Sticky Prices vs. Sticky Information: A Cross-Country Study of Inflation Dynamics

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Abstract

This paper provides an empirical comparison of the sticky-price and the sticky-information Phillips curves considering second moments of inflation for six countries, the US, the UK, Germany, France, Canada, and Japan. We evaluate the models' abilities to match empirical second moments of inflation evaluating moment distributions generated by model simulation. The absolute deviation of model moments to data moments is lower for sticky information but the joint density of the observed moments is higher for sticky prices. We also find that sticky information would need an unusually low degree of real rigidity to perform similar to sticky prices.

Keywords: Phillips curve; Sticky information; Sticky prices
JEL Classification: E31; E32; E37

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1 Introduction

Mankiw and Reis (2002) proposed sticky information as an alternative to the then workhorse of monetary analysis, the sticky-price approach. The basic idea of sticky information is that information spreads slowly through the economy. Mankiw and Reis argue that this approach is favorable to the sticky prices because it is able to produce certain empirical observations that can not be generated by sticky prices: hump-shaped responses of inflation to monetary impulses, contractionary disinflations and the acceleration phenomenon.[1]

Reis (2006) examines the second-moment performance of the sticky-information Phillips curve in an otherwise very simple model. In this model, the sticky-information Phillips curve represents the supply side while the model is closed by exogenous stochastic processes on the demand side. Reis finds that even such simple sticky-information model matches selected second moments of US inflation reasonably well.

The quantitative analysis of Reis is augmented in several respects in this paper. First, we consider five more countries, the UK, Germany, France, Canada, and Japan. Second, we examines whether the finding of Reis is unique to a sticky-information Phillips curve or whether it can also be achieved using a sticky-price Phillips curve. We third extend the analysis methodologically by comparing not only absolute deviations between model moments and empirical observations but also evaluating the likelihood of the observed moments given the two models. We finally use the models to estimate the degrees of real rigidity in our countries on the basis of second moments of inflation using a Maximum-Likelihood approach.

In order to compare the two Phillips curves on a leveled playing field, we need models which are, except for the Phillips curves, identical. Furthermore, the other equations of the model should be dichotomous from the Phillips curve. Otherwise, when estimating the model parameter estimates for the other equations would be influenced by the specific Phillips curve chosen.

[1] In the analysis of Mankiw and Reis (2002), demand is exogenously given. Trabandt (2007) demonstrates that, in a general-equilibrium model, sticky prices perform equally well in producing these three observations.

We thus need models which can be estimated recursively.[2]

The model used by Mankiw and Reis (2002) fulfills these criteria. Therefore the model we use is a generalization of the Mankiw and Reis model. They use exogenous autoregressive process of order 1 to characterize the demand side. For certain countries, we find that dynamics on the demand side are better captured by higher order stochastic processes. We thus allow for higher order dynamics.

When we estimate the models, we make use of the recursive structure of the model and estimate in a first step equations on the demand side and then the Phillips curves. This ensures that, when comparing models, both have identical demand sides. The model do not only have the same demand-side equations but also the same parameter estimates here.

Our empirical procedure builds on the generation of a distribution of moments generated by simulations of the two respective models. To derive such distributions, we first estimate stochastic processes for demand-side variables. We then solve for inflation as a rational-expectations equilibrium response to innovations in these variables. For a set of selected second moments of inflation, we generate distributions of model moments by repeated simulations of the quantified models. We evaluate the densities of the observed moments within these distributions as a measure of the likelihood of the observation given the quantified model. When estimating degrees of real rigidity, we determine the parametrization under which the joint density of the observed moments is maximized.

Our results confirm the relative closeness of the sticky-information model to empirical second moments. In an extended US sample, we find results that are similar to those of Reis (2006). However, comparing the two models puts the empirical success of the sticky-information model into perspective. The absolute deviation of model moments to data moments is lower for sticky information in all countries. But the joint density of the observed moments is higher for sticky prices, also in all countries.

