

Monetary policy, banking and heterogeneous agents

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Abstract

The influence of heterogeneous expectations on monetary policy performance has gained a lot of attention in the recent years. It proved to be an important factor that, under some circumstances, may even destabilize the economy (Massaro, 2012). This paper investigates the phenomenon of heterogeneous expectations further, analyzing its role in the monetary policy conduct with an active banking sector. In our analysis we assume a constant fraction of boundedly rational agents who use simple heuristics to form their expectations. The impact of those biased beliefs is studied in the framework, originally developed by Goodfriend and McCallum (2007). We first show that the presence of the banking sector changes the determinacy structure of the system and, depending on the heuristics used, the presence of the boundedly rational agents might have either stabilizing or destabilizing effect. In particular, when boundedly rational agents extrapolate the past performance in forming their expectations, the range of the stable (determinate) monetary policy instruments is narrowed.

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1 Introduction

The need for a framework which would incorporate financial frictions in the DSGE models was stressed long before the 2007-2009 financial crisis (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). The body of literature in this topic has grown substantially thereafter, bringing significant changes to the monetary policy conduct (Rotemberg and Woodford, 1997; Woodford, 2003). Surprising it is, as argued by Goodfriend and McCallum (2007) and Casares, Miguel and Poutineau (2010), the role of the banking sector was left unexplored in the monetary policy analysis until recently.

The framework used in this study clarifies this oversight. First, by introducing profit-maximizing bankers at the micro level, one may explicitly study the impact of their individual behavior on the macro aggregates. Second, the differentiation of the capital market allows to investigate the relationship between various types of interest rates (Goodfriend, 2005). Third, by having government bonds that serve for collateral purposes, one observes the direct influence of public policy on the monetary aggregates.

Most noticeably however, a banking sector *per se* is an important, if not the most important (Levine, 1996), part of each economy. Since it is a general source of liquidity, its problems may easily spread over the other sectors, bringing them down eventually. Especially, the recent history proves that the banking sector disturbances might result in sovereign crises, as it recently took place in the Eurozone (Grammatikos and Vermeulen, 2012). Therefore, a detailed study of the banking's role in the monetary framework is required in order to (i) understand its transmission mechanism and (ii) endow the monetary authorities with the sufficient preventive tools.

The goal of this paper is twofold. First, we assess the determinacy properties of different monetary policies in the DSGE model with a banking sector of Goodfriend and McCallum (2007). The model is built within the standard New-Keynesian framework where the aggregate dynamics is a direct consequence of individual utility maximizing behavior of forward-looking agents. Second, we relax the assumption of agents' homogeneity and investigate how the presence of the backward-looking agents influences the determinacy of the equilibrium.

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The latter part of this study is motivated by a growing body of research which shows explicitly that agents differ in forming expectations. This phenomenon was confirmed by both survey data analysis (Carroll, 2003; Mankiw et al., 2003; Branch, 2004) as well as laboratory experiments with human subjects (Hommes et al., 2004; Hommes, 2011; Pfajfar and Zakelj, 2011). The heterogeneity among agents was proved to have important implications on the determinacy properties in the New-Keynesian models (Branch and McGough, 2009; Massaro, 2012). We follow this approach and assess its implication within the system with a banking sector.

The paper is organized as follows. Section 2 describes the workhorse model and discusses the implications of the banking sector on the monetary policy conduct. In section 3 we relax the assumption of representative agent structure and introduce boundedly rational backward-looking agents. Section 4 presents the numerical results and Section 5 concludes.

2 The model

In this section we develop the workhorse version of the model. Since the complete derivation, with the first order conditions and aggregation, is described in detail in the original paper of Goodfriend and McCallum (2007), we skip it in the main part of this text. However, for reader's convenience, the complete derivation is given in the Appendix.

The model space consists of a continuum of farmers who provide labor supply to the production and banking sectors at the same time (n_t and m_t respectively). Additionally, each farmer manufactures a differentiated product and sells it in the monopolistically competitive environment. The same as in the standard New-Keynesian framework, it is assumed that only a fraction $(1 - \omega)$ of all farmers can adjust their prices fully flexibly. The remaining part takes the prices from the previous period (Calvo, 1983). Given these conditions, the goal of each farmer is to maximize his expected utility, which is the linear combination of consumption and leisure, over the infinite horizon.

In the utility maximization problem, each farmer has to take into account three constraints: (i) the budget constraint, (ii) the production constraint and (iii) the banking constraint. The first one is the standard intertemporal budget constraint which ensures that the net income and bond/money holdings in one period are being transmitted to the next period. The second constraint is a direct consequence of the production technology, which in this case, is of the Cobb-Douglas type. Assuming market clearing, the production (Y_t) in each period is the consequence of the amount of capital (K_t) and labor (n_t^d) involved, corrected for their output elasticities: η and $(1 - \eta)$ respectively. The banking constraint assumes that the level of consumption (C_t) has to be rigidly related to the level of deposits held at a bank. One may view this as if all the transactions were being facilitated through the banking sector and each agent may consume a part V of his wealth only. A bank is then allowed to use $(1 - rr)$ fraction of the deposits to produce loans using the Cobb-Douglas production function with collateral (col_t) and labor (m_t^d) as production factors and α and $(1 - \alpha)$ being the output elasticities. The collateral consists of two parts: the discounted level of real bond holdings $B_{t+1}/(P_t^A(1 + r_t^B))$, with P_t^A being the aggregate price level and r_t^B the interest rate on bonds, and real level of capital $q_t K_{t+1}$, corrected for the inferiority of capital to bonds for collateral purposes v . The last term results from the fact that bonds, contrary to capital goods, do not require substantial monitoring effort in order to verify their market value (Goodfriend and McCallum, 2007).

There are two main simplifications of the original model. First, we abstract from the capital shocks in the loan production function. We assume that the capital level is at its steady state level and the productivity shocks are transmitted through the labor channels only. This simplification does not affect the final results as in the determinacy analysis the stochastic terms do not play a role (Blanchard and Kahn, 1980). Second, we assume zero tax rate. Eventually, the role of government is to issue bonds in each period at some exogenously given level, and pay the interest.

Given the specification above, we may now turn to derive the three model equations: the Investment-Savings (IS) curve, the Phillips curve and the banking curve. The first two of them build the standard New-Keynesian model. The last one is the direct consequence of the presence of the banking sector and describes its role in the aggregate dynamics.

