# Balance-Sheet Shocks and Recapitalizations[*]

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PRELIMINARY

## Abstract

We develop a dynamic stochastic general equilibrium model with financial frictions on both financial intermediaries and goods-producing firms. In this context, due to high leverage of financial intermediaries, balance sheet disruptions in the financial sector are particularly detrimental for aggregate output. We show that the welfare gains from recapitalizing the financial sector in response to large but rare net worth losses are as large as those from eliminating business cycle fluctuations. We also find that these gains are increasing in the size of the aggregate shock, are larger when recapitalization comes from the household rather than the real sector, and may increase with a reduction in financial intermediaries idiosyncratic risk.

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# 1 Introduction

The recent crisis has been characterized by a large collapse in asset values and a severe deterioration of financial intermediaries' balance sheets. This has been associated with the freezing of credit markets and a dramatic contraction in output, which have put pressure on governments and central banks to forcefully step in the economy. Usual monetary and fiscal expansionary policies have been complemented with more unconventional interventions, including the public recapitalization of banks. Inspired by these events, we develop a dynamic stochastic general equilibrium model to quantitatively assess the impact of rare but large shocks to the net worth of financial intermediaries and explore the merit of recapitalization policies.

The model is composed of three sectors, representing households, financial intermediaries, and firms in the non-financial corporate sector that we will refer to as the real sector. The household sector is made of a continuum of risk-averse workers which offer their labor endowment to real firms and deposit their savings in financial intermediaries. The financial sector includes a continuum of financial intermediaries subject to idiosyncratic shocks that raise deposit from households and lend to real firms. Finally, firms in the real sector borrow from financial intermediaries, hire workers and earn a rate of return subject to idiosyncratic shocks. All three sectors face the risk of exogenous net worth (or wealth) shocks and both financial and real firms pay dividends to households at an exogenous rate.

The key feature of the model is presence of financial frictions not only on real firms, as already analyzed in the literature, but also on financial intermediaries. In the absence of financial frictions, the distribution of net worth across sectors is inconsequential for aggregate dynamics since capital freely flows to its most productive use through borrowing and lending. Financial frictions – that we model with a costly state verification problem *à la* Robert M. Townsend (1979) - prevent this efficient market-based reallocation of capital by generating spreads between the lending and borrowing rates. This implies that a given wealth loss in the economy has different economic implications depending on which sector it impacts.

Calibrating the model to U.S. data, we analyze the responses to a negative shock to the net worth of each of the three sectors. The model generates rich implications for the dynamics of both financial (leverage, spreads, and credit flows) and real variables (GDP, wages and profitability).

By considering financial frictions on financial intermediaries, our model reveals the particularly disruptive consequences of a shock to the financial sector. A given net worth loss involves indeed a much larger GDP contraction if it falls on the balance sheets of financial intermediaries rather than on households or real firms. This is essentially due to the high leverage of financial intermediaries that severely impairs their ability to withstand large shocks.

The fact that output losses depend on which sector is directly affected by the net worth shock provides the rationale for recapitalization policies that aim to shift losses to the sectors that can better absorb them. These transfers can in principle be implemented in many different ways, for example by easing monetary policies to reduce borrowing costs, by altering fiscal policies, by purchasing assets to support their values, or more directly through equity injections financed with tax-payer money. In this paper, we do not narrowly focus on any particular form of implementation, but consider more generally the welfare gains from exogenously redistributing wealth in response to shocks. Comparing the merit of different implementation procedures becomes an interesting topic of research only after establishing that the potential gains from recapitalization are reasonably large as we find in this paper.

We assess the benefits of recapitalization policies in response to infrequent but large shocks that when affecting the financial sector trigger a crisis similar to the recent one. We find that the gains from recapitalizing the financial sector with funds from the household sector are equivalent to a permanent increase in consumption by 0.19%. To put this number in context, these are the gains that a similar model without financial friction would predict from the elimination of business cycle fluctuations. Therefore, designing and implementing recapitalization policies in case of rare but large shocks to the financial sector could be equally important as the design of countercyclical policies aiming at smoothing business cycle fluctuations.

By including financial frictions on both the financial and the real sector, the model also allows us to consider the merit of recapitalizing the financial sector with funds from the real sector. At a first glance, this may seem inappropriate since the ultimate purpose of policy intervention is to support the level of investment in the real sector. However, since financial intermediaries are more leveraged than real firms we find that financing the recapitalization with resources from the real sector is still welfare enhancing.

We also use the model to study how changes in idiosyncratic risk affect the welfare gains from

recapitalization policies. [1]. This is an interesting issue in the context of the paper since under costly state verification the level of idiosyncratic volatility controls the importance of financial frictions and therefore also the merit for recapitalization. The model would indeed boil down to a standard frictionless world if idiosyncratic volatility were set to zero. A reduction in volatility should thus be beneficial for the economy since it reduces financial frictions. However, the model reveals that lower uncertainty leads to a gradual increase in the leverage of financial intermediaries, which can actually imply larger welfare gains from recapitalization policies.

The modeling of financial frictions adopted in the paper is borrowed from Charles T. Carlstrom and Timothy S. Fuerst (1997) and Ben Bernanke, Mark Gertler and Simon Gilchrist (1999) who incorporated Townsend (1979)'s costly state verification framework in a dynamic macro model. The purpose of this early literature on financial frictions, also including Nobuhiro Kiyotaki and John Moore (1997), was to study the role of agency problems on good-producing firms in amplifying the responses to productivity shocks. Differently from our work, these papers did not feature a financial sector. The recent crisis has demonstrated that a credit tightening may not only result from a deterioration in the balance sheets of ultimate borrowers, but also from problems affecting the financial sector itself. Recent papers have thus incorporated a role for the lender either implicitly by shocking the tightness of credit constraints (Gilchrist, Sim and Zakrajšek (2010), Vincenzo Quadrini and Urban Jermann (2011), and Vasco Cúrdia and Michael Woodford (2009)), or by imposing financial frictions only on financial intermediaries (Mark Gertler and Nobuhiro Kiyotaki (2009)).

The main innovation of our paper is to consider financial friction on both financial intermediaries and good-producing firms. Apart from improving the realism of the model, our setup allows to compare the responses to shocks in the financial and real sector and assess the impact of wealth transfers between these two sectors. A double layer of financial frictions was first introduced by Bengt Holmstrom and Jean Tirole (1997) but in a static setting and with an exogenous lending rate by households. The models by Nan-Kuang Chen (2001) and Césaire A. Meh and Kevin Moran (2010) also feature financial frictions on both banks and firms, but using an agency problem based on limited enforcement rather than on costly state verification. The advantage of the latter

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[1]Recent papers that give a prominent role to uncertainty shocks include Christina Arellano, Yan Bai and Patrick Kehoe (2011), Nicholas Bloom, Max Floetotto and Nir Jaimovich (2011), Lawrence Christiano, Roberto Motto and Massimo Rostagno (2009), and Simon Gilchrist, Jae W. Sim and Egon Zakrajšek (2010)

approach adopted in our paper is to allow for the analysis of time-varying borrowing spreads and for the role of recapitalization between the real and financial sector which has instead no impact on output in the just cited papers. Finally, our paper shares with Mark Gertler and Peter Karadi (2010) and Marco Del Negro, Gauti Eggertsson, Nubuhiro Kiyotaki and Andrea Ferrero (2011) the purpose of analyzing unconventional policies in response to financial crises, but our focus in on recapitalizations instead of credit policies and liquidity support.