[2]This feature distinguishes our work from previous studies comparing Phillips curves resulting from different approaches, such as Gali et al. (2005), Korenok (2005), Sbordone (2005), Paustian and Pytlarczyk (2006), Kiley (2007), and Döpke et al. (2008a).

Considering specific countries, the sticky-price model is capable of matching second moments of US and German inflation well, sticky information performs reasonably in these countries. No model performs well for UK inflation dynamics. French, Canadian, and Japanese inflation dynamics matched poorly by the sticky-price model, whereas sticky information performs even worse.

Considering specific moments, it is a robust finding across countries that both models are especially successful in generating cross-correlations of inflation to productivity and demand that are close to empirical observations. However, our analysis suggests that both models have difficulties to generate an autocorrelation of inflation in the magnitude found in the data. Both models overpredict the autocorrelation substantially in all countries.

Parameter variations indicate that rather extreme calibrations for the degree of rigidity would be needed to bring the autocorrelation to its observed value. This finding is also reflected in our estimation results. Using the sticky-information model, we estimate rather low degree of real rigidities, especially for France. By contrast, using the sticky-price model we, in general, estimate values in the magnitude discussed in the literature.

The remainder of the paper is organized as follows. Section 2 presents the theoretical models used in this paper. The underlying data, parameter choices, and our empirical strategy are presented in Section 3. The results of the analysis can be found in Section 4. Finally, Section 5 concludes.

2 Models

2.1 Sticky-Price and Sticky-Information Phillips Curves

This section contains a brief sketch of the sticky-price and sticky-information concepts and presents the resulting Phillips curves. For the following empirical analysis, we use only the two Phillips curves and close the model in a simple way, which is described in Sections 2.2 and 2.3.

The sticky-price model bases on the concept of Calvo pricing (Calvo

(1983)). In every period, only a fraction λ of firms is able to change prices. When enabled to do so, a firm sets a price that is then fixed until the firm is drawn again. The firm uses all current information and sets one price to maximize expected profits over the next T periods, where T is uncertain and $1/\lambda$ in expectation. Therefore, in any period t, changes in the aggregate price level only occur due to the fraction λ of firms that are allowed to change prices in t. These firms plan with all current information but need to use expectations over future inflation since they set prices for more than one period. In consequence, the sticky-price Phillips curve takes the form

$$\pi_t = \left[\frac{\alpha\lambda^2}{1-\lambda}\right] y_t + E_t\pi_{t+1}, \tag{1}$$

where π_t denotes inflation, y_t is the log output gap and E_t the expectations operator based on the information set of period t.[3] The parameter α is a measure of real rigidities that measures how much an individual firm's optimal price depends on the output gap. A higher α means less real rigidity. As equation (1) states, in the sticky-price model, inflation depends on current expectations of future inflation, because that is the information used by firms that currently change prices.

In the sticky-information model proposed by Mankiw and Reis (2002), all firms can change prices every period. However, in every period, only a fraction λ of them is given access to new information. When receiving new information, firms make plans for current and future prices which can also include plans to change prices in the future. Firms do change prices every period but some of these changes have been decided upon in the past using past information. Other than in the sticky-price model, prices for period t are set to maximize only expected profits in period t. However, some of these expectations are build with not-up-to-date information. Consequently, changes in the aggregate price level are due to all firms changing prices in order to maximize profits in this particular period, but fraction $1-\lambda$ of firms change prices based on obsolete information of different age. The sticky-

[3] This particular form of the Phillips curve results from the sticky-price model used in Mankiw and Reis (2002).

information Phillips curve reflects this by containing all past expectations of current inflation:

$$\pi_t = \left[\frac{\alpha\lambda}{1-\lambda}\right] y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} (\pi_t + \alpha \Delta y_t), \tag{2}$$

where Δ is the difference operator, i.e. $\Delta y_t = y_t - y_{t-1}$.[4]

2.2 Closing the Models

The Phillips curve represents a relationship between two endogenous variables, inflation π_t and the log output gap y_t. In order to close the model, a second relationship between these two variables is needed. Assuming that natural output Y_t^n is equal to labor productivity A_t, $Y_t^n = A_t$, the log output gap can be written as

$$y_t = m_t - p_t - a_t,$$

where m_t is log nominal income, p_t is the log price level.[5] We follow the empirical analysis of Reis (2006) and use his assumptions regarding m_t and a_t: We assume that these variables are exogenous to inflation, that natural output is equal to productivity, and that m_t and a_t follow independent stochastic processes.