2.1 The IS curve

The model implies the presence of two Lagrange multipliers: λ_t for the budget constraint and ξ_t for the production constraint. They represent the shadow values, or the utility gains, of unit values of consumption and production respectively (Casares, Miguel and Poutineau, 2010). In particular, from the banking labor demand optimality condition we know that

$$\lambda_t^i = \frac{\xi_t^i}{\varphi_t^i} = \frac{\frac{\phi}{C_t^i}}{1 + \left(\frac{1-rr}{V}\right) \chi_t^i}, \quad (1)$$

where φ_t^i is the individual marginal production cost, ϕ is the utility weight on consumption and we explored the fact that the χ_t^i might be viewed as the individual marginal loan management cost, or simply the marginal banking cost (Goodfriend and McCallum, 2007; Casares, Miguel and Poutineau, 2010)¹. To put it more formally, imagine the cost minimization problem of a representative bank in a situation without collateral cost. The total cost function may be rewritten as $TC_t = m_t^d w_t$, where w_t is the real wage. The minimization problem includes the loan production constraint with a Lagrangian multiplier (here perceived as a marginal cost (Walsh, 2010)), denoted by χ_t . The first order condition implies that $\chi_t = V w_t m_t^d / ((1 - rr)(1 - \alpha)C_t)$. χ_t^i is parallel to the individual marginal production cost that is being often referred to in the standard New-Keynesian framework (Walsh, 2010). One may view that as a general variable describing the situation in the banking sector - the higher it is the less effective the loan management is. As it is shown later, this variable is of crucial importance as it becomes a link between a standard New-Keynesian model and the banking system.

Eq. (1) gives the first overview of the model behavior. First, the shadow value of production is equal shadow value of consumption corrected for the marginal production cost. In other words, additional consumption has to turn up in either increased production or decreased production costs. Second, λ_t is the marginal utility of consumption corrected for the marginal banking cost. Put it differently, each additional unit of consumption requires more deposits, which may be raised at the cost χ_t . It is straightforward to notice that the lower the marginal banking cost, the relatively cheaper the additional consumption. On the contrary, highly inefficient banking sector limits the incentives to increase consumption.

Substituting Eq. (1) to the bond optimality condition, we finally arrive at the familiar Euler equation

$$\beta E_t^i \left(\frac{\frac{\phi}{C_{t+1}^i}}{1 + \left(\frac{1-rr}{V}\right) \chi_{t+1}^i} \right) = \frac{\frac{\phi}{C_t^i}}{1 + \left(\frac{1-rr}{V}\right) \chi_t^i} (1 + E_t^i \pi_{t+1}) \left(\frac{1 - \frac{1-rr}{V} \chi_t^i \Omega_t^i}{1 + r_t^B} \right), \quad (2)$$

where $(1 + E_t^i \pi_{t+1}) = P_{t+1}^A / P_t^A$ is the inflation rate and $\Omega_t^i = \alpha C_t^i / col_t^i$.

Following Goodfriend (2005), let us introduce a one-period default free security with the nominal rate denoted by r_t^T . We additionally assume that it cannot serve for collateral purposes. r_t^T represents a pure intertemporal rate of interest and serves as a benchmark for other interest rates. From the agent optimization problem, we know that $1 + r_t^{i,T} = E_t^i \lambda_{t+1}^i P_{t+1}^i / (\beta \lambda_{t+1}^i P_t^i)$ so that it includes the discounted difference between expected changes in shadow prices and actual prices. An important distinction is that the pricing of this fictitious security is done at the individual level which is not strange given its completely artificial and agent-dependent nature. Eventually, the last term of Eq. (2) might be rewritten as the reciprocal of $(1 + r_t^{i,T})$.

At the same time, let us assume that each bank can obtain funds from the interbank market at the common rate r_t^{IB} . It can then loan them to agents at the rate $r_t^{i,T}$. The profit maximization of a bank implies that the marginal costs of obtaining funds has to be equal their marginal profit so that

$$(1 + r_t^{IB})(1 + \chi_t^i) = (1 + r_t^{i,T}). \quad (3)$$

Inserting Eq. (3) into Eq. (2) and taking the log approximation around the steady state we have

$$\hat{Y}_t^i = E_t^i \hat{Y}_{t+1}^i + \left(\frac{1 - rr}{V} \right) E_t^i \tilde{\chi}_{t+1}^i - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t^i - (\hat{r}_t^{IB} - E_t^i \pi_{t+1}), \quad (4)$$

where tildes and hats denote deviations and percentage deviations from the steady state, respectively, and we explored the market clearing condition².

The same as in the standard New-Keynesian framework, we define the potential output as the output under completely flexible prices and wages (Walsh, 2010). We additionally assume that in such a situation there is a fixed proportion between employment in the production and banking sector, $n_t^d \propto m_t^d$. Following Walsh (2010), price flexibility implies that all agents can adjust their prices immediately, which gives that the marginal cost of production φ_t is equal $(\theta - 1)/\theta$ across all individuals, where θ is the elasticity of substitution between consumption goods. The labor optimality condition implies that the real wage has to be equal the marginal rate of substitution between leisure and consumption, corrected for the presence of the banking sector. Combining the above mentioned points with Eq. (1) and the production constraint, we finally get that under flexible prices and wages, the supply of labor of each individual is fixed so that if the capital stock is in the steady state (as we assume throughout the model) the log deviations of the potential product depend only on exogenous disturbances, $\hat{Y}_t^f = (1 - \eta)(A1_t - \bar{A}1)$. Subtracting them from both sides of Eq. (3) and omitting the i subscript, we finally arrive at the aggregate IS curve corrected for the presence of a banking sector

$$x_t = E_t x_{t+1} + \left(\frac{1 - rr}{V} \right) E_t \tilde{\chi}_{t+1} - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t - [\hat{r}_t^{IB} - E_t \pi_{t+1}] + u_t, \quad (5)$$

¹We include subscript i to underline the individual level of the relationship which is explored in detail later. In the representative agent structure it may be omitted as every agent behaves the same.

²Following literature, we take the zero inflation steady state.

where $x_t = \hat{Y}_t^i - \hat{Y}_t^{f,i}$ is the output gap measure and u_t is the disturbance term that depends only on exogenous productivity shocks.

It is straightforward to notice that when skipping the banking sector variables from Eq. (5) we get the standard New-Keynesian IS curve. What is important, is that the aggregate dynamics is affected not only by the current, but also expected situation of the banking variables. In other words, the way the agents form their expectations about future banking sector conditions seems to play a role in determining current production. The impact of the banking sector is limited by (i) the reserve requirement, rr , and (ii) the proportion of consumption that has to be covered by deposits, V . Clearly, the lower the minimum reserve requirement, the larger the loan production so that the importance of the banking sector increases, *ceteris paribus*. At the same time, if the consumption-to-deposits coverage ratio is large, relative size of the banking sector is smaller so that its impact decreases.