This paper is organized as follows. The next section presents the model. Section 3 discusses calibration. Section 4 presents quantitative experiments showing how the model responds to networth shocks. Section 5 discusses recapitalization policies and their welfare effects. Section 6 assesses how welfare gains from recapitalization policies are affected by idiosyncratic risk. Section 7 concludes.

## 2  The Model

We proceed to formally describe the optimization problems faced by households and firms in the financial and real sector.

### 2.1  Real Sector Firms

The real sector ($r$ superscript) is composed of a continuum of competitive firms with mass one. Each firm is managed a risk-neutral entrepreneur that maximizes expected profits. Real sector entrepreneurs invest in productive capital $k_t$ using their own net worth $e_t^r$ and taking out loans $l_t^r$ from financial entrepreneurs. They have access to a Cobb-Douglas production function combining capital and labor and returning revenues net of depreciation $\delta$ equal to

$$\left((1-\delta)k_t + k_t^\alpha n_t^{1-\alpha} - n_t W_t\right) \omega_{t+1}^r \tag{1}$$

where $\omega_{t+1}^r$ is a mean one idiosyncratic shock with cumulative distributive function $\Omega^r$, $W_t$ is the competitive market wage, and $n_t$ is the amount of labor hired from the household sector. [2] The

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[2]Notice that we assume idiosyncratic risk affects profits and not revenues as it would under a traditional productivity shock interpretation. This is done for two reasons: first, it allows us to preserve linearity of the problem, facilitating aggregation as it will be seen momentarily, and second, it means the wage is not affected by the idiosyncratic risk since workers collect it before the shock is realized.

optimal demand for labor is given by

$$n_t^* = k_t \left( \frac{1-\alpha}{W_t} \right)^{1/\alpha} \qquad (2)$$

which, if substituted out, allows us to rewrite expected net revenues in equation (1) as

$$k_t \overbrace{\left( 1 + \alpha \left( \frac{1-\alpha}{W_t} \right)^{(1-\alpha)/\alpha} \right)}^{A_t} \omega_{t+1}^r \qquad (3)$$

where $A_t$ is the expected return to capital common to all entrepreneurs because it does not depend on any firm specific variable.

As in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999), we introduce financial frictions through an agency problem caused by costly state verification *à la* Townsend (1979). In particular, external investors can observe the realization of the idiosyncratic shock $\omega_{t+1}^r$ only after paying a monitoring cost equal to a fraction $\mu^r$ of their loan. As shown by Douglas Gale and Martin Hellwig (1985) and Stephen Williamson (1987), the optimal contract in this setting involves risky debt, so financial entrepreneurs will monitor real sector entrepreneurs only if they declare to be insolvent. This happens any time liabilities exceed assets:

$$l_t^r (1 + r_t^r) \geq k_t A_t \omega_{t+1}^r \qquad (4)$$

or in other words any time the idiosyncratic shock $\omega_{t+1}^r$ is lower than

$$\underline{\omega}^r = \frac{l_t^r (1 + r_t^r)}{k_t A_t} \qquad (5)$$

where $r_t^r$ is the firm-specific interest rate which ensures an expected return to financial intermediaries equal to the average market return $r_t$

$$(1 + r_t) = (1 - \mu^r) \int_0^{\underline{\omega}^r} \frac{k_t}{l_t^r} A_t \omega^r \mathrm{d}\Omega^r(\omega^r) + (1 - \Omega^r(\underline{\omega}^r))(1 + r_t^r) \qquad (6)$$

where the first and second terms on the right-hand side capture respectively the return to financial

intermediaries if the real sector firm goes bankrupt or is solvent.[3]

The entrepreneur's objective is to maximize the expected profit

$$e_t^r \overbrace{\int_{\underline{\omega}^r}^{\infty} \left( \frac{k_t}{e_t^r} A_t \omega^r - \frac{l_t^r}{e_t^r}(1 + r_t^r) \right) d\Omega^r(\omega^r)}^{\Pi_t^r} \tag{7}$$

subject to definition (5) and the constraint (6). As shown in Bernanke, Gertler and Gilchrist (1999), this optimization problem boils down to choose the optimal leverage $\lambda_t^r = k_t^r / e_t^r$, which maximizes the expected profit rate $\Pi_t^r$ given the ratio $A_t/(1 + r_t)$ .[4] Note that expected profits are concave in leverage due to the increasing risk of default and the borrowing cost $r_t^r$. Quite importantly for the tractability of the general equilibrium solution, optimal leverage is not a function of internal equity $e_t^r$ because of the assumption of linear investment and monitoring technology. This implies that all firms will have equal leverage, so that we have to include in the model's state space only the aggregate equity in the real sector $E_t^r$, instead of the entire distribution of equity across firms.

After the idiosyncratic shock $\omega_{t+1}^r$ is realized, similarly to Bernanke, Gertler and Gilchrist (1999), a share $\mathfrak{D}^r$ of entrepreneurs die and are replaced by newly entrants. This exit assumption is needed to rule out complete self-financing of the real sector which would make financial frictions irrelevant. Exiting entrepreneurs distribute their net worth in dividends to the household.[5]

Summing up and focusing on the aggregate dynamics of the real sector, entrepreneurs observe at the beginning of the period their net worth $e_t^r$ and based on the average returns on borrowing $r_t$ and investing $A_t$ choose the optimal leverage $\lambda_t^r$. The fact that leverage is common across firms implies that the aggregate borrowing and investment in the real sector (denoted by upper case variables) are simply given by:

$$L_t^r = E_t^r(\lambda_t^r - 1) \tag{8}$$

$$K_t = E_t^r \lambda_t^r \tag{9}$$

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[3]The average market return $r_t$ will be determined in equilibrium and is the return earn by the financial intermediary on a fully diversified lending portfolio.

[4]This follows simply from the fact that Equation (6) can be expressed it in terms of spreads (or ratios) relative to $A_t$. Similarly, Equation (7) can also be normalized in the same way, resulting in an equivalent problem in which the key argument is the ratio $A_t/(1 + r_t)$.

[5]In Bernanke, Gertler and Gilchrist (1999) entrepreneurs consumed their net worth at the moment of death, we instead assume that money flows back to the household so that there is a unique value function in the model that is used to assess the welfare gains from recapitalization.

Dividends for the household from dying entrepreneurs are equal to

$$\chi_t^r = \mathfrak{D}^r E_t^r \Pi_t^r \tag{10}$$

where we also use the law of large numbers to write ex-post returns $\Pi_t^r$ as the average across idiosyncratic risk realizations. Finally, aggregate net worth in the real sector evolves according to

$$E_{t+1}^r = (1 - \mathfrak{D}^r) E_t^r \Pi_t^r - \Upsilon_{t+1}^r \tag{11}$$

where $\Upsilon_{t+1}^r$ denotes aggregate net worth shocks. These can be interpreted as a reduced-form modeling of asset value shocks that eventually have an impact on net worth. We choose not to model asset shocks more explicitly because their realization would generate immediate losses in other sectors since a larger fraction of firms would go bankrupt on impact. Our modeling choice prevents this immediate spreading of the shocks by assuming that they hit at the end of the period after debts have been repaid. Using net worth shocks is also instrumental to more naturally capture the effects of recapitalization policies which aim to redistribute capital between sectors.