While Reis (2006) finds that first-order auto-regressive processes are sufficient for US data, we need processes of higher order in order to describe the growth rates of nominal income and productivity in other countries adequately. We therefore allow the growth rates Δa_t and Δm_t to follow auto-regressive processes of up to order four. Given such processes, we write Δm_t and Δa_t as a moving average of past shocks:

$$\Delta a_t = \sum_{i=0}^{\infty} \omega_i \epsilon_{t-i}^a \tag{3}$$

[4] Again, this particular form results from the sticky-information model in Mankiw and Reis (2002).

[5] We follow the convention that lower-case letters denote the logarithms of the respective capital letters.

and

$$\Delta m_t = \sum_{i=0}^{\infty} \rho_i \epsilon_{t-i}^m \tag{4}$$

While assuming that productivity follows an exogenous stochastic process as in (3) is standard in the literature, assuming this also for nominal income is rather unusual. There are however ways monetary policy can ensure that nominal income M_t follows such process. For details about these policies see Mankiw and Reis (2010).

Modeling the dynamics of nominal income and productivity as such implies being ignorant to any structural relationships governing these dynamics. However, this modeling strategy ensures that the model can be estimated recursively and hence the choice of the Phillips curve does not influence estimates for other equations of the model.

2.3 Solving the Models

Both models consist of a Phillips curve and the exogenous stochastic processes for nominal income and productivity growth described above. Shocks to Δm_t and Δa_t are thus the only driving forces for dynamics in the models. We determine the model solution by a guess-and-verify approach. We guess that inflation is a moving average of past shocks,

$$\pi_t = \sum_{i=0}^{\infty} \gamma_i^z \epsilon_{t-i}^m + \sum_{i=0}^{\infty} \xi_i^z \epsilon_{t-i}^a, \tag{5}$$

where $z = SI, SP$. Then we solve for the coefficients γ_i^{SI} and ξ_i^{SI}, or γ_i^{SP} and ξ_i^{SP} respectively, using matching coefficients.

In the sticky-information model, the coefficients on shocks to Δm_t have to fulfill

$$\begin{aligned}\sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m \quad &= \quad \left[\frac{\alpha\lambda}{1-\lambda}\right] \left\{ \sum_{i=0}^{\infty} \rho_i \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m\right] - \sum_{i=0}^{\infty} \gamma_i^{SI} \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m\right]\right\} \\ &\quad +\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \left\{(1-\alpha) \sum_{i=j+1}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m + \alpha \sum_{i=j+1}^{\infty} \rho_i \epsilon_{t-i}^m \right\}\end{aligned}$$

which is the sticky-information Phillips curve (2) with the stochastic processes (4) and (3) and the guess (5) plugged in. Matching coefficients yields

$$\gamma_k^{SI} = \begin{cases} \frac{\alpha\lambda}{1-\lambda+\alpha\lambda}, & \text{if } k=0 \\ \alpha\lambda\left(1-\lambda(1-\alpha)\sum_{i=0}^{k}(1-\lambda)^i\right)^{-1} & \\ \cdot\left[1-\sum_{i=0}^{k-1}\gamma_i^{SI}+\sum_{i=1}^{k}\rho_i+\rho_k\sum_{i=1}^{k}(1-\lambda)^i\right] & \text{else} \end{cases}. \tag{6}$$