2.2 The Phillips curve

The model allows us also to derive the explicit formula for the Phillips (or Aggregate Supply) curve. We know that all the farmers share the same production technology and face the same constant demand elasticities. We know from the Calvo lottery that fraction ω of agents cannot adjust their prices in a given period t . Profits of some future date $t+k$ are affected only if an agent did not receive a chance to adjust prices between t and $t+k$. Therefore, the probability of having lower expected profits in period k is ω^k . Having pointed that out, the price optimality condition has to be corrected for the nominal price rigidities in the long run and by iterating forward it might be viewed as

$$E_t^i \sum_{j=0}^{\infty} \beta^j \omega^j \left[(1-\theta) \left(\frac{P_t^i}{P_{t+j}^A} \right) + \theta \left(\frac{\xi_{t+j}^i}{\lambda_{t+j}^i} \right) \right] \left(\frac{1}{P_t^i} \right) \left(\frac{P_t^i}{P_{t+j}^A} \right) C_{t+j}^A = 0. \quad (6)$$

Solving for optimal price setting, we arrive at

$$\frac{P_t^i}{P_t^A} = \frac{E_t^i \sum_{j=0}^{\infty} \beta^j \omega^j C_{t+j}^A \varphi_{t+j}^i \left(\frac{P_{t+j}^A}{P_t^A} \right)^{\theta}}{E_t^i \sum_{j=0}^{\infty} \beta^j \omega^j C_{t+j}^A \left(\frac{P_{t+j}^A}{P_t^A} \right)^{\theta-1}}, \quad (7)$$

where we explored $\varphi_t^i = \xi_t^i / \lambda_t^i$ is the individual marginal production cost (Goodfriend and McCallum, 2007). Skipping the i subscript and taking log approximation, after some algebra we have³

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\varphi}_t, \quad (8)$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$. We further explore the fact that given the Cobb-Douglas production function, the steady state log deviations of the marginal production cost might be viewed as the output gap measure (Goodfriend and McCallum, 2007). Finally, we arrive at the standard New-Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \quad (9)$$

What is important, the situation in the banking sector does not affect the inflation level directly but only through the consumption channel. The absence of the banking variables in Eq. (9) is the consequence of banking sector specification. The level of consumption is rigidly related to the amount of deposits in the banking sector. Therefore, changes in the banking sector would result in the different deposit level what would shake the consumption eventually. However, there is no direct link to the inflation in the meantime.

2.3 The banking sector curve

Since the presence of the banking sector affects the aggregate evolution of of the IS and (indirectly) Phillips curves, it is also necessary to describe its dynamics. Observing that $\varphi_t = q_t K_t / (\eta C_t)$, the capital optimality condition implies

$$1 - \frac{v(1-rr)}{V} \Omega_t^i \lambda_t^i = \beta(2-\delta) E_t^i \left[\frac{\left(1 + \left(\frac{1-rr}{V}\right) \lambda_t^i\right) \varphi_{t+1}^i}{\left(1 + \left(\frac{1-rr}{V}\right) \lambda_{t+1}^i\right) \varphi_t^i} \right]. \quad (10)$$

Observe that the LHS of Eq. (10) is almost identical with the numerator of the last term in Eq. (2). The only difference comes from the inferiority of capital to bonds for collateral purposes, v . Applying the same interest

³For detailed derivation see the appendix of chapter 8 from Walsh (2010).

rate reasoning to the log approximation of LHS of Eq. (10), we get that $-v(1-rr)\Omega_t^i\chi_t^i/V = -v(r_t^{IB} - r_t^B + \chi_t^i)$. Since the interbank rate r_t^{IB} and the government bond rate r_t^B are both short-term rates, they should be close to each other (Goodfriend and McCallum, 2007). Additionally, given the fact that v is small, we neglect the influence of $v(r_t^{IB} - r_t^B)$. Eventually, after taking the deviations from the steady state of Eq. (10), iterating forward and skipping the i subscript, we get

$$\left(v + \frac{1-rr}{V}\right)\tilde{\chi}_t = \frac{1-rr}{V}E_t\tilde{\chi}_{t+1} - (E_t x_{t+1} - x_t). \quad (11)$$

Given Eq. (11) it is clear that the marginal cost of banking depends on (i) expectations about banking situation in the future and (ii) the current and expected production. In particular, the expectations about higher next period marginal banking cost work as a self fulfilling prophecy, increasing also today's cost. On the other hand, given the link between banking sector and consumption, high expectations about next period output gap decrease today's marginal banking cost. Imagine that people expect that there is going to be a decrease in production in the next period. Since the banking sector is a source of funding, there will be gradually less effort involved in the loan production, bringing the today's marginal cost down, eventually.

The effects on the current banking situation are proportional to the size of the banking sector, expressed by $(1-rr)/V$, being more prominent for smaller banking sectors. Smaller banking sectors are more vulnerable to the changes in the production sector as the relatively higher part of the banking capital is involved. On the other side, the bigger banking sector might be viewed as being more stable in the sense that production sector affects it to the lower extent. It should be kept in mind, however, that the model does not say that big banks are ultimately stable as a high drop in today's production can cause the marginal banking cost to skyrocket. Eq. (11) predicts only that this effect will be more prominent in the environment with a smaller banking sector.

At the same time, the inferiority of capital to bonds for collateral purposes v also plays a role in determining current marginal banking cost. In particular, let us consider the extreme case when capital cannot serve as a collateral, i.e. $v = 0$. Banks do not have access to capital then so that the only link between them and production sector is through loans. If there is a production shock, it affect the bond holdings and labor in the banking sector, making it relatively more severe. In this sense, using capital as a collateral serves as a hedge against production sector disturbances. When banks can access capital, in the presence of a production shock, its magnitude is being partially absorbed by the capital part.

3 The influence of heterogeneity

So far, we assumed that all the agents are the same and each of them faces the same optimization problem. Before turning to the numerical results, let us first consider what happens in the environment with heterogeneous agents. Contrary to the standard representative agent framework, we allow a part $(1-\gamma)$ of agents to be boundedly rational in forming their expectations⁴. In other words, we assume that a constant proportion of agents is uniformed or unable to form rational expectations. This implies that we may divide our continuum of farmers into two groups: those with rational expectations (E^{RE}) producing good $j \in [0, \gamma]$ and those with boundedly rational expectations (E^{BRE}) producing good $j \in [\gamma, 1]$. By rational agents we mean forward-looking fundamentalists who try to analyze the economy and form their expectations accordingly.