## 2.2 Financial sector

The financial sector ($f$ superscript) is modeled in the same way as the real sector, with the exception that external financing is raised from households (rather than from financial intermediaries) and that resources are lent to real sector firms (rather than invested in final production). While in business, financial entrepreneurs use their own net worth $e_t^f$ and the deposits raised from workers $d_t^f$ to lend funds $l_t^f = e_t^f + d_t^f$ to firms in the real sector. Revenues from lending are given by:

$$l_t^f (1 + r_t) \omega_{t+1}^f \tag{12}$$

where $\omega_{t+1}^f$ is a mean-one idiosyncratic shock with cumulative distributive function $\Omega^f$, and $r_t$ if the average market return on loans to the real sector after averaging across individual loans. We interpret these shocks as some form of managerial ability in collecting loans. Note that $r_t$ is a safe return since financial entrepreneurs are assumed to fully diversify their lending and insure against idiosyncratic shocks in the real sector. Furthermore, $r_t$ is also independent from aggregate shocks

to the net worth of the real sector since they are realized after loans have been repaid.

The same agency problem described for real sector entrepreneurs applies to the entrepreneurs in the financial sector. Financial firms become insolvent if the idiosyncratic shock $\omega_{t+1}^f$ is lower than

$$\underline{\omega}^f = \frac{d_t^f(1 + i_t^f)}{l_t^f(1 + r_t)} \tag{13}$$

where $i_t^f$ is the firm-specific interest rate which needs to ensure an expected return to the lender (i.e. depositors) equal to the risk-free return $i_t$ that households gain by fully diversifying their deposits. This participation constraint requires

$$(1 + i_t) = (1 - \mu^f) \int_0^{\underline{\omega}^f} \frac{l_t^f}{d_t^f}(1 + r_t)\omega^f \mathrm{d}\Omega^f(\omega^f) + (1 - \Omega^f(\underline{\omega}^f))(1 + i_t^f) \tag{14}$$

where the first and second terms on the right-hand side capture respectively the return to depositors if the financial intermediary goes bankrupt or is solvent.

The entrepreneur's objective is to choose leverage $\lambda_t^f = l_t^f/e_t^f$ as to maximize the expected profit

$$e_t^f \overbrace{\int_{\underline{\omega}^f}^{\infty} \left( \frac{l_t^f}{e_t^f}(1 + r_t)\omega^f - \frac{d_t^f}{e_t^f}(1 + i_t^f) \right) \mathrm{d}\Omega^f(\omega^f)}^{\Pi_t^f} \tag{15}$$

subject to definition (13) and the constraint (14). Optimal leverage is only a function of the ratio $(1 + r_t)/(1 + i_t)$ and is thus common to all entrepreneurs, so that we only have to keep track of the aggregate net worth $E_t^f$ in the financial sector to solve for the model dynamics. After the realization of the idiosyncratic shock $\omega_{t+1}^f$, solvent entrepreneurs repay their debt to depositors and then a share $\mathfrak{D}^f$ exits the market and distribute their net worth in dividends to the household.

The aggregate dynamics of financial sector are analogous to those of the real sector. Aggregate borrowing and lending are given by:

$$D_t^f = E_t^f(\lambda_t^f - 1) \tag{16}$$

$$L_t^f = E_t^f \lambda_t^f \tag{17}$$

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After the realization of the idiosyncratic shocks and the repayment of debt contracted with the household, aggregate dividends distributed to the household from dying entrepreneurs are given by

$$\chi_t^f \;\; = \;\; \mathfrak{D}^f E_t^f \Pi_t^f \tag{18}$$

Finally, next period net worth is given by

$$E_{t+1}^f \;\; = \;\; (1 - \mathfrak{D}^f) E_t^f \Pi_t^f - \Upsilon_{t+1}^f \tag{19}$$

where $\Upsilon_{t+1}^f$ is an aggregate net worth shock affecting the financial sector.

## 2.3 Equilibrium in the financial and real sectors

Before closing the model with the household sector, it is useful to describe the equilibrium in credit markets for a given level of the risk-free rate. From Section 2.1 we obtained an optimal solution for leverage, common to all entrepreneurs in the real sector, which was a function of $A_t/(1 + r_t)$. In equilibrium, the marginal product of capital is determined by the total amount of capital that is used in production by all entrepreneurs. The assumption of competitive markets and constant returns to scale production function at the individual level ensures that the aggregate production function is just $K_t^\alpha N^{1-\alpha}$, where $N$ denotes the aggregate and fixed labor supply, implying that $A_t = (\daleth + \alpha K_t^{\alpha-1})$. The equilibrium return on capital, $A_t$, for a given $r_t$ and $E_t^r$ is given by the solution to $A_t = \daleth + \alpha E_t^r (\lambda^r (A_t/(1 + r_t)))^{\alpha-1}$.

With this solution on hand, the amount of loans demanded by the real sector is a function of the amount of equity in the real sector and the average market return on loans demanded by the financial sector, $L^r(E_t^r, r_t) = E_t^r(\lambda^r(A(E_t^r, r_t)/(1 + r_t)) - 1)$, where $A(E_t^r, r_t)$ is the equilibrium return on capital obtained above.

Similarly, the financial sector optimal leverage is a function of $r_t/(1+i_t)$, which yields an optimal supply of loanable funds that is a function of $r_t$, $i_t$, and $E^f$, $L^f(E_t^f, r_t, i_t) = (\lambda^f(r_t/(1 + i_t))E_t^f$.

The equilibrium in the credit markets, given the risk-free rate $i_t$, satisfies $L^r(E_t^r, r_t) = L^f(E_t^f, r_t, i_t)$, which yields a solution for the equilibrium average return on loans as a function of $E_t^r$, $E_t^f$, and $i_t$. Notice that with this function on hand, we can express all endogenous variables up to now, as a

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function of $E_t^r$, $E_t^f$, and $i_t$.

## 2.4 Household sector

The representative household ($h$ superscript) maximizes the present discounted value from the utility derived from consumption $c_t^h$ with a CRRA utility function with risk aversion coefficient $\gamma$. The household supplies labor inelastically to the real sector, deposits savings $d_t$ into financial intermediaries, and receives dividends from both, exiting real $\chi_t^r$ and financial $\chi_t^f$ entrepreneurs. Net worth in the sector then evolves according to:

$$E_{t+1}^h = \overbrace{(E_t^h - C_t^h)}^{D_t^h}(1 + i_t) + \chi_t^r + \chi_t^f + W_t - \Upsilon_{t+1}^h \tag{20}$$

where $E_t^h$ is aggregate net worth, $D_t^h$ is the supply of deposits, $W_t$ is the market competitive wage, and $\Upsilon_{t+1}^h$ is a mean-one aggregate shock affecting household wealth. The intra-period rate of return earned on deposits $i_t$ is safe since households diversify their deposits to insure against the idiosyncratic risk in the financial sector.[6] This return is also protected from aggregate risk since we assume that aggregate shocks to the net worth of financial intermediaries are realized after deposits have been honored.