The corresponding calculation of the coefficients on shocks to productivity growth gives[6]

$$\xi_k^{SI} = \begin{cases} \frac{\alpha\lambda}{1-\lambda+\alpha\lambda}, & \text{if } k=0 \\ \alpha\lambda\left(1-\lambda(1-\alpha)\sum_{i=0}^{k}(1-\lambda)^i\right)^{-1} & \\ \cdot\left[1-\sum_{i=0}^{k-1}\xi_i^{SI}+\sum_{i=1}^{k}\omega_i+\omega_k\sum_{i=1}^{k}(1-\lambda)^i\right] & \text{else} \end{cases}. \tag{7}$$

In the sticky-price model, coefficients on shocks to Δm_t have to fulfill

$$\begin{aligned}\sum_{j=0}^{\infty}\gamma_j^{SP}\sum_{k=0}^{\infty}\epsilon_{t-j-k}^{m} =& \theta\left[\sum_{j=0}^{\infty}\gamma_j^{SP}\sum_{k=0}^{\infty}\epsilon_{t-j-k-1}^{m}\right] \\ &+(1-\theta)^2\sum_{i=0}^{\infty}\theta^i\sum_{n=0}^{\infty}\sum_{j=\max(i-n,0)}^{\infty}\rho^j\epsilon_{t-n-j+i}^{m}\end{aligned}, \tag{8}$$

where $\theta = \frac{\beta}{2}+1-\frac{\sqrt{(\beta+2)^2-4}}{2}$ and $\beta = \left[\alpha\lambda^2/(1-\lambda)\right]$. Again we use matching coefficients which yields

$$\gamma_k^{SP} = \begin{cases} (1-\theta)\left[\frac{1}{1-\theta\rho_1}\right] & \text{if } k=0 \\ (\theta-1)\left\{\sum_{j=0}^{k-1}\gamma_j^{SP}-\sum_{i=0}^{k-1}\rho_i-\sum_{i=k}^{\infty}\rho_i\theta^{i-k}\right\} & \text{else} \end{cases}. \tag{9}$$

[6]Equations (6) and (7) are derived in detail in Appendix A.1.1.

for the coefficients on shocks to nominal income and

$$\xi_k^{SP} = \begin{cases} (1-\theta)\left[\frac{1}{1-\theta\omega_1}\right] & \text{if } k=0 \\ (\theta-1)\left\{\sum_{j=0}^{k-1}\xi_j^{SP} - \sum_{i=0}^{k-1}\rho_i - \sum_{i=k}^{\infty}\rho_i\theta^{i-k}\right\} & \text{else} \end{cases}. \tag{10}$$

for the coefficients on shocks to productivity growth.[7]

3 Data and Methodology

3.1 Data

For our empirical analysis, data on inflation, productivity, and nominal income is needed. We use CPI inflation, output per working hours, and nominal GDP per capita. We have quarterly data on these three variables for a sufficiently long period for the following six countries: the US, the UK, Germany, France, Canada, and Japan. However, the period for which we have complete data varies considerably between the different countries.

The longest sample is available for the US, where data from the first quarter of 1947 onwards is available. For Canada, the shortest sample in our data set, only data from the first quarter of 1981 is available for all three variables. All data is taken from the *OECD*, *Datastream*, and national statistical offices. In the first quarter of 1990, a linear extrapolation is used for Germany in consideration of the re-unification.[8]

3.2 Parameter Choices

In order to calculate values of the coefficients $\{\gamma_i^z\}$ and $\{\xi_i^z\}$, values for the rigidity parameters α and λ are needed. We take values for these parameters from Reis (2006), Carroll (2003), Döpke et al. (2008b), and Khan and Zhu (2002). In principle, α and λ may differ across countries. Concerning λ, Carroll (2003) finds $\lambda_{US} = 0.25$ for the US using survey data. Döpke et al. (2008b) who also use survey data report similar values for European

[7] Equations (9) and (10) are derived in detail in Appendix A.1.2.

[8] Data sources and details are described in Appendix A.2.

countries, $\lambda_{UK} = \lambda_{FR} = 0.27$ and $\lambda_{DE} = 0.26$, which we take. For Canada, we use $\lambda_{CA} = 0.25$ which we take from Khan and Zhu (2002) who use VAR predictions. Since their is no study estimating λ for Japan, but there are no substantial differences across the other countries, we use the US value for this country, i.e. $\lambda_{JP} = 0.25$.