To be able to aggregate the results over both groups, we follow the methodology proposed by Branch and McGough (2009) and we impose similar seven axioms on expectation operators:

1. expectations operators fix observables,
2. if z is a forecasted variable and has a steady state, then $E^{RE}\bar{z} = E^{BRE}\bar{z} = \bar{z}$,
3. expectations operators are linear,
4. if for all $k \geq 0$, z_{t+k} and $\sum_{k=0}^{\infty} \beta^{t+k} z_{t+k}$ are forecasted variables then $E_t^\tau (\sum_{k=0}^{\infty} \beta^{t+k} z_{t+k}) = \sum_{k=0}^{\infty} \beta^{t+k} E_t^\tau z_{t+k}$ for $\tau \in \{RE, BRE\}$,
5. expectation operators satisfy the law of iterative expectations,
6. if z is a forecasted variable at time t and time $t+k$ then $E_t^\tau E_{t+k}^{\tau'} z_{t+k} = E_t^\tau z_{t+k}$ for $\tau \neq \tau'$,
7. all agents have common expectations on expected differences in limiting wealth and marginal banking cost.

⁴Throughout the paper we use the term rational to refer to forward-looking expectations and boundedly rational to express backward-looking expectations.

Our contribution to the original methodology comprises axiom 7, which describes the limiting behavior of the expectation operators. Since we add the banking sector to the model, we have to include it also in the expectation formation. Branch and McGough (2009) assume that both types of agents have common expectation on their limiting wealth. It allows to represent the aggregate expectations as a weighted average of group expectations. Otherwise, there is an extra term on the limiting behavior of expectations that complicates the dynamics (see Eq. (B7) from the Appendix). The similar pattern might be observed when aggregating the banking sector (Eq. (B15) from the Appendix). The aggregate dynamics of the system is therefore influenced by how agents predict the banking sector behaves over the infinite horizon.

Axiom 7 might be viewed as an agreement among all agents that in the limiting future their banking sectors will be equivalent or will at least generate the same marginal costs. From the macroeconomic perspective, one may think of it as if both groups of agents were trying to reach the banking sector technological frontier. Since there is a common technology, both types of agents should be heading towards the same frontier eventually, satisfying axiom 7.

Proposition 1. *In the presence of fraction $(1 - \gamma)$ of boundedly rational agents, if agents' expectations satisfy axioms 1-7 then the model from Eq. (5), (9) and (11) might be rewritten as*

$$x_t = \bar{E}_t x_{t+1} + \left(\frac{1 - rr}{V} \right) \bar{E}_t \tilde{\chi}_{t+1} - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t - [\hat{r}_t^{IB} - \bar{E}_t \pi_{t+1}] + u_t, \quad (12)$$

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t, \quad (13)$$

$$\left(v + \frac{1 - rr}{V} \right) \tilde{\chi}_t = \frac{1 - rr}{V} \bar{E}_t \tilde{\chi}_{t+1} - (\bar{E}_t x_{t+1} - x_t), \quad (14)$$

where $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$.

The proof of Proposition 1 can be found in the Appendix.

4 Numerical analysis

As opposed to the standard framework, the central bank policy instrument is the interbank interest rate \hat{r}_t^{IB} (not the bond rate). In fact, this is the monetary policy tool used in practice (Goodfriend and McCallum, 2007). As argued by Bernanke and Woodford (1997), to close the model we use the forward-looking Taylor rule of the form

$$\hat{r}_t^{IB} = \rho_x E_t^{RE} x_{t+1} + \rho_\pi E_t^{RE} \pi_{t+1}, \quad (15)$$

where ρ_x and ρ_π are constant weights on output and inflation variability, respectively. We follow a common approach and assume that the central bank does not target the situation in the banking sector directly. Including a banking sector variable in the monetary rule would extend the monetary policy analysis to the three-dimensional so that the interpretation of the results would not be straightforward anymore. Instead, the purpose of this study is to observe how the standard monetary policy rule behaves in the environment with a present banking sector.

4.1 Formation of expectations

Throughout the model, we assumed that the economy consists of two types of agents that are homogeneous within each group. The first type of agents, $i = RE$, are those who form rational expectations. We abstract here from the standard understanding of rationality, where agents have full knowledge and capacities to perfectly predict the future. Instead, we rather view them as being forward-looking fundamentalists, who collect information and form their expectations accordingly. They are not aware of the presence of the other type of agents so that their form their expectations as if everybody in the economy was rational in forming the expectations (Branch and McGough, 2009).

The second type of agents is not able to form rational expectations and use simple backward-looking heuristics instead to predict the future. Following Evans and Honkapohja (2001) we assume them to form adaptive expectations of the form

$$E_t^{BRE} \square_t = \mu \square_t, \quad (16)$$

where \square is either x , π or $\tilde{\chi}$. Parameter $\mu > 0$ describes the magnitude and the direction of the expectations. If $\mu > 1$, the influence of the past is being extrapolated to the future so that we would call those expectations

Parameter	V	rr	v	κ	β
Value	0.31	0.005	0.2	0.05	0.99

Table 1: The calibration values for the model parameters.

extrapolative. On the other hand, when $\mu < 1$, this influence disappears over time and we would call those expectations adaptive⁵. When $\mu = 1$, the boundedly rational agents form purely naive expectations (Evans and Honkapohja, 2001).

Given the expectation operators for both groups of agents, we may rewrite the aggregate expectations as

$$\bar{E}_t \square_{t+1} = \gamma E_t^{RE} \square_{t+1} + (1 - \gamma) \mu^2 \square_{t-1}, \quad (17)$$

with \square being either x , π or $\tilde{\chi}$.

4.2 Calibration and numerical results

DSGE models often exhibit indeterminacy of equilibrium, i.e. the system does not converge to one equilibrium, but may end up in multiple states or sunspot equilibria in the long run. In such a situation, the quantities and prices might not be even locally determinate, making the monetary policy conduct more unstable (Woodford, 1994). Therefore, it is important to make sure that the monetary tools are bring the system to the determinate state eventually.

Let us write the complete model in the matrix form

$$\begin{pmatrix} B & 0 \\ 0 & I_3 \end{pmatrix} \begin{pmatrix} y_{t+1} \\ y_t \end{pmatrix} = \begin{pmatrix} F & -C \\ I_3 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix}, \quad (18)$$

where $y = (x, \pi, \tilde{\chi})'$, $\varepsilon = (u, 0, 0)$ is a vector of exogenous shocks and B , F and C are the coefficient matrices described in detail in Appendix.

