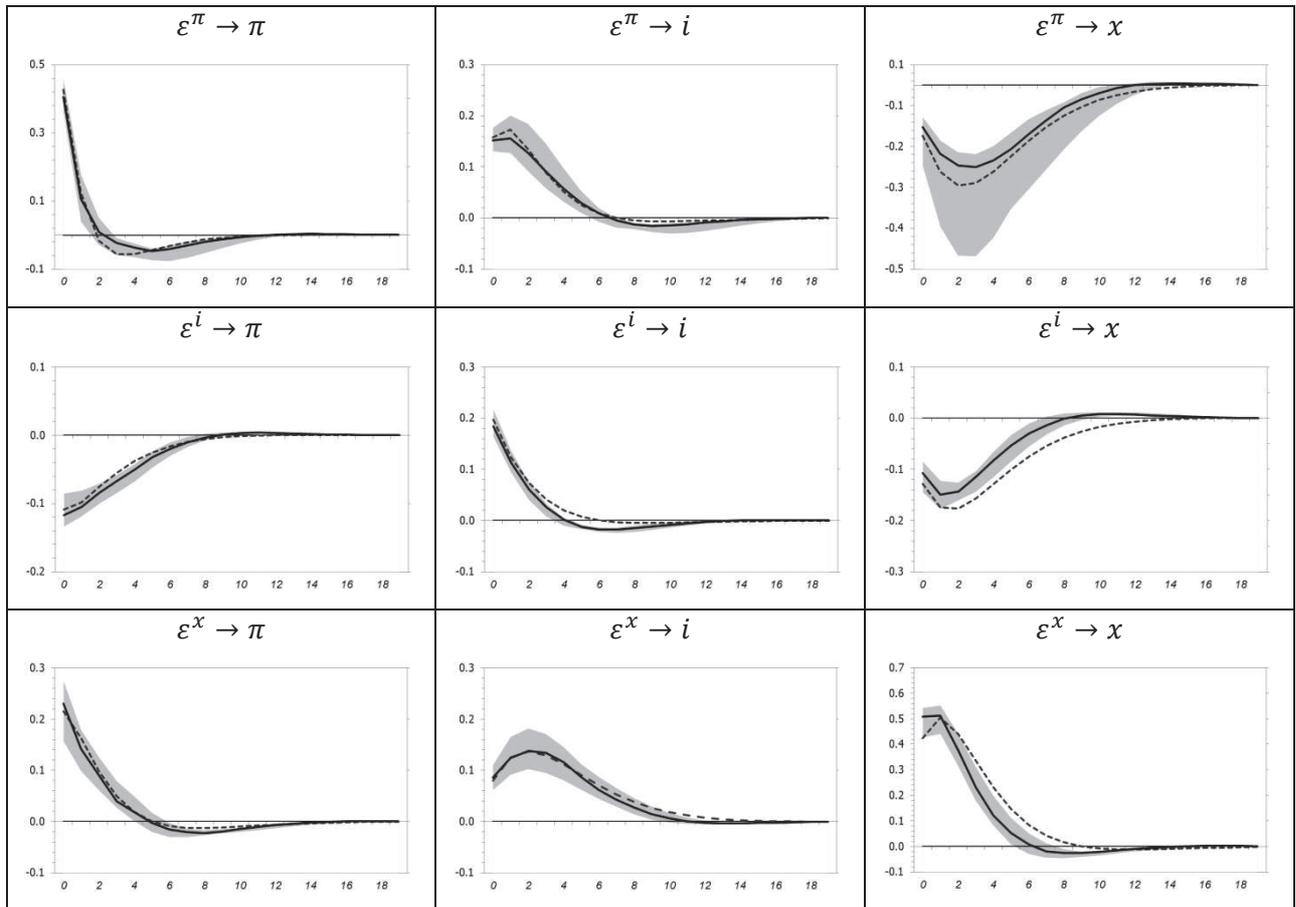
Figure 7. Prior (grey lines) and posterior (black lines) distributions of fraction of firms in price vintages  $\omega_j$  at steady state in the estimated models with state-dependent (dotted lines) and time-dependent (full lines) pricing.



Source: Own calculations.

Note: in the time-dependent model  $\omega_j = (1 - \theta)\theta^j$

Figure 8. Impulse response functions of inflation  $\pi$ , interest rate  $i$ , and output gap  $x$  to one-standard-deviation shocks to technology  $\varepsilon^\pi$ , monetary policy  $\varepsilon^i$  and preferences  $\varepsilon^x$ .