The model is closed by solving for the $i_t$ that ensures equality between supply and demand for deposits. The state space of the model boils down to keeping track only of aggregate net worth positions in each sector. This allows us to write the household's value function as follows

$$V^h(E_t^h, E_t^r, E_t^f) = \max_{\{c_t\}} \left\{ u(C_t^h) + \beta \mathbb{E}_t \left[ V^h(e_{t+1}^h, E_{t+1}^r, E_{t+1}^f) \right] \right\} \tag{21}$$

$$\text{s.t.}$$

$$E_{t+1}^h = \overbrace{((E_t^h - C_t^h)}^{D_t^h}(1 + i_t) + \chi_t^r + \chi_t^f + W_t) - \Upsilon_{t+1}^h \tag{22}$$

$$D_t^h = E_t^f(\lambda^f - 1) \tag{23}$$

$$E_{t+1}^f = (1 - \mathfrak{D}^f)E_t^f \Pi_t^f - \Upsilon_{t+1}^f \tag{24}$$

$$E_{t+1}^r = (1 - \mathfrak{D}^r)E_t^r \Pi_t^r - \Upsilon_{t+1}^r \tag{25}$$

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[6]This can be thought as a mutual fund investment.

where $\beta$ is the intertemporal discount factor. Note that the household knows the dynamic evolution of aggregate variables but takes prices as given. Figure 1 summarizes the sequence of events for the whole model. We solve the model with global solution methods building on Christopher D. Carroll (2006)'s method of endogenous gridpoints.
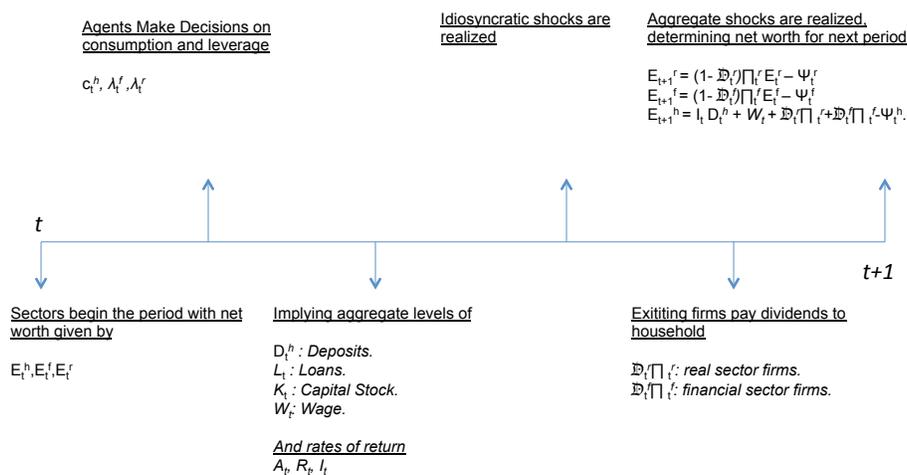
Agents Make Decisions on
consumption and leverage

$c_t^h, \lambda_t^f, \lambda_t^r$

Idiosyncratic shocks are
realized

Aggregate shocks are realized,
determining net worth for next period

$E_{t+1}^r = (1- \mathcal{D}_t^r)\prod_t^r E_t^r - \Psi_t^r$
$E_{t+1}^f = (1- \mathcal{D}_t^f)\prod_t^f E_t^f - \Psi_t^f$
$E_{t+1}^h = I_t D_t^h + W_t + \mathcal{D}_t\prod_t^r + \mathcal{D}_t\prod_t^f - \Psi_t^h.$

$t$

$t+1$

Sectors begin the period with net
worth given by

$E_t^h, E_t^f, E_t^r$

Implying aggregate levels of

$D_t^h$ : Deposits.
$L_t$ : Loans.
$K_t$ : Capital Stock.
$W_t$: Wage.

And rates of return
$A_t, R_t, I_t$

Exititing firms pay dividends to
household

$\mathcal{D}_t\prod_t^r$: real sector firms.
$\mathcal{D}_t\prod_t^f$: financial sector firms.

Figure 1: Sequence of Events

# 3    Calibration

We calibrate the model at quarterly frequency setting the depreciation rate, $\delta$ and share of capital, $\alpha$, in the production function to standard levels of 2.5 % and 36 % respectively. The discount factor for the household is set equal to 0.994, which generates a steady state level for $i_t$ of roughly 2 percent in annual terms. We calibrate the remaining parameters to match statistics of leverage, borrowing spreads, and bankruptcies in the real and financial sectors in the deterministic steady state. This is computed as the point in the state space where the model converges to, if no net worth shock hits the economy. This is also the initial point from which all our impulse response functions originate.

The data show that spreads and bankruptcy rates are roughly similar across financial and non-financial firms, but leverage is much higher in the financial sector. We could generate these differences in leverage by using a higher death rate in the financial sector which would exogenously shrink net worth. However, this would artificially inflate the rate of return on financial firms and thus possibly the welfare gains from its recapitalization. Therefore, we constrain ourselves to use

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the same death rate in both sectors, which leaves us with five parameters – the death rate and the standard deviation of idiosyncratic shocks and monitoring costs in each sector – to match six moments. This implies that to achieve higher leverage in the financial sector we need to reduce the extent of financial frictions on financial intermediaries, which plays against finding large gains from its recapitalization. In this context, we chose a lognormal distribution for idiosyncratic risk in both sectors with mean 1 and standard deviations equal to 0.22 for the real sector and 0.06 for financial sector firms. The exogenous death rate is set at 4.7 % in both sectors. Bankruptcy costs are set $\mu^r$=0.36 and $\mu^f = 0.12$, which are within the range of values considered as empirically plausible (Carlstrom and Fuerst (1997)). The resulting steady state outcomes and data moments are shown below

Table 1: Steady State Values and Targets

| Variable | Model | Data |
|---|:---:|---:|
| Leverage Financial Sector | 7.5 | 9[1] |
| Leverage Real Sector | 2.4 | 2.2[2] |
| Risk Spread Financial Sector | 40 bps | 44 bps[3] |
| Risk Spread Real Sector | 127 bps | 110 bps [3] |
| Bankruptcy Rate Financial Sector | 3.0 % | 2.1 % [4] |
| Bankruptcy Rate Real Sector | 3.1 % | 3.0 % [5] |

[1] Average capital-to-assets ratio for U.S. banks over the period 1995-2008
[2] United States Flow of Funds, Nonfarm Nonfinancial Corporate Business average for 2004-2008
[3] Average spreads on one-year BBB bonds over the period 2002-2008
[4] Market share of failed U.S. banks over the period 1985-2010
[5] Historic average of business failures Bernanke, Gertler and Gilchrist (1999)

Regarding the calibration of the aggregate shock, we assume that each sector net worth shock, $\Upsilon^f, \Upsilon^r, \Upsilon^h$ involve the same amount of loss equal to a proportion $\Phi$ of the total amount of net worth in the economy $E^h + E^f + E^r$. $\Phi$ is set equal to 4 % and is chosen so that a net worth loss that if concentrated in the financial sector involves a fall in GDP of about 4 % as experienced by the US in 2008Q4. These shocks hit with equal probability each of the three sectors every 25 years. This implies that each sector is expected to receive a shock every 75 years, roughly equivalent to the time gap between the Great Depression and the Great Recession.

Figure 2 shows the relationship between the key financial variables in the model, that is optimal leverage, as well as the interest rate spreads in each sector as a function of the net worth in the

real and financial sector, setting $i_t$ at its deterministic steady state level. Optimal leverage for real sector firms declines as net worth in the sector increases, since more net worth implies higher investment, higher capital, and thus lower returns which reduce the incentives to lever up. However, it increases in net worth in the financial sector because the larger the supply of credit the lower the borrowing costs. Financial firms' leverage also decreases with net worth in the sector since larger net worth increases the supply of credit and reduces returns, similar to what happens with real sector firms. The figures depicting the behavior of risk spreads show that they move in the same direction as leverage, since higher leverage implies higher default risk. The last two figures show the evolution of capital and wages in the economy.
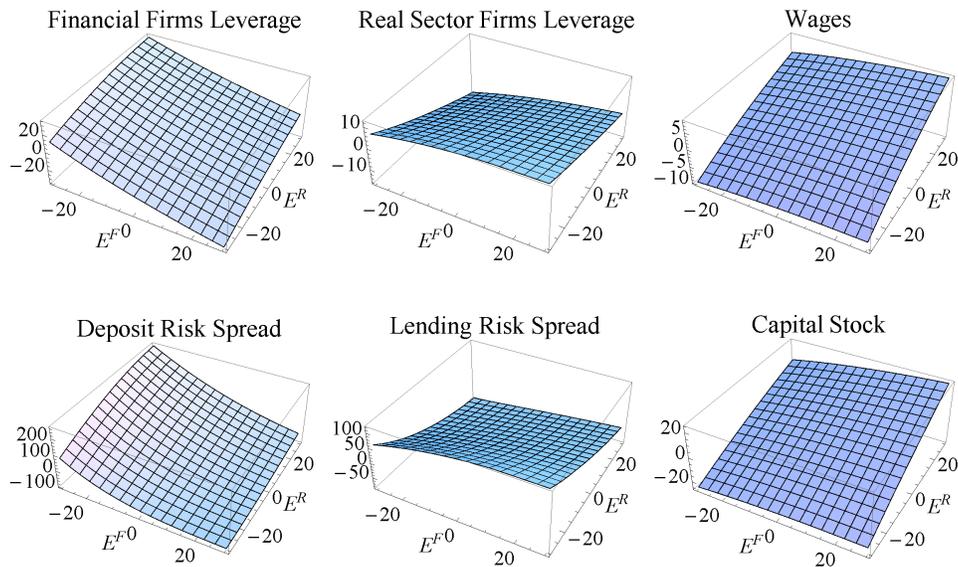