To study the determinacy properties, we apply the methodology developed by Blanchard and Kahn (1980). Since it does not depend on the exogenous disturbances, we omit ε in our further analysis. The determinacy is a result of the properties of the solution matrix M , where

$$M = \begin{pmatrix} B^{-1}F & -B^{-1}C \\ I_3 & 0 \end{pmatrix}. \quad (19)$$

The equilibrium of the system is determinate only if the number of eigenvalues that are outside the unit circle is equal to the number of non-predetermined variables (or the forward-looking variables (Walsh, 2010)), which is 3 in this case. Having more eigenvalues outside the unit circle implies explosiveness and fewer of them implies indeterminacy. The degree of indeterminacy is equal to the number of non-predetermined variables less the number of eigenvalues outside the unit circle (Evans and McGough, 2005).

We calibrate our model accordingly to Goodfriend and McCallum (2007). The detailed values are presented in Table 4.2.

The determinacy properties are studied for extrapolative and adaptive expectations separately. For the former, the μ parameter is set for 1.1 and for the latter for 0.9 (Branch and McGough, 2009). The ranges for policy parameters ρ_x and ρ_π are set at 5 and 10, respectively, in order to show the complete behavior of the system. The results are presented in Fig. 1 and 2.

First, the results confirm the 'rotating' behavior of the system from Branch and McGough (2009). With adaptive expectations the system rotates counterclockwise so that the determinacy area increases. With extrapolative expectations the system rotates clockwise decreasing the determinacy area.

Second, the location of the indeterminacy of order one and two is in line with figures presented in Branch and McGough (2009). In fact, the only difference lies in the size of the those areas, comparing with the original paper. This, however, is the consequence of the banking calibration parameters.

Third, the presence of the banking sector has one important impact on determinacy properties. When agents form extrapolative expectations ($\mu = 1.1$), a new region of indeterminacy of order 2 arises for too lenient inflation targeting. In fact, this effect additionally narrows the determinacy area, suggesting more aggressive inflation targeting. In the case with adaptive expectations ($\mu = 0.9$) there is no similar effect.

⁵In the literature, adaptive expectations are being recognized as the whole group of operators of the form similar to Eq. (16). However, for clarity purposes, we distinguish here between extrapolative and adaptive expectations when $\mu > 1$ and $\mu < 1$, respectively.

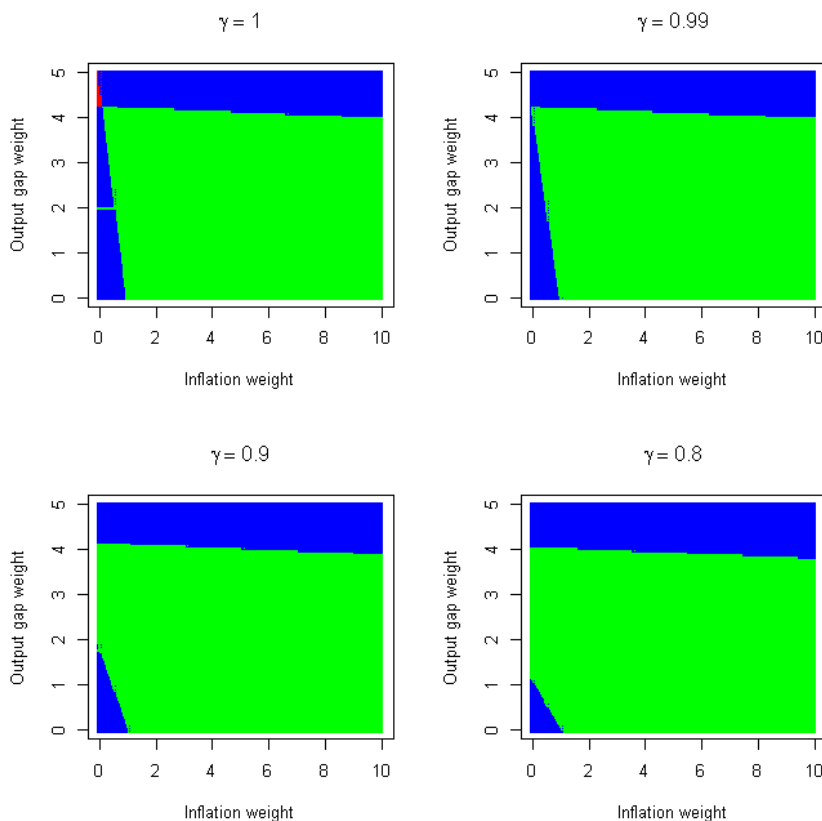


Figure 1: Determinacy properties ($\mu = 0.9$). Green color describes determinacy, blue order 1 indeterminacy and red order 2 indeterminacy.

5 Conclusions

The goal of the paper was twofold. First, we derived the workhorse model for the monetary policy analysis with the present banking sector. Second, we relaxed the assumption of the representative agent structure and investigated the effects of the presence of boundedly rational agents.

The results suggest that the presence of a banking sector brings stability to the system with only forward-looking agents. The determinacy area is visibly larger than in the model of Branch and McGough (2009). The pattern remains the same, even in the environment with the backward-looking agents who form adaptive expectations. Additionally, adaptive expectations seem to stabilize the system even more, bringing extra determinacy area for lenient output gap and inflation targeting.

The problem arises when backward-looking agents extrapolate the past performance over their future forecasts. The presence of the banking sector brings additional indeterminacy area for lower inflation targeting. In other words, in the environment with a fraction of extrapolative agents, the monetary policy that does not fight inflation sufficiently well, may not reach the equilibrium in the long run.

The results prove that households' expectations play an important role in determining the monetary policy, especially when a banking sector is present. However, this research shows just the top of an iceberg and more study is needed in order to understand fully the phenomenon of banking in the modern economy. In particular, future work should investigate the role of systemic risk and the network structure of the banking sector.