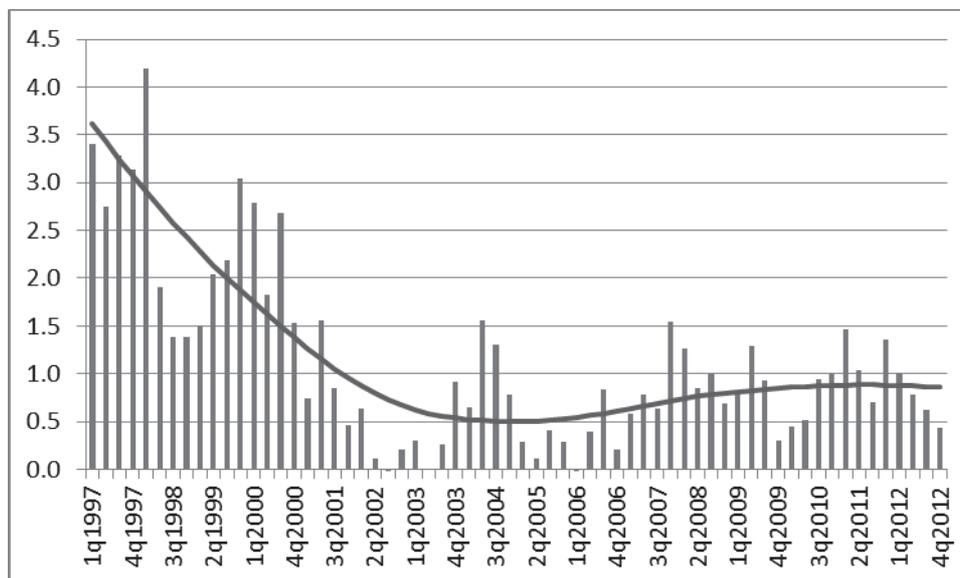


Source: Own calculations with Dynare 4.3.

Note: Black line and a shaded area denote, respectively, medians in posterior IRFs and their 90% HPD intervals from the state-dependent pricing model. Dashed lines are means of posterior IRFs from the time-dependent pricing model.

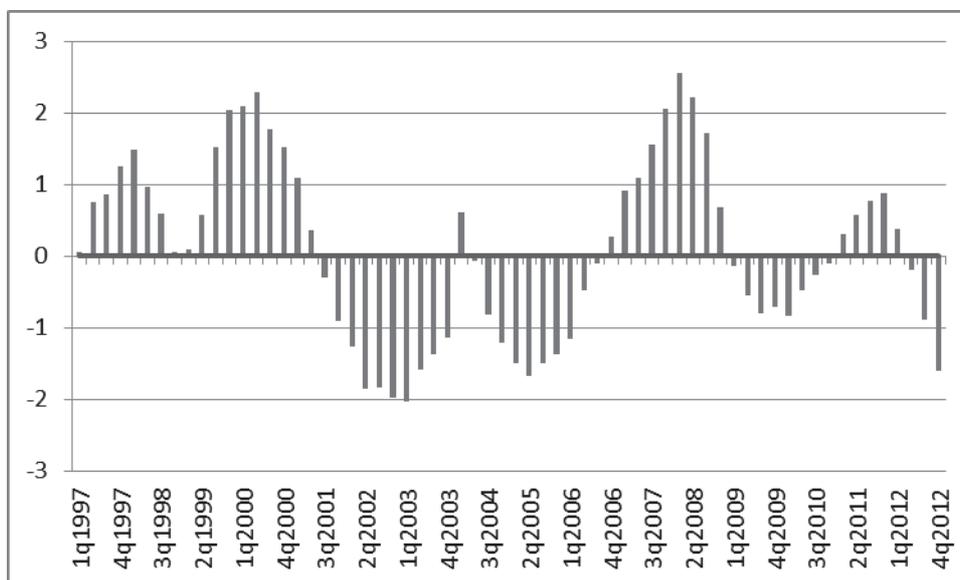
## Appendix A. Data for Polish economy (1997-2012)

Figure 1A. Inflation (seasonally adjusted, in percentage points per quarter) and its HP-filter trend.