Figure 2: Financial Variables

# 4    Responses to net worth shocks

In this section we analyze the model responses to shocks reducing the net worth in the household, financial and real sector. We begin by analyzing in section 4.1 the role of the financial sector in amplifying shocks originating in the real sector. We find no amplification effects, when shocks generate in the real sector, since the intermediation margin remains fairly stable. But the financial sector can become itself a strong generator of shocks. In section 4.2 we show that the economic

disruption caused by a given aggregate destruction of net worth is much more severe if the losses are concentrated on the balance-sheets of the financial sector.

## 4.1  Financial sector as amplifier

We examine how the addition of an intermediary changes the dynamic adjustment in the economy relative to a benchmark in which financial frictions are present only on real sector firms. This benchmark is comparable to Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (1997), models in which the financial's intermediary's financial structure does not matter because the lender is perfectly diversified and it faces no risk of bankruptcy. In conducting this experiment, we shut down the possibility of aggregate shocks to the household and financial entrepreneur's net worth and solve the problem under the assumption that only the real sector is subject to such shocks. We are interested in shocks that hit the real sector directly and compare the dynamic response of the economy with and without financial frictions in the financial sector. In the next section we will examine how the economy responds to shocks hitting the financial sector directly. Figure 3 shows the resulting dynamics as a percentage deviation from the deterministic steady state of the model for a subset of variables.
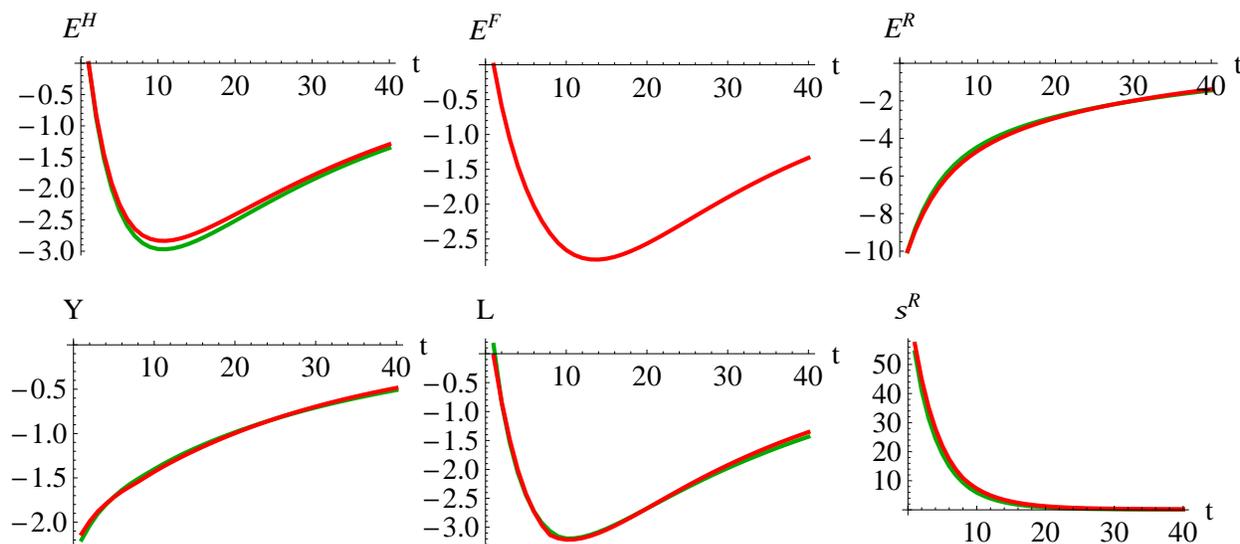


Figure 3: Shocks to real sector with (red) and without (green) financial sector (% deviations from Steady State)

Financial frictions on the financial sector do not amplify shocks to the real sector.[7] In absence

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[7]It is important to note however that our model does not incorporate endogenous asset prices, which in the financial

of financial frictions on the lender, the borrowing rate for real sector firms includes a wedge over the rate of return on deposits,$i_t$, driven only by its risk of default and bankruptcy costs. When financial frictions on the intermediary are introduced, its risk of bankruptcy increases this wedge because now the lender's risk of default and bankruptcy costs are also priced in. By increasing this wedge between these two rates, financial frictions on the intermediaries effectively raise the cost of borrowing for the real sector and consequently lowers its leverage and the capital stock in the economy. Therefore, there is a change in the steady state, but the response to shocks does not change significantly, as seen in the graph, at least for the baseline calibration. What this exercise implies is that for shocks originating outside the financial sector, there is no much gain in adding a third sector. As we shall see in the next section, it is only when we want to study shocks originating in the financial sector where significant effects.

## 4.2 Financial sector as shock generator

In the previous section we have exclusively considered the role of the financial sector in amplifying shocks affecting the real sector. But the financial sector can itself be a source of shocks. More specifically, we are interesting in learning about how the effects of a given reduction in aggregate net worth depends on the sector that is directly absorbing the losses. As discussed in the presentation of the model, we consider an exogenous reduction in total net worth of 4 percent and compare the impulse response functions of the model when this loss is allocated to either the household, the financial or the real sector. Figure 4 shows the resulting dynamics as a percentage deviation from the deterministic steady state of the model. The top row shows the evolution of net worth in each sector, the second row traces the responses in consumption, loans and output, and the third and fourth rows focus respectively on the financial and real sector by considering the risk-free borrowing rates, the borrowing spreads and the leverages. Each period of the simulation represents one quarter.

Consider first the responses when the net worth loss is concentrated on the real sector (green lines). These are similar to those considered in the previous section, but not identical since now the household's policy functions are solved taking into account the possibility of net worth shocks

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accelerator literature is a key mechanism to generate quantitatively important amplification effects (Bernanke, Gertler and Gilchrist (1999) and others).

also on the household and financial sector. The fall in the capitalization of the real sector reduces invested capital and output, and drives up the marginal return on investment. The real sector's leverage therefore increases leading to higher borrowing spreads. From the perspective of the household, the lower capitalization of the real sector implies lower future wages. In an effort to smooth consumption over time, the household immediately reduces consumption which keeps falling over time since the household's net worth shrinks due to lower wages and rates of return. The process of recovery is slow and characterized by a gradual de-leveraging of the real sector.