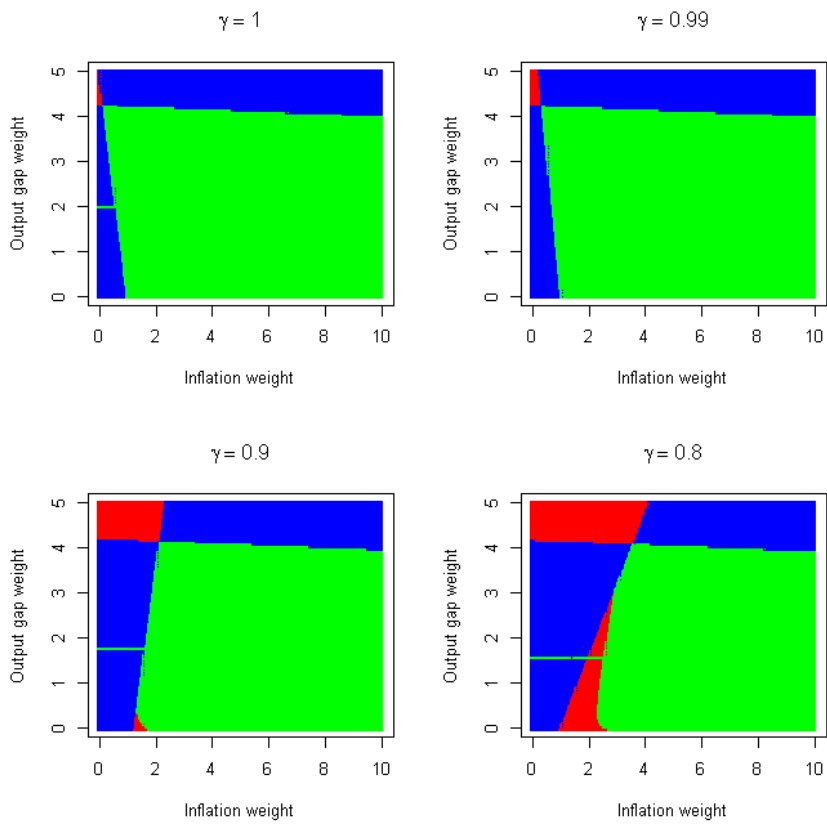


Figure 2: Determinacy properties ($\mu = 1.1$). Green color describes determinacy, blue order 1 indeterminacy and red order 2 indeterminacy.

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Appendix A

The utility of a farmer is defined as a weighted average of the consumption and leisure and takes the form

$$U^i(C_t^i, n_t^i, m_t^i) = \phi \log(C_t^i) + (1 - \phi) \log(1 - n_t^i - m_t^i), \quad (\text{A1})$$

where ϕ is the relative preference weight on consumption and t is the time subscript. C_t^i represents a composite consumption good and is of the standard Constant Elasticity of Substitution (CES) form, as in Dixit and Stiglitz (1977)

$$C_t^i = \left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (\text{A2})$$

with θ being the elasticity of substitution.

The farmer's decision problem is to maximize her discounted expected utility subject to the budget and technology constraints. Assuming a cashless limit (Woodford, 2003; Branch and McGough, 2009), we may define the former in real terms as

$$w_t(n_t^i + m_t^i) + q_t(1 - \delta)K_t^i + \frac{Y_t^i P_t^i}{P_t^A} + \frac{B_t^i}{P_t^A} = w_t(n_t^{i,d} + m_t^{i,d}) + C_t^i + q_t K_{t+1}^i + \frac{B_{t+1}^i}{P_t^A(1 + r_t^B)}, \quad (\text{A3})$$

where K_t^i is capital level with q_t being its real price and δ the depreciation rate, w_t is the real wage and B_t^i are the nominal bond holdings with the nominal interest equal r_t^B . Y_t^i is the production level, P^i is the price of the individual good and P_t^A is the aggregate price level, as in the Dixit-Stiglitz setup. Subscript d denotes the amount of labor demanded by a given farmer. Subscripts i and t relate to the agent and time dimensions, respectively.

Contrary to the standard New-Keynesian framework, there is a capital market in the model. Its role is twofold. First, capital serves as a production factor in the farmers' technology. Second, it is used as a collateral in the banking sector to produce loans. For simplicity, it is assumed that the aggregate capital stock is on a steady state growth path (Goodfriend and McCallum, 2007). What is important is that farmers are allowed to trade it so that its market price q_t may fluctuate.

The production constraint requires that

$$Y_t^i = K_t^{i\eta} \left(e^{A1_t} n_t^{i,d} \right)^{1-\eta}, \quad (\text{A4})$$

where $A1_t$ is an aggregate productivity disturbance and η is the capital elasticity measure.

A novelty in the model is the presence of the banking sector. Its main role is to facilitate transactions between production and consumption sides of the economy. Since the medium of exchange is the crucial role of the monetary policy analysis, the model does not distinguish between transaction balances and time deposits at the banks. In this simple form, this implies that the farmer's consumption in each period has to be rigidly related to the deposits held at a bank Goodfriend and McCallum (2007). In other words, in each period, the level of consumption (C_t^i) has to be covered by some constant fraction of the real deposits (VD_t^i/P_t^A). Since each bank has to hold a given level of reserves at the central bank (rr), the nominal amount of loans it may produce from deposits held by farmer i is constrained by $L_t^i = (1 - rr)D_t^i$. At the same time, the real loan production depends on the collateral and loan monitoring, and is assumed to be of a Cobb-Douglas form

$$\frac{L_t^i}{P_t^A} = F \left(\frac{B_{t+1}^i}{P_t^A(1 + r_t^B)} + vq_t K_{t+1}^i \right)^\alpha \left(e^{A2_t} m_t^{i,d} \right)^{1-\alpha}. \quad (\text{A5})$$

The loan monitoring is assumed to be proportional to the labor supplied to the banking sector by farmer i and $A2_t$ is the productivity disturbance similar to the one in the production sector. Since capital stock require a substantial monitoring effort to confirm its physical condition, its inferiority to bonds for collateral purposes is expressed by v (Goodfriend and McCallum, 2007).

The complete intertemporal farmers's maximization problem (with the banking sector present) may be written as

$$\max_{n_t^i, m_t^i, n_t^{i,d}, m_t^{i,d}, P_t^i, K_{t+1}^i, B_{t+1}^i} E_t^i \sum_{j=0}^{\infty} \beta^j [\phi \log(C_{t+j}^i) + (1 - \phi) \log(1 - n_{t+j}^i - m_{t+j}^i)]. \quad (\text{A6})$$

subject to the budget constraint (Eq. A3) and production constraint (Eq. A4).

Before solving the optimization problem, from Eq. (A5) we know that

$$C_t^i = \frac{VF}{1 - rr} (b_{t+1}^i + vq_t K_{t+1}^i)^\alpha \left(e^{A2_t} m_t^{i,d} \right)^{1-\alpha}, \quad (\text{A7})$$

where $b_{t+1}^i = B_{t+1}^i / (P_t^A(1 + r_t^B))$. Additionally, by imposing market clearing we know that the good produced by farmer i is equal to its demand

$$Y_t^i = \left(\frac{P_t^i}{P_t^A} \right)^{-\theta} C_t^A, \quad (\text{A8})$$

where C_t^A is the aggregate consumption level that each individual takes as given.