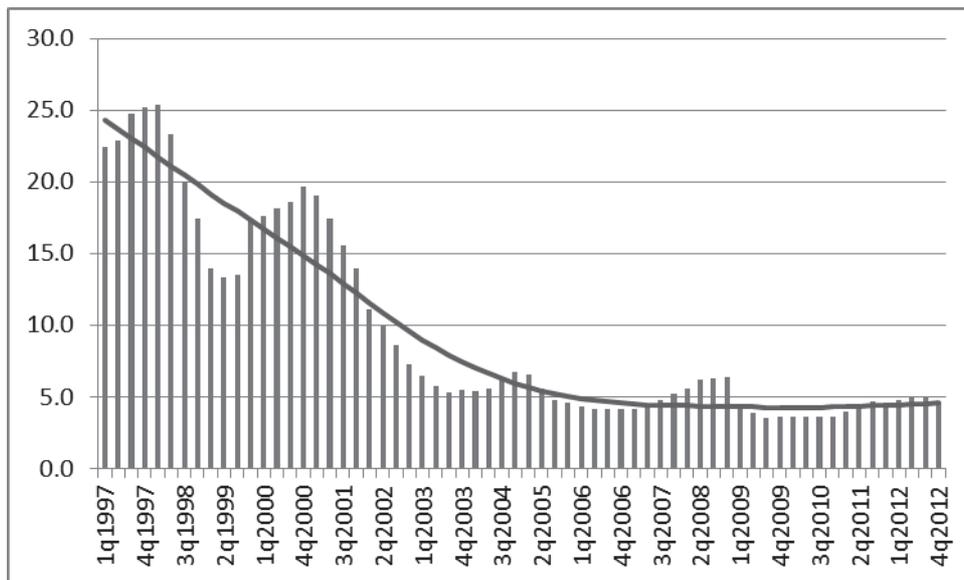
Source: Central Statistical Office (GUS). Own calculations.

Figure 2A: Output gap (% deviations from HP-filter trend).



Source: Central Statistical Office (GUS). Own calculations.

Figure 3A: Short-term interest rate (WIBOR 1m) and its HP-filter trend.



Source: Central Statistical Office (GUS). Own calculations.

## Appendix B. The estimation results of a small-scale model with the Gali - Gertler (1999) pricing mechanism

Table 1B. Characteristics of prior and posterior distribution in the time-dependent pricing model.

Parameter names	Prior distribution			Posterior distribution	
	Type of distribution	Mean	Standard deviation	Mean	HPD interval (90%)
$\theta$	Beta	0.5	0.25	0.869	[0.813; 0.927]
$\tau$	Beta	0.5	0.25	0.158	[0.006; 0.297]
$h$	Beta	0.5	0.25	0.924	[0.846; 0.999]
$\varphi$	Gamma	2.0	1.50	1.780	[0.053; 3.491]
$\sigma$	Gamma	3.0	1.50	6.290	[4.008; 8.518]
$\phi_\pi$	Normalny	2.0	0.10	2.001	[1.917; 2.082]
$\phi_x$	Normalny	0.0	0.10	0.001	[-0.015; 0.018]
$\lambda$	Beta	0.5	0.15	0.808	[0.773; 0.844]
$\rho_\pi$	Beta	0.5	0.15	0.306	[0.137; 0.469]
$\rho_x$	Beta	0.5	0.15	0.554	[0.428; 0.681]
$\sigma^\pi$	Gamma <sup>-1</sup>	1.0	$\infty$	0.340	[0.271; 0.408]
$\sigma^x$	Gamma <sup>-1</sup>	0.4	$\infty$	0.132	[0.096; 0.166]
$\sigma^i$	Gamma <sup>-1</sup>	1.0	$\infty$	0.232	[0.197; 0.266]

Source: Own calculations with Dynare 4.3 (Adjemian et al. 2011). Note: HPD stands for highest posterior density.

## Appendix C. DSGE model structure and SDPC derivation

### Households

We consider representative households maximizing intertemporal utility from their consumption ( $C_t(i)$ ), and disutility of labor ( $N_t$ )<sup>8</sup>:

$$U(C_t(i), N_t) = e^{v_t^p} \left( \frac{[C_t(i)/(\bar{C}_{t-1})^h]^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad (11)$$

where  $\sigma > 0$  is a constant relative risk aversion,  $\varphi > 0$  is the inverse of Frisch elasticity of labour, and  $0 < h < 1$  is a measure of external habit persistence (Abel 1990), which is measured in relation to the average consumption (across all households) from the previous period  $(\bar{C}_{t-1})^h$ , and  $v_t^p$  is an AR(1) process, which we interpret as a preference shock in period  $t$  (i.e. a shock that shifts consumer tastes).