Let us now turn to the case in which the net worth shock is concentrated on the financial sector (red line). Note that the capitalization of the financial sector suffers a percentage loss much larger than the one experienced by the real sector when receiving the shock. This is because the financial sector's leverage is very high and its net worth quite low, so that if an equal-size shock hits the financial sector it produces a much larger percentage reduction in capitalization. The capital loss leads to much higher leverage and borrowing spreads for the financial sector, whose ability to intermediate funds is severely impaired. The demand for deposit and supply for loans contract considerably leading to a strong reduction in the deposit risk-free rate and increase in the lending rate. Lower lending coupled with higher bankruptcies in the financial sector leads to a large contraction in GDP similar in magnitude to the recent great recession.
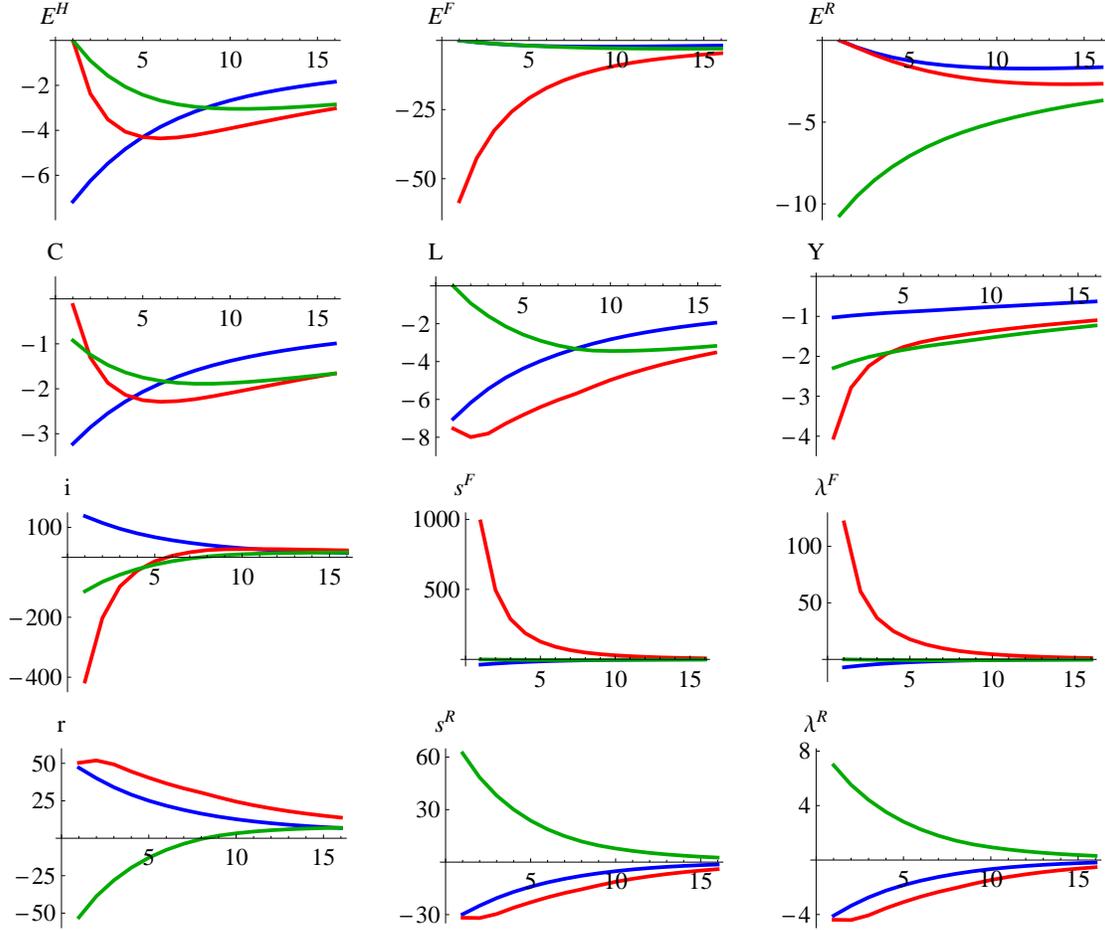
Figure 4: Impulse response functions from net worth shocks to the household (blue), financial (red), and real (green) sectors (% deviations from the steady state)

Finally, we consider the model responses if the net worth loss falls on the household (blue lines). Lower household's wealth implies lower deposits, lower lending and ultimately lower output. The reduction in output is however very moderate compared to when the shocks are concentrated on the financial and real sectors. This largely depends on the behavior of the rate earned on deposits. Shocks on the financial and real sectors reduce the deposit rate and thus discourage household's saving which is needed to speed up the recovery. Conversely, when the shock is concentrated on the household the deposit rate increases and the household responds by cutting consumption and supporting new investment.

Summing up, the model responses reveal that output losses are much more pronounced if the net worth shock is concentrated on the highly leveraged financial sector. This observation provides

the motivation for policies targeted to recapitalize the financial sector after a negative shock.

# 5 Recapitalization policies

How large are the potential welfare gains from recapitalizing the financial sector after a negative net worth shock? Can recapitalization funds be taken from the real sector or only from the household sector? How do gains vary with the size of the shock? We address these questions by comparing the household's welfare under different recapitalization policies. These recapitalizations may take different forms, including government equity injections, debt-to-equity conversion, asset swaps at above market rates, or simply overly lax monetary policy for a prolonged period of time. All of these interventions can ultimately be seen as wealth transfers across sectors, generally from savers to borrowers. In this section, we compare welfare when we implement these wealth transfers in response to shocks, without taking a stand on the particular form of implementation.

To assess welfare in the presence of recapitalization policies, we solve for the household's value function under the expectation that the financial sector will be fully recapitalized in case of a net worth shock. Recapitalization funds may come from the household sector, from the real, or from both. We express welfare gains as the permanent percentage increase that the household would require to voluntary remain in a world without recapitalization. This is computed by solving for the $\nu$ such that:

$$(1+\nu)^{1-\gamma} v^h(E^h, E^f, E^r) = \tilde{v}^h(\tilde{E}^h, \tilde{E}^f, \tilde{E}^r) \tag{26}$$

where tilde denotes variables in the presence of recapitalization. Regarding at which point in the state space to compare welfare, we will consider two cases. First, we compute the welfare gains at the determinist steady state without recapitalization policies, defined at the point in the state space the model converges to if not disturbed by net worth shocks. These can be thought as the gains from *introducing* recapitalization policies in an economy that has not had them so far. Second, we assess the welfare gains using for each value function its own deterministic steady state. So we will compare the welfare at the deterministic steady state for the model without recapitalization with the welfare at the steady state in an economy with recapitalization policies. This captures

the gains from *having* recapitalization policies, by comparing the steady state welfare in economies with and without recapitalization.