Concerning α, we use the US value, $\alpha = 0.11$, used by Reis (2006) for all countries in a baseline parametrization. Later on, we estimate this parameter using Maximum Likelihood and compare models with the estimated value for α.

3.3 Empirical Strategy

Model Comparison under Benchmark Parametrization. Our first empirical analysis studies the second-moment performance of the two competing Phillips curves under the benchmark parametrization which implies $\alpha = 0.11$ for all countries. For each country c and model z, the analysis consists of a complete model parametrization and a repeated model simulation.

1. In the parametrization phase, we first estimate processes for nominal income growth and productivity growth from the data. In any country and for both time series, we start with estimating the parameters of an AR(4) process by OLS. If the coefficient on the last lag is not significantly different from zero, we drop that lag and re-estimate an auto-regressive process of order 3. We drop insignificant lags until we arrive at a process with a significant last lag. Having found such an auto-regressive process, we invert it into its MA representation.[9] We collect the values for the coefficients $\{\rho_i^c\}$ and $\{\omega_i^c\}$ and the innovation variances $\sigma^2_{m,c}$ and $\sigma^2_{a,c}$ governing the dynamics of nominal income growth and productivity growth for this country in $\Omega_c = \left\{\{\rho_i^c\}_{i=0}^{\infty}, \sigma^2_{m,c}, \{\omega_i^c\}_{i=0}^{\infty}, \sigma^2_{a,c}\right\}$. The model is now completely quantified, the parametrization is described by α, λ_c, and Ω_c.

[9] In this step, we have to modify the procedure of Reis (2006) in order to capture country-specific patterns in the behavior of Δm and Δa. In Reis' study it is sufficient to use AR(1) processes, we need processes of higher order in order to describe the growth rates of nominal income and productivity in other countries adequately.

Using the values for the coefficients $\{\rho_i^c\}$ and $\{\omega_i^c\}$ and the parameters α and λ_c, we calculate the coefficients $\{\gamma_i^{c,z}\}$ and $\{\xi_i^{c,z}\}$ in the MA representation of inflation (5) as given by equations (6) and (7), or (9) and (10) respectively.

2. We simulate the model 10,000 times. In each simulation, we draw sequences of innovations $\{\epsilon_t^{m,c}\}$ and $\{\epsilon_t^{a,c}\}$ with mean zero and variances $\sigma_{m,c}^2$ and $\sigma_{a,c}^2$ and feed them into the model. Combining the innovations $\{\epsilon_t^{m,c}\}$ and $\{\epsilon_t^{a,c}\}$ and the MA coefficients of inflation $\{\gamma_i^{c,z}\}$ and $\{\xi_i^{c,z}\}$, we generate a sequence of inflation rates $\{\pi_t^{c,z}\}$ as predicted by the respective model z given Ω_c.

   For each simulation, we calculate the standard deviation of inflation, its auto-correlation, and its cross-correlations to current values, leads, and lags of nominal income and productivity growth. We thus generate a distribution of model moments by simulation. Denote by X a set of second moments of inflation, specifically, in our case:

$$X = \left\{ \begin{array}{c} S.D.(\Delta p_t), Corr(\Delta p_t, \Delta p_{t-1}), Corr(\Delta p_t, \Delta m_t), \\ Corr(\Delta p_t, \Delta m_{t-1}), Corr(\Delta p_t, \Delta m_{t+1}), Corr(\Delta p_t, \Delta a_t), \\ Corr(\Delta p_t, \Delta a_{t-1}), Corr(\Delta a_t, \Delta a_{t+1}) \end{array} \right\}$$

   For each moment $x \in X$, we estimate a density function $f_x^{c,z}(x|\alpha, \lambda_c, \Omega_c)$ from the 10,000 generated observations using Maximum Likelihood.