### Technology and aggregation

Firms indexed with  $j \in (0,1)$  transform labour to products given initial technology level  $A_0$ , and aggregate technology shocks  $v_t^m$ :<sup>9</sup>

$$Y_t(j) = A_0 e^{v_t^m} N_t^{1-\alpha}. \quad (12)$$

where  $v_t^m$  is a stationary AR(1) stochastic process, and  $0 < (1 - \alpha) < 1$  is a labor share.

With Dixit - Stiglitz (1977) monopolistic competition conditions we define aggregate consumption and price as:

$$C_t = \left( \int_0^1 C_t(j)^{(\varepsilon-1)/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}. \quad (13)$$

$$P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}. \quad (14)$$

where  $\varepsilon > 1$  is constant elasticity of substitution between goods or price elasticity of consumption

$$C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t.$$

### Firm pricing decisions

In period  $t$  the firm decision requires calculation the firm's real values:

<sup>8</sup> We assume competitive labor market with firms renting labour time of a representative household  $N_t$  at an economy wide real wage rate  $W_t$ . The household decisions are also subject to standard budget constraint.

<sup>9</sup> Alternatively, one can assume shocks to the monopolistic markup, which is hard to distinguish from negative technology shocks in the reduced Phillips curve relation (see Smets - Wouters 2003).

$$\begin{aligned}
v_{0,t} &= \max_P \{ z_{0,t}(P) + E_t Q_{t,t+1} \cdot (1 - \alpha_{1,t+1}) \cdot v_{1,t+1} + E_t Q_{t,t+1} \alpha_{1,t+1} \\
&\quad \cdot (v_{0,t+1} - W_{t+1} \cdot K_{1,t+1}) \} \\
v_{j,t} &= z_{j,t}(P_{t-j}^*) + E_t Q_{t,t+1} \cdot (1 - \alpha_{j,t+1}) \cdot v_{j+1,t+1} + E_t Q_{t,t+1} \alpha_{j,t+1} \\
&\quad \cdot (v_{0,t+1} - W_{t+1} \cdot K_{j,t+1}), \\
&\quad j = 1, 2, \dots, J-1,
\end{aligned} \tag{15}$$

where  $z_{j,t}(P_{t-j}^*) = \left(\frac{P_{t-j}^*}{P_t}\right)^{-\varepsilon} \cdot Y_t \cdot \frac{P_{t-j}^*}{P_t} - \Psi_{t,j}$  is the firm's current period real profit if its nominal price is  $P_{t-j}^*$ ,  $K_{j,t+1}$  is the average menu cost in vintage  $j$  and the term  $Q_{t,t+k} = \beta^k U'(C_{t+k})/U'(C_t)$  represents stochastic discount factor for the future real profits (see Campbell - 1999).

The solution to the problem of maximization the firm's real value  $v_{0,t}$  is given by (cf. Dotsey – King - Wolman 1999, p. 665):

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j \cdot E_t Q_{t,t+j} \cdot \frac{\omega_{j,t+j}}{\omega_{0,t}} \cdot MC_{t+j} \cdot P_{t+j}^\varepsilon \cdot Y_{t+j}}{\sum_{j=0}^{J-1} \beta^j \cdot E_t Q_{t,t+j} \cdot \frac{\omega_{j,t+j}}{\omega_{0,t}} \cdot P_{t+j}^{\varepsilon-1} \cdot Y_{t+j}}, \tag{16}$$

where  $MC_t$  is the real marginal cost and  $\frac{\omega_{j,t+j}}{\omega_{0,t}}$  is the probability of nonadjustment of the price from period  $t$  to period  $t + j$ . In the case of flexible prices ( $B = 0$ ) the formula (16) can be rewritten as  $\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} MC$ . Hence, the term  $\frac{\varepsilon}{\varepsilon-1}$  can be interpret as a monopolistic markup.

### Central bank monetary rule

The last type of shocks  $(\varepsilon_t^i)_{t=1,2,\dots}$  is a monetary policy shock. It is interpreted in terms of deviations of a central bank from a current Taylor (1993) rule augmented with an interest rate smoothing:

$$\tilde{i}_t = \lambda \tilde{i}_{t-1} + (1 - \lambda)(\phi_\pi \tilde{\pi}_t + \phi_x x_t) + \varepsilon_t^i.$$