The welfare gains are plotted in Figure 5 as a function of the recapitalization share financed by the household sector. The red and blue lines refer respectively to the comparison at the non-recapitalization steady state and at each value function's steady state. Considering first the gains at the non-recapitalization steady state, we observe that if the financial sector is entirely recapitalized with funds from the household sector, the welfare gains are equivalent to a permanent increase in consumption of 0.16%. To put this number into context, these gains are similar to those derived by Robert E., Jr. Lucas (1987) from the elimination of the US business cycle. We do not want to enter the debate on whether these gains are small or large, but we simply want to highlight that recapitalization polices in response to large financial shocks are equally deserving of consideration as policies to reduce business cycle fluctuations. The model implies indeed that the potential gains from recapitalizing the financial sector in case of large but rare net worth shocks (occurring in our calibration only every 75 years) could lead to gains as large as those from completely removing business cycle fluctuations.
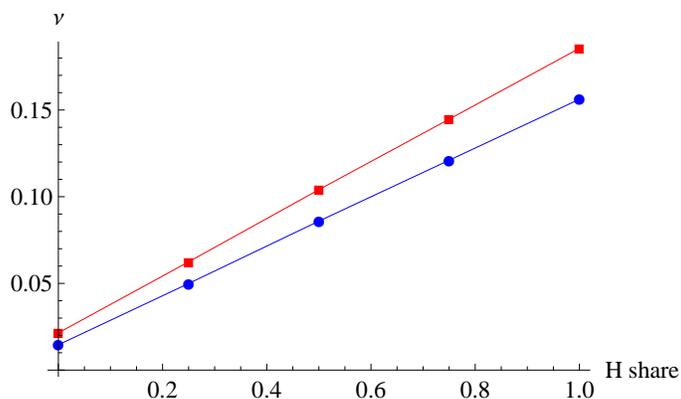


Figure 5: Welfare gains from introducing (blue) and having (red) recapitalization policies as a share of the recapitalization financed by the household sector

Figure 5 also shows that the gains from recapitalization are decreasing if the household provides a lower share of the recapitalization funds which are thus taken from the real sector. Interestingly the gains from recapitalizing the financial sector remain positive even if the recapitalization is entirely financed by subtracting resources to the real sector. This may seem counterintuitive since the purpose of the recapitalization is ultimately to support investment by the real sector. The

difference in leverage between the two sectors explains the result. The much higher leverage in the financial sector, particularly after a negative net worth shock, implies that funds moved from the real to the financial sector are able to support much larger lending and thus increase final investment. Furthermore, the recapitalization of the financial sector reduces bankruptcies which allow for a faster re-accumulation of aggregate capital.

Let us now turn to compare welfare at the steady states of economies with and without recapitalization policies (red line, Figure 5). Welfare gains are now even somewhat larger, essentially because the presence of recapitalization policies moves the economy to a higher steady state level of net worth in all sectors. At a first glance, this may seem surprising since recapitalization policies should reduce the household's precautionary savings by removing the risk of wage reduction during financial crisis. This is indeed the case, but recapitalizations also eliminate the big decrease in the risk-free deposit rate associated with net worth shocks on financial intermediaries.[8] As a consequence, the expected higher return on savings stimulates capital accumulation and leads the economy towards a higher welfare steady state. This is a second channel through which the adoption of recapitalization policies can improve welfare.

Finally, we consider how the welfare gains vary with the size of the net worth shock. Figure 6 traces the welfare gains from fully recapitalizing the financial sector with funds taken from the household sector. Gains are presented again both by comparing welfare at the non-recapitalization steady state (blue) or at each value function's steady state (red). We observe that the gains are exponentially increasing in the size of the shock. This result is relevant to think about the merit of recapitalization policies if there are costs associated with transferring resources across sectors. We have so far neglected that implementing recapitalization policies can be costly, since they could create distortions and and require a more active government role. In so far as these costs involve a fixed component, recapitalization policies should thus be confined to respond only to large shocks. The exact quantification of this threshold is beyond the purpose of our analysis and would be contingent on how recapitalizations are implemented.

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[8]Note that the knowledge that shocks affecting the financial sector will be passed to the household through recapitalization policies reduces the expected rate of return on savings. However, this negative impact on expected returns is more than compensated from avoiding the negative real interest rates that would be associated with a financial crisis.
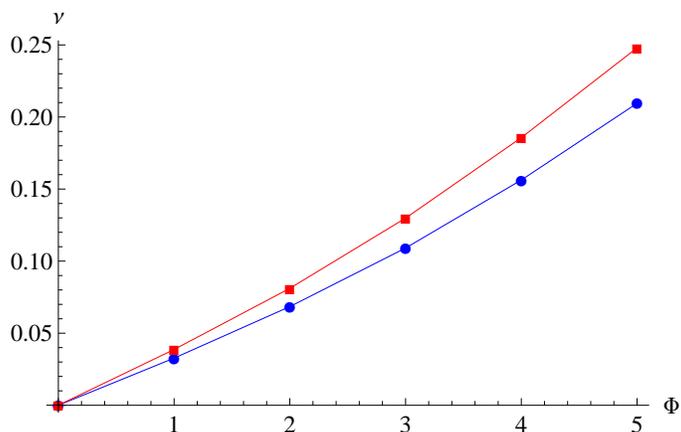
Figure 6: Welfare gains from introducing (blue) and having (red) recapitalization policies as a functions of the size of the net worth shock

# 6    Uncertainty shocks

Notice that in our model, idiosyncratic risk is a key ingredient in generating agency costs, since the latter arise from the interplay between costly monitoring of projects and the risk of bankruptcy. We now turn to examine the dynamic responses in our model to changes in idiosyncratic risk. We focus on idiosyncratic risk in the financial sector, but similar results can be obtained when examining similar changes in the real sector. Recent literature has discussed the importance of uncertainty shocks of similar nature like the ones we described here and argue that they contribute importantly to business cycle fluctuations. Some examples include: Christiano, Motto and Rostagno (2009), Bloom, Floetotto and Jaimovich (2011), Gilchrist, Sim and Zakrajšek (2010), and Arellano, Bai and Kehoe (2011).

Figure 7 depicts the dynamic responses of the model following a permanent increase (red line) and decrease (green line) in idiosyncratic uncertainty in the financial sector equivalent to 20 percent of the baseline standard deviation. In absence of limited liability in the financial sector, the change in its idiosyncratic risk would be inconsequential, given the linearity of the financial entrepreneur's problem. However, limited liability implies that the risk of default changes at the individual firm level, and hence the average defaults at the aggregate level.

Consider for instance the case of a decrease in idiosyncratic risk, the lower risk of default implies that the spread between the interest rate charged by the household for his deposits at the financial intermediary over the risk-free rate decreases. With borrowing costs lower, the financial

intermediary finds it profitable to increase leverage and increase lending, which in turn raises the demand for deposits, pushing the risk-free rate up, discouraging consumption and increasing savings by households, which increases the supply of deposits. Lending supply increases, pushing down also the borrowing costs for firms in the real sector, which in turn increases the incentives for real sector firms to lever up as well. Consequently, the capital stock and output rise. Net worth in the financial sector decreases because limited liability implies that changes in idiosyncratic risk change only upside returns, therefore, profitability declines below the rate at which financial entrepreneurs exit the market, inducing a gradual decline in aggregate net worth in the financial sector. In contrast, net worth in the real sector increases because profitability in that sector goes up since the borrowing costs have gone down, while the marginal product of capital is initially high and gradually adjust downwards towards its new steady state. All these effects carry through symmetrically when the shock is an increase in idiosyncratic uncertainty.

One interesting implication of this exercise is that a reduction in the idiosyncratic uncertainty increases leverage in the financial sector, which in turn increases its vulnerability to aggregate shocks. It is useful to draw a parallel between this exercise and what happened prior to the crisis that started in the U.S. in 2007. The reduction in idiosyncratic uncertainty in our model could be interpreted as the impact of financial innovation. For instance, the increasing use of credit default swaps and securitization that grew rapidly over the decade that preceded the crisis, had precisely the effect of reducing idiosyncratic risk. Over this same period, we witnessed a significant increase in leverage, which in turn increased the economy's vulnerability to systemic risk, or aggregate shocks. The outcome of the simulation performed here is consistent with these stylized facts.