   We evaluate for each moment x the density of the actual observation $x^{c,data}$ from the data in the distribution of simulated model moments, $f_x^{c,z}\left(x^{c,data}|\alpha, \lambda_c, \Omega_c\right)$ and the corresponding cumulated density $F_x^{c,z}\left(x^{c,data}|\alpha, \lambda_c, \Omega_c\right)$. Finally, we calculate

$$\prod_{x \in X} f_x^{c,z}\left(x^{c,data}|\alpha, \lambda_c, \Omega_c\right)$$

   as a measure of accuracy of the respective model. The product of densities can be understood as the likelihood of the set of observed moments given the model z and the parametrization $\alpha, \lambda_c, \Omega_c$.

Estimation of Real Rigidities and Second-Moment Performance under Estimated Parametrization. In our second empirical analysis, we seek to estimate the Phillips-curve parameter on the output gap, α, by matching our two models to the observed second moments of inflation. For each country c and model z, we determine the value for α which maximizes the joint density of observed second moments in the distribution of model moments.

The estimated parameter $\alpha_{c,z}$ is the solution to the problem

$$\max_{\alpha} \prod_{x \in X} f_x^{c,z} \left(x^{c,data} | \alpha, \lambda_c, \Omega_c \right),$$

where we determine the density functions $f_x^{c,z}$ as described above. We determine $\alpha_{c,z}$ numerically using a grid search technique.

4 Results

4.1 Nominal Income and Productivity Processes

Table 1 reports the estimated auto-regressive processes for nominal income and productivity growth for the six countries in our sample. To determine the coefficients $\{\rho_i^c\}$ and $\{\omega_i^c\}$, which are needed for the further analysis, the AR processes have to be inverted to their MA representations. All estimated processes are stationary and inverting is thus possible.

In half of the cases, it is sufficient to use not more than one lag to describe the dynamics in productivity and nominal income growth in the various countries. Growth in nominal income can be described as an AR(1) process for the United States and Canada with comparable persistence. For the UK, nominal income growth seems to be i.i.d. For Germany, France, and Japan, growth of nominal income is best described by auto-regressive processes of higher order. While for France our estimation procedure results in an AR(2), even the fourth lag is of significant influence for nominal income growth in Germany.

Productivity growth can be described as i.i.d. with positive mean for the

nominal income growth						
	cons $\cdot 10^2$	t-1	t-2	t-3	t-4	$\sigma^2_{m,c} \cdot 10^4$
US	0.87	0.47				1.03
UK	0.61					0.79
Germany	0.48	-0.04	0.08	0.14	0.37	1.58
France	0.13	0.48	0.40			0.18
Canada	0.28	0.55				0.23
Japan	0.10	0.21	0.37	0.29		3.31
productivity growth						
	cons $\cdot 10^2$	t-1	t-2	t-3	t-4	$\sigma^2_{a,c} \cdot 10^4$
US	0.48					0.98
UK	0.50					0.93
Germany	0.91					1.16
France	0.28	-0.03	0.23			0.28
Canada	0.22	0.09	-0.07	0.26		0.40
Japan	0.14	0.52	0.18	0.02	-0.27	1.11

Table 1: Estimated coefficients and shock variances for productivity and nominal income growth processes.

US, the UK, and Germany. French, Canadian, and Japanese growth rates show some significant auto-regressive components. This is most pronounced for Japan where the coefficient on the fourth lag is significantly negative at the one percent level.[10]

4.2 Impulse Responses

Figures 1 and 2 display the impulse responses of inflation to nominal income or productivity shocks in both models for our six countries. The lines represent the coefficients $\{\gamma_i^{c,z}\}$ and $\{\xi_i^{c,z}\}$ of the MA representation of inflation in the respective model z for country c.

The impulse responses in the sticky-information model (represented by the solid lines) show the characteristic hump-shaped patterns for both kind of shocks. The largest effect occurs several periods after the shock. In contrast, in the sticky-price model a shock has its strongest influence on inflation

[10]Detailed results of the estimation procedure of the processes are available on request.