### Non-zero steady state and derivation of SDPC

The steady state of the economy is defined as the constant level of inflation  $\Pi^{SS}$ , total production  $Y$  and stationary distribution of prices  $\omega_0, \omega_1, \dots, \omega_{J-1}$ . Moreover, denote by  $RP_t^* = \frac{P_t^*}{P_t}$  relative optimal price in period  $t$  and notice that the steady state value of  $RP_t^*$  is constant in time and given

$$\text{by } RP^* = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{j=0}^{J-1} \beta^j \cdot \omega_j \Pi^j \varepsilon}{\sum_{j=0}^{J-1} \beta^j \cdot \Pi^j (\varepsilon-1)} MC.$$

Hence, DKW pricing mechanism in the steady state is described by time-homogenous stationary Markov chain with states  $1, \dots, J$  denoting the price vintages and with transition matrices,  $M$ :

$$M = \begin{bmatrix} \alpha_1 & 1 - \alpha_1 & 0 & \dots & 0 \\ \alpha_2 & 0 & 1 - \alpha_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{j-1} & 0 & 0 & \dots & 1 - \alpha_{j-1} \\ 1 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (17)$$

The probabilities  $\alpha_1, \alpha_2, \dots, \alpha_j$  with  $\alpha_j = 1$  are the solution to the constrain optimization problem:

$$\begin{aligned} v_0 &= \max_{RP} \{z_0(RP) + (1 - \alpha_1) \cdot v_1 + \alpha_1 \cdot (v_0 - W \cdot K_1)\} \\ v_j &= z_j(RP^*) + (1 - \alpha_{j,t+1}) \cdot v_{j+1} + \alpha_j \cdot (v_0 - W \cdot K_j), \\ \alpha_j &= G((v_0 - v_j)/W), \\ j &= 1, 2, \dots, J - 1, \end{aligned} \quad (18)$$

where  $z_j(RP^*) = (RP^*)^{1-\varepsilon} \cdot Y \cdot \Pi^{j(\varepsilon-1)} - MC \cdot Y$ . In consequence, the sums of conditional discounted current and future profits  $v_0, v_j$  are constant over time.

To derive the Phillips Curve one has to solve the optimal problem for firms and aggregate the price distribution. The Dixit - Stiglitz (1977) price aggregation entails:  $P_t^{1-\varepsilon} = \sum_{j=0}^{J-1} \omega_{j,t} (P_{t-j}^*)^{1-\varepsilon}$ . Then, substituting the formula for relative optimal prices  $RP_t$  into the above equation, one obtains:

$$1 = \sum_{j=0}^{J-1} \omega_{j,t} \left( RP_{t-j}^* \frac{P_{t-j}}{P_t} \right)^{1-\varepsilon} \quad (19)$$

Log-linearization around steady-state leads to:

$$\begin{aligned} \widetilde{r\bar{p}}_t^* &= \frac{1}{\omega_0} \left[ \sum_{j=0}^{J-2} \widetilde{\pi}_{t-j} \sum_{i=j+1}^{J-1} \omega_i \Pi^{i(\varepsilon-1)} - \sum_{j=1}^{J-1} \omega_j \Pi^{j(\varepsilon-1)} \widetilde{r\bar{p}}_{t-j}^* \right. \\ &\quad \left. + \frac{1}{\varepsilon - 1} \sum_{j=0}^{J-1} \omega_j \widetilde{\omega}_{j,t} \Pi^{j(\varepsilon-1)} \right]. \end{aligned} \quad (20)$$

where variables with a tilde are deviations from steady-state:  $\widetilde{\omega}_{j,t} = \ln \omega_{j,t} - \ln \omega_j$ ,  $\widetilde{\pi}_{t-j} = \ln \Pi_{t-j} - \ln \Pi$ ,  $\widetilde{r\bar{p}}_{t-j}^* = \ln(RP_{t-j}^*) - \ln(RP^*)$ ,  $j = 0, 1, \dots, J - 1$ .

After log-linearization of formula for relative optimal price (see Appendix A to Bakhshi – Khan – Rudolf 2006) and assuming  $\widetilde{q}_{t,t+j} = 0$ , we obtain:

$$\begin{aligned}
\tilde{r}\tilde{p}_t^* &= E_t \sum_{j=0}^{J-1} \left[ \tilde{\omega}_{j,t+j} - \tilde{\omega}_{0,t} + \tilde{m}\tilde{c}_{t+j} + \varepsilon \sum_{k=1}^j \tilde{\pi}_{t+k} + x_{t+j} \right] \rho_j \\
&\quad - E_t \sum_{j=0}^{J-1} \left[ \tilde{\omega}_{j,t+j} - \tilde{\omega}_{0,t} + (\varepsilon - 1) \sum_{k=1}^j \tilde{\pi}_{t+k} + x_{t+j} \right] \delta_j,
\end{aligned} \tag{21}$$

where  $x_t$  is an output gap,  $\delta_j = \frac{\beta^j \omega_j \Pi^{j(\varepsilon-1)}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i(\varepsilon-1)}}$  and  $\rho_j = \frac{\beta^j \omega_j \Pi^{j\varepsilon}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i\varepsilon}}$ .