## 6.1   Recapitalization Policies and Idiosyncratic Risk

We now re-examine the welfare gains obtained earlier with the goal of studying how they vary with idiosyncratic risk. Notice that on the one hand, one would expect recapitalization of the financial sector to become less relevant as we approach zero idiosyncratic risk in the financial sector because at that point financial frictions in the sector become irrelevant. However, as shown in the previous section, there is also the fact that for lower (but still positive) idiosyncratic risk leverage rises, increasing the vulnerability of the economy to aggregate shocks. It is therefore not obvious at first the direction of the net effect of these two effects on the welfare gains.
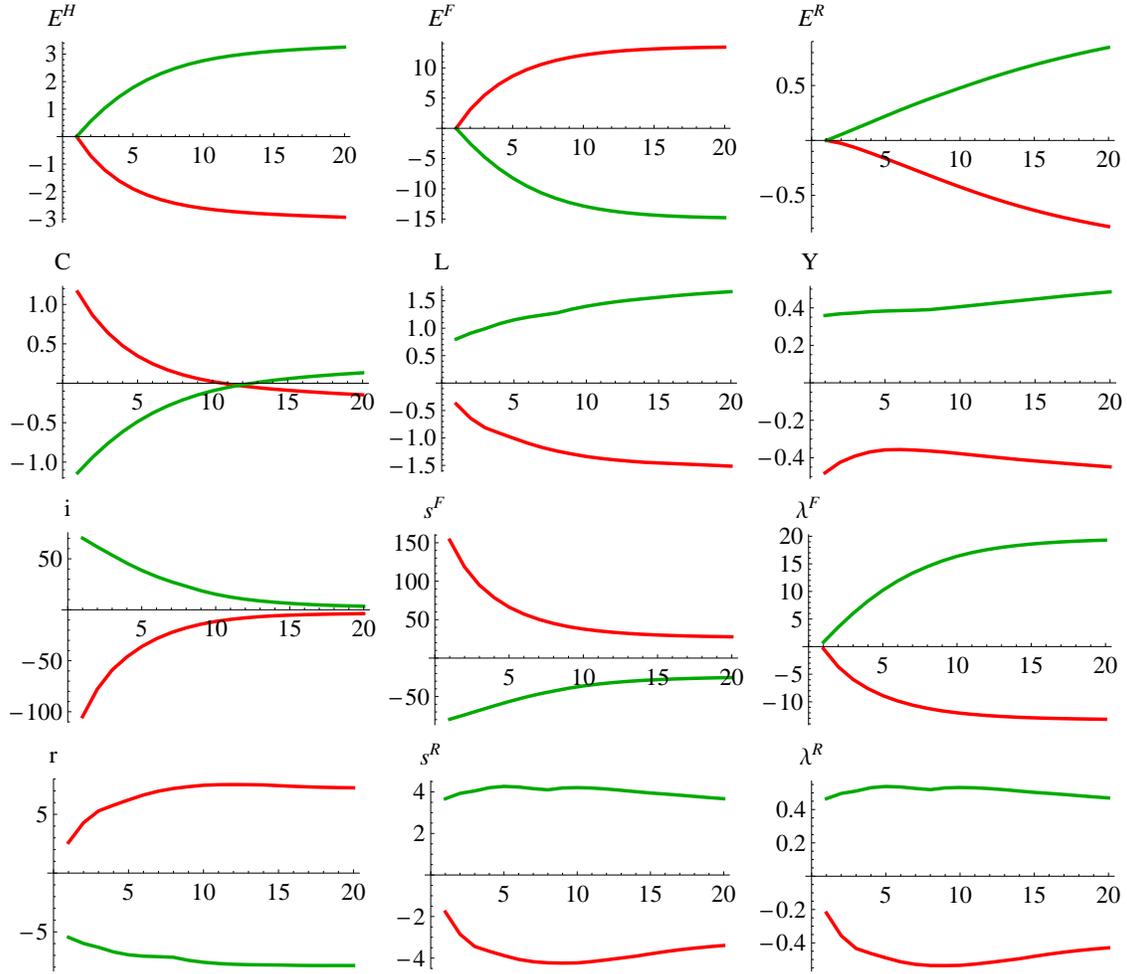
Figure 7: Impulse response functions from an increase (red) and decrease (green) in idiosyncratic risk in the financial sector (% deviations from the steady state)

Figure 8 shows the outcome of computing the welfare gains from recapitalization policies as indicated in the previous section for versions of the model solved under different calibrations of financial sector idiosyncratic risk. Notice that as idiosyncratic risk decreases, the gains increase, which results from the increase leverage in the financial sector highlighted earlier. However, for further reductions in idiosyncratic risk, the gains start decreasing because financial frictions become less relevant enough to trump the increased leverage effect. Consequently, the model implies a non-monotonic relationship between welfare gains the financial sector idiosyncratic risk.
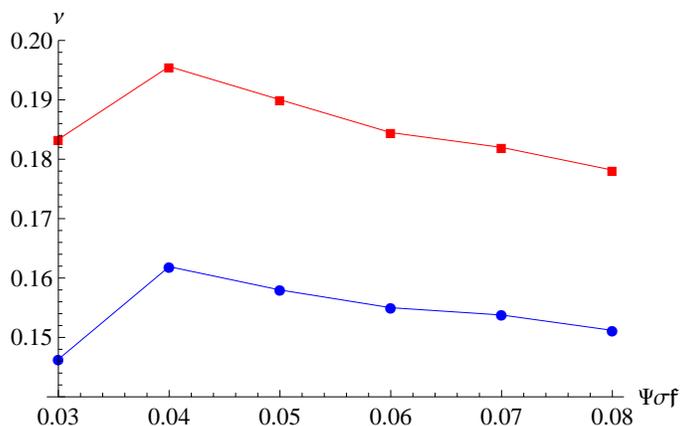
Figure 8: Welfare gains from introducing (blue) and having (red) recapitalization policies as a function of idiosyncratic risk in financial sector

# 7    Conclusion

The crisis that started in the U.S. in 2007 brought about significant deterioration in balance sheets, in particular of households and financial institutions, and later a sharp economic contraction in the U.S. and in many other countries to which the crisis spread. In response to these events, massive policy intervention was deployed aiming at containing the real effects of the crisis, restoring balance sheets, and speeding up the recovery. These events have revamped significantly the interest of academic research in studying what the welfare-improving policies to deal with financial crises are. This paper contributes to the growing literature on financial crises by developing a dynamic-stochastic general equilibrium framework to assess how large shocks that impair balance sheets in the different sectors in the economy propagate and affect output and welfare. At the same time, it allows us to conduct a quantitative assessment of welfare gains that are obtained if policies oriented towards restoring balance sheets are implemented. We do not take a position regarding how these policies could or should be implemented, which in practice can take many forms: through monetary policy, through fiscal policy, through direct equity injections, etc. Instead, we take a general stance and consider wealth redistributions, since all these policies can ultimately be seen as such. The key departure of our paper relative to existing models in the literature include the explicit modeling of a financial intermediary, which allows us to have an intermediate sector between savers (households) and borrowers (real sector firms), with different financial conditions than other firms in the economy. By exploiting this heterogeneity in financial conditions, we can examine how financial variables such

as leverage and spreads influence the magnitude on the economy of the initial shock as well as shocks that originate in the financial industry. Since we are interest in studying the effects of large shocks, we pursue a global solution over the entire state space, which becomes another salient feature of our model when compared with the standard approach in the DSGE literature of linearizing around the steady state. Calibrating the model with U.S. data we find first that shocks that originate in the financial sector tend to generate the largest contraction in output because of the high leverage in the sector. Second, recapitalization policies improve welfare more when they are funded with the household than when they are funded with the real sector. Third, we also assess the impact of changes in idiosyncratic risk and find that a decrease in idiosyncratic risk can increase welfare gains from recapitalization policies. In terms of quantitative implications, we find that the welfare gains are roughly similar to those that would be obtained if business cycle fluctuations were entirely removed.

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