Let Lagrange multipliers be λ_t and ξ_t for the budget and production constraints respectively. By including Eq. (A7) and Eq. (A8) into the maximization problem and assuming market symmetry (Goodfriend and McCallum, 2007), the first order conditions provide

$$\frac{-(1-\phi)}{1-n_t^i-m_t^i} + \lambda_t^i w_t = 0, \quad (\text{A9})$$

$$-\lambda_t^i w_t + \xi_t^i e^{A1_t} (1-\eta) \left(\frac{K_t^i}{e^{A1_t} n_t^i} \right)^\eta = 0, \quad (\text{A10})$$

$$\left(\frac{\phi}{C_t^i} - \lambda_t^i \right) \frac{C_t^i (1-\alpha)}{m_t^i} - \lambda_t^i w_t = 0, \quad (\text{A11})$$

$$C_t^A \left(\frac{P_t^i}{P_t^A} \right)^{-\theta} \left(\frac{(1-\theta)\lambda_t^i}{P_t^A} + \frac{\theta\xi_t^i}{P_t^i} \right) = 0, \quad (\text{A12})$$

$$\left(\frac{\phi}{C_t^i \lambda_t^i} - 1 \right) \Omega_t^i v q_t - q_t + \beta(1-\delta) E_t^i \left(\frac{\lambda_{t+1}^i}{\lambda_t^i} q_{t+1} \right) + \beta \eta E_t^i \left(\frac{\xi_{t+1}^i}{\lambda_t^i} \left(\frac{e^{A1_{t+1}} n_{t+1}^i}{K_{t+1}^i} \right)^{1-\eta} \right) = 0, \quad (\text{A13})$$

$$\left(\frac{\phi}{C_t^i \lambda_t^i} - 1 \right) \Omega_t^i - 1 + \beta E_t^i \left(\frac{\lambda_{t+1}^i}{\lambda_t^i} \frac{P_t^A}{P_{t+1}^A} (1+r_t^B) \right) = 0, \quad (\text{A14})$$

where Ω_t^i is the partial derivative of the deposit constraint $C_t^i = \frac{VL_t^i}{(1-rr)P_t^A}$ with respect to collateral

$$\Omega_t^i = \frac{\alpha C_t^i}{b_{t+1}^i + v q_t K_{t+1}^i}. \quad (\text{A15})$$

Appendix B

Throughout the following derivation, we assume that each agent belongs to one of the two groups, i.e. $i = \tau \in \{RE, BRE\}$. By subscript A we will refer to the aggregate values.

1 The heterogeneous IS curve

Let us first introduce a benevolent financial institution that helps farmers in hedging the risk associated with the Calvo lottery (Shi, 1999; Mankiw and Reis, 2007). In each period it collects all the income from the market and then redistribute it evenly across farmers. Given this property and assuming cashless limit, the agents' budget constraint becomes

$$w_t(n_t^i + m_t^i) + q_t(1 - \delta)K_t^i + \frac{Y_t^i P_t^i}{P_t^A} + \frac{B_t^i}{P_t^A} + I_{r,t}^i = w_t(n_t^{i,d} + m_t^{i,d}) + C_t^i + q_t K_{t+1}^i + \frac{B_{t+1}}{P_t^A(1 + r_t^B)} + I_{p,t}^i, \quad (B1)$$

where $I_{r,t}^i$ and $I_{p,t}^i$ are the real receipts from and payments to the insurance agency.

We know that the average real income (denote it by Ψ_t^τ) and the average marginal banking cost χ_t^τ obtained by rational and boundedly rational agents are

$$\Psi_t^{RE} = \frac{1}{\gamma P_t^A} \int_0^\gamma P_t^i Y_t^i di \quad \text{and} \quad \Psi_t^{BRE} = \frac{1}{(1-\gamma)P_t^A} \int_\gamma^1 P_t^i Y_t^i di, \quad (B2)$$

$$\chi_t^{RE} = \frac{1}{\gamma} \int_0^\gamma \chi_t^i di \quad \text{and} \quad \chi_t^{BRE} = \frac{1}{1-\gamma} \int_\gamma^1 \chi_t^i di. \quad (B3)$$

From the above equations it is clear that we may view the aggregate product and aggregate real marginal banking cost as a weighted average of their components, i.e. $Y_t^A = \gamma Y_t^{RE} + (1-\gamma)Y_t^{BRE}$ and $\chi_t^A = \gamma \chi_t^{RE} + (1-\gamma)\chi_t^{BRE}$.

Following Branch and McGough (2009), if an agent is of type τ , then her real receipts from and payments to the insurance agency are $I_{r,t}^i = \Psi_t^\tau$ and $I_{p,t}^i = Y_t^i P_t^i / P_t^A$. By market clearing and axiom A2 the steady states of consumption and production are equal at individual and group levels. By imposing market symmetry, the budget constraint (Eq. B1) yields

$$\hat{C}_t^\tau = \hat{\Psi}_t^\tau + \frac{B_t^\tau / P_t^A}{\bar{Y}_t^A} - \frac{B_{t+1}^\tau / (P_t^A(1 + \bar{r}_t^B))}{\bar{Y}_t^A} + \frac{q_t(1 - \delta)K_t^\tau}{\bar{Y}_t^A} - \frac{q_t K_{t+1}^\tau}{\bar{Y}_t^A}, \quad (B4)$$

where bars indicate the steady state levels. Bond and capital market clearing requires that $\alpha B_t^{RE} = -(1 - \alpha)B_t^{BRE}$ and $\alpha K_t^{RE} = -(1 - \alpha)K_t^{BRE}$. After multiplying Eq. (B4) by γ for rational and by $(1-\gamma)$ for boundedly rational agents and summing up, we arrive at

$$\hat{Y}_t^A = \gamma \hat{\Psi}_t^{RE} + (1 - \gamma) \hat{\Psi}_t^{BRE}. \quad (B5)$$

From Eq. (4), (B3) and (B4) we have

$$\hat{\Psi}_t^\tau = E_t^\tau \hat{\Psi}_{t+1}^\tau + \left(\frac{1 - rr}{V} \right) E_t^\tau \tilde{\chi}_{t+1}^\tau - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t^\tau - (\hat{r}_t^{IB} - E_t^\tau \pi_{t+1}). \quad (B6)$$

Iterate this equation forward and substitute into Eq. (B5) we finally get

$$\begin{aligned} \hat{Y}_t^A &= \bar{E}_t \hat{Y}_{t+1}^A + \left(\frac{1 - rr}{V} \right) \bar{E}_t \tilde{\chi}_{t+1}^A - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t^A - (\hat{r}_t^{IB} - \bar{E}_t \pi_{t+1}) + \\ &+ \left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right) - \bar{E}_t \left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right), \end{aligned} \quad (B7)$$

with $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$ and $\hat{\Psi}_\infty^\tau = \lim_{k \rightarrow \infty} E_t^\tau \hat{\Psi}_{t+k}^\tau$. In fact, Eq. (B7) is of exactly the same form as in Branch and McGough (2009) but with a banking sector present. Axiom 7 indicates that agents predict their limiting wealth identically, which makes

$$\left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right) = \bar{E}_t \left(\gamma \hat{\Psi}_\infty^{RE} + (1 - \gamma) \hat{\Psi}_\infty^{BRE} \right). \quad (B8)$$