Consumer's habit persistence leads to the following relationship between percentage deviation of real marginal cost from its steady-state value and output gap:

$$\tilde{m}\tilde{c}_t = \kappa_1 x_t + \kappa_2 x_{t-1} + \frac{\phi + 1}{1 - \alpha} d_t + \frac{\phi + 1}{\alpha - 1} v_t^m,$$

where  $\kappa_1 = \frac{\phi + \alpha + (1 - \alpha)\sigma}{1 - \alpha}$ ,  $\kappa_2 = h(\sigma - 1)$ .

Substituting this equation together with  $E_t v_{t+j}^m = 0$  for  $j = 1, 2, \dots, J - 1$  into (13) gives:

$$\begin{aligned}
\tilde{r}\tilde{p}_t^* &= E_t \sum_{j=1}^{J-1} \tilde{\pi}_{t+j} \sum_{k=j}^{J-1} (\varepsilon \rho_k - (1 - \varepsilon) \delta_k) + E_t \sum_{j=0}^{J-1} (\rho_j - \delta_j) (\tilde{\omega}_{j,t+j} \\
&\quad - \tilde{\omega}_{0,t}) + \kappa_2 \rho_0 x_{t-1} \\
&\quad + E_t \sum_{l=0}^{J-1} \hat{\psi}_l x_{t+l} + \frac{\phi + 1}{1 - \alpha} d_t + \frac{\phi + 1}{\alpha - 1} v_t^m,
\end{aligned} \tag{22}$$

where:

$$\hat{\psi}_l = \begin{cases} \kappa_1 \rho_j + \rho_j - \delta_j + \kappa_2 \rho_{j+1} & l = 0, 1, \dots, J - 2 \\ \kappa_1 \rho_j + \rho_j - \delta_j & l = J - 1 \end{cases}$$

Comparing (20) and (22) we obtain:

$$\begin{aligned}
\tilde{\pi}_t = E_t \sum_{k=1}^{J-1} \tilde{\pi}_{t+k} \delta'_k + E_t \sum_{j=0}^{J-1} \gamma_j (\tilde{\omega}_{j,t+j} - \tilde{\omega}_{0,t}) + \frac{1}{\mu_0} \kappa_2 \rho_0 x_{t-1} \\
+ E_t \sum_{l=0}^{J-1} \frac{1}{\mu_0} \hat{\psi}_l x_{t+l} \\
- \sum_{k=1}^{J-2} \frac{\mu_k}{\mu_0} \tilde{\pi}_{t-k} + \sum_{j=1}^{J-1} \frac{\omega_j}{\mu_0 \omega_0} \Pi^{j(\varepsilon-1)} \tilde{r}\tilde{p}_{t-j}^* + \frac{1}{1-\varepsilon} \hat{\Omega}_t \\
+ \frac{1}{\mu_0} \frac{\phi+1}{1-\alpha} d_t + \frac{1}{\mu_0} \frac{\phi+1}{\alpha-1} v_t^m
\end{aligned} \tag{23}$$

$$\delta'_k = \frac{1}{\mu_0} \sum_{n=j}^{J-1} (\varepsilon \rho_n - (1-\varepsilon) \delta_n) \text{ for } k = 1, 2, \dots, J-1, \gamma_j = \frac{1}{\mu_0} (\rho_j - \delta_j) \text{ for } j = 0, 2, \dots, J-1,$$

$$\mu_k = \frac{1}{\omega_0} \sum_{i=k+1}^{J-1} \omega_i \Pi^{i(\varepsilon-1)} \text{ for } k = 0, 1, 2, \dots, J-2, \hat{\Omega}_t = \sum_{j=0}^{J-1} \frac{1}{\omega_0} \Pi^{j(\varepsilon-1)} \omega_j \tilde{\omega}_{j,t}.$$

To derive State Dependent Phillips Curve from equation (23) one needs to recurrently substitute  $\tilde{r}\tilde{p}_{t-1}^*, \tilde{r}\tilde{p}_{t-2}^*, \dots$ , formulas from (22) (see Appendix B in Bakhshi - Khan - Rudolf 2006).