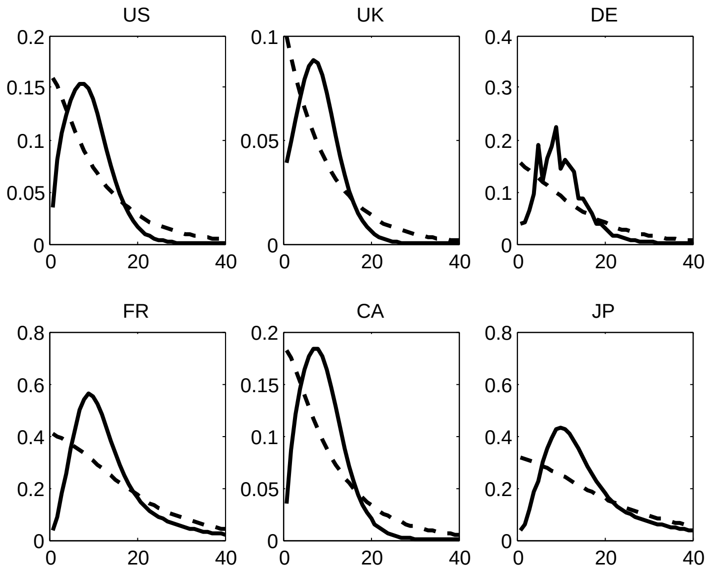

Figure 1: Impulse response of inflation to a one standard deviation shock to nominal income growth in the sticky information (solid lines) and the sticky price (dashed lines) model for different countries.

instantaneously, the respective impulse response functions are downward (for the case of nominal income) or upward (for the case of productivity) sloping.

4.3 Second-Moment Performance under Baseline Parametrization

One specific simulation. Before turning to a comparison of the two Phillips curves using simulated likelihoods, we present an illustrative example. Table 2 contains model moments which arise from one specific simulation. In this simulation, we used the sequences of residuals from the OLS estimations of stochastic processes in Table 1 as innovations $\{\epsilon_t^{m,c}\}$ and $\{\epsilon_t^{a,c}\}$. This procedure was used by Reis (2006) to examine the ability of the sticky-information Phillips curve to match second moments of US inflation.

Reis concludes that "the model fits the data remarkably well" because "with only one exception, all of the model's predictions do not differ from the

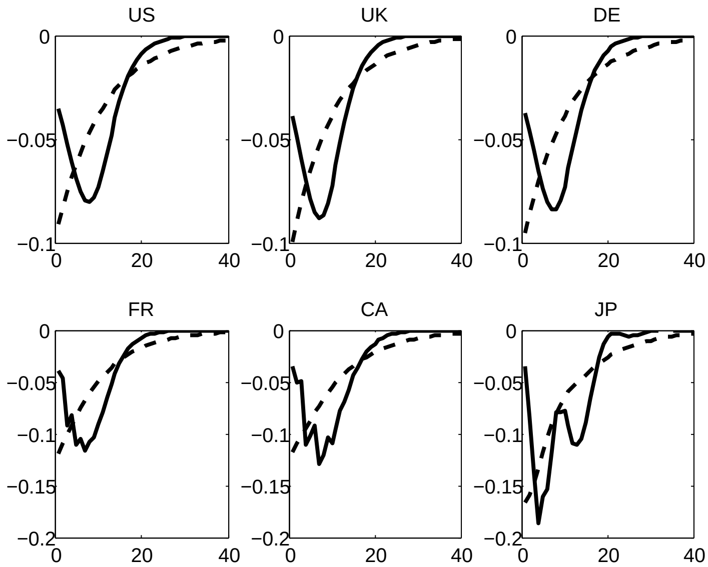

Figure 2: Impulse response of inflation to a one standard deviation shock to productivity growth in the sticky information (solid lines) and the sticky price (dashed lines) model for different countries.

empirical moments by more than 0.05". Here we augment the quantitative analysis of Reis in two respects. First, we do a similar exercise also with a sticky-price Phillips curve and compare the models' empirical performances. Second, we rerun the analysis for five more countries, the UK, Germany, France, Canada, and Japan. In the next section, we will further deepen the analysis and compare models using simulated likelihoods.

For each country and moment, three numbers are reported in Table 2, the observed value from the data and two numbers calculated from the series of predicted inflation from the sticky-information model (S.I.) and the sticky-price model (S.P.).

In the results of Reis (2006) no model prediction differs from its empirical counterpart by more than 0.1 with a total absolute deviation of 0.3. With our extended US sample, similar results emerge. Comparing the two competing models for the US, sticky information seems to perform better than sticky prices. Six out of eight moments are better matched by the sticky-information model in terms of absolute deviation.

	United States			United Kingdom		
	S.I.	S.P.	data	S.I.	S.P.	data
S.D.(Δp_t)	**0.0062**	0.0057	0.0081	**0.0026**	0.0021	0.0150
Corr($\Delta p_t, \Delta p_{t-1}$)	0.9958	**0.9861**	0.7172	0.9831	**0.9716**	0.5991
Corr($\Delta p_t, \Delta m_t$)	**0.3419**	0.4695	0.3447	**-0.1001**	0.0238	-0.2747
Corr($\Delta p_t, \Delta m_{t-1}$)	**0.3890**	0.4365	0.3557	**-0.0505**	0.0791	-0.1206
Corr($\Delta p_t, \Delta m_{t+1}$)	**0.3189**	0.3616	0.2198	**-0.1475**	-0.0801	-0.1745
Corr($\Delta p_t, \Delta a_t$)	**-0.2295**	-0.1881	-0.2815	**-0.2314**	-0.2710	-0.1470
Corr($\Delta p_t, \Delta a_{t-1}$)	**-0.2228**	-0.2437	-0.2279	**-0.2524**	-0.2637	-0.0657
Corr($\Delta a_t, \Delta a_{t+1}$)	-0.2019	**-0.2341**	-0.2779	**-0.1962**	-0.2528	-0.0522
total abs. dev.	0.5487	0.7714		1.0833	1.5008	
(without auto-corr.)	0.2701	0.5026		0.6992	1.1282	
	Germany			**France**		
	S.I.	S.P.	data	S.I.	S.P.	data
S.D.(Δp_t)	0.0059	**0.0053**	0.0046	0.0162	**0.0047**	0.0043
Corr($\Delta p_t, \Delta p_{t-1}$)	0.9572	**0.9544**	0.5644	0.9991	**0.9413**	0.3697
Corr($\Delta p_t, \Delta m_t$)	**0.3670**	0.4352	0.1712	**0.4901**	0.8095	0.3505
Corr($\Delta p_t, \Delta m_{t-1}$)	**0.3263**	0.4251	0.2980	**0.5245**	0.7288	0.2553
Corr($\Delta p_t, \Delta m_{t+1}$)	0.2816	**0.2540**	0.1975	**0.4977**	0.6206	0.2931
Corr($\Delta p_t, \Delta a_t$)	**-0.0987**	-0.1646	-0.0176	**0.1786**	0.4242	0.1637
Corr($\Delta p_t, \Delta a_{t-1}$)	**-0.1536**	-0.1569	0.1118	**0.1809**	0.3106	0.1273
Corr($\Delta a_t, \Delta a_{t+1}$)	**-0.0549**	-0.0629	0.0005	**0.2062**	0.2347	0.1634
total abs. dev.	1.1042	1.3175		1.3658	2.3468	
(without auto-corr.)	0.7114	0.9275		0.7364	1.7753	
	Canada			**Japan**		
	S.I.	S.P.	data	S.I.	S.P.	data
S.D.(Δp_t)	**0.0029**	0.0023	0.0063	0.0138	**0.0072**	0.0086
Corr($\Delta p_t, \Delta p_{t-1}$)	0.9840	**0.9471**	0.2384	0.9913	**0.8651**	0.1576
Corr($\Delta p