Subtracting the log deviations of the potential product from both sides, we finally arrive at the heterogeneous IS curve with a present banking sector

$$x_t = \bar{E}_t x_{t+1} + \left(\frac{1 - rr}{V} \right) \bar{E}_t \tilde{\chi}_{t+1}^A - \left(\frac{1 - rr}{V} + 1 \right) \tilde{\chi}_t^A - [\hat{r}_t^{IB} - \bar{E}_t \pi_{t+1}] + u_t, \quad (B9)$$

where $x_t = \hat{Y}_t^A - \hat{Y}_t^{f,A}$ is the output gap measure, expectation operator is the weighted average of the group expectations $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$ and u_t is the disturbance term that depends only on exogenous productivity shocks.

.2 The heterogeneous Phillips curve

It is important to note that when farmers may hedge against the Calvo risk their production level would be 0 in equilibrium as the result of the free-riding problem. Therefore, following Branch and McGough (2009), we assume that farmers make their pricing decisions as if there was no insuring agency.

Let us take the log approximation of Eq. (7)

$$\log P_t^\tau - \log P_t^A = (1 - \omega\beta)\hat{\varphi}_t^\tau + \omega\beta E_t^\tau \pi_{t+1} + \omega\beta E_t^\tau \log P_{t+1}^\tau / P_{t+1}^A. \quad (\text{B10})$$

Branch and McGough (2009) show that the Calvo lottery implies aggregate inflation to follow

$$\pi_t = \frac{1 - \omega}{\omega} (\gamma \log P_t^{RE} / P_t^A + (1 - \gamma) \log P_t^{BRE} / P_t^A). \quad (\text{B11})$$

As long as the pricing decisions are homogeneous within each group τ , by multiplying Eq. (B10) by γ for rational and by $(1 - \gamma)$ for boundedly rational agents and adding up, after some algebra we arrive at the final aggregate heterogeneous Phillips curve

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa \varphi_t^A, \quad (\text{B12})$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$.

Finally, noting that the aggregate marginal production cost is the aggregate output gap measure, the heterogeneous New-Keynesian Phillips curve amended for the banking sector may be viewed as

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t, \quad (\text{B13})$$

where $\bar{E}_t = \gamma E_t^{RE} + (1 - \gamma) E_t^{BRE}$.

.3 The heterogeneous banking sector curve

Taking the steady state log deviations of Eq. (10) and iterating forward we get for each group of agents

$$\tilde{\chi}_t^\tau = \left(\frac{1 - rr}{V} \right)^{-1} \left[-v E_t^\tau \sum_{j=0}^{\infty} \tilde{\chi}_{t+j}^\tau + \left(\frac{1 - rr}{V} \right) \tilde{\chi}_\infty^\tau - (x_\infty^\tau - x_t^\tau) \right], \quad (\text{B14})$$

where $\tilde{\chi}_\infty^\tau = \lim_{k \rightarrow \infty} E_t^\tau \tilde{\chi}_{t+k}^\tau$ and $x_\infty^\tau = \lim_{k \rightarrow \infty} E_t^\tau x_{t+k}^\tau$.

Given Eq. (B3) and (B14), we get

$$\begin{aligned} \tilde{\chi}_t^A &= \gamma \tilde{\chi}_t^{RE} + (1 - \gamma) \tilde{\chi}_t^{BRE} \\ &= \left(\frac{1 - rr}{V} \right)^{-1} \left[-v \left(\gamma E_t^{RE} \sum_{j=0}^{\infty} \tilde{\chi}_{t+j}^{RE} + (1 - \gamma) E_t^{BRE} \sum_{j=0}^{\infty} \tilde{\chi}_{t+j}^{BRE} \right) + x_t \right. \\ &\quad \left. + \left(\frac{1 - rr}{V} \right) (\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) - (\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE}) \right] \\ &= \bar{E}_t \chi_{t+1}^A - \frac{vV}{1 - rr} \chi_t^A - \left(\frac{1 - rr}{V} \right)^{-1} (\bar{E}_t x_{t+1} - x_t) \\ &\quad + [(\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) - \bar{E}_t (\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE})] \\ &\quad - \left(\frac{1 - rr}{V} \right)^{-1} [(\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE}) - \bar{E}_t (\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE})]. \end{aligned} \quad (\text{B15})$$

The last two lines disappear due to the axiom 7 which implies the same limiting expectations of both groups of agents

$$(\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) = \bar{E}_t (\gamma \tilde{\chi}_\infty^{RE} + (1 - \gamma) \tilde{\chi}_\infty^{BRE}) \quad (\text{B16})$$

$$(\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE}) = \bar{E}_t (\gamma x_\infty^{RE} + (1 - \gamma) x_\infty^{BRE}) \quad (\text{B17})$$

so that the final banking curve equation may be written as Eq. (14).

Appendix C

The condensed model can be viewed as

$$\begin{pmatrix} B & 0 \\ 0 & I_3 \end{pmatrix} \begin{pmatrix} y_{t+1} \\ y_t \end{pmatrix} = \begin{pmatrix} F & -C \\ I_3 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix}, \quad (\text{C1})$$

where $y = (x, \pi, \tilde{\chi})'$ and

$$B = \begin{pmatrix} \gamma - \rho_x & \gamma - \rho_\pi & \frac{\gamma(1-rr)}{V} \\ 0 & \beta\gamma & 0 \\ -\gamma & 0 & \frac{\gamma(1-rr)}{V} \end{pmatrix}, \quad (\text{C2})$$

$$F = \begin{pmatrix} 1 & 0 & \frac{(1-rr)}{V} + 1 \\ -\kappa & 1 & 0 \\ -1 & 0 & v + \frac{(1-rr)}{V} \end{pmatrix}, \quad (\text{C3})$$

$$C = \begin{pmatrix} (1-\gamma)\mu^2 & (1-\gamma)\mu^2 & \frac{(1-\gamma)\mu^2(1-rr)}{V} \\ 0 & \beta(1-\gamma)\mu^2 & 0 \\ -(1-\gamma)\mu^2 & 0 & \frac{(1-\gamma)\mu^2(1-rr)}{V} \end{pmatrix}. \quad (\text{C4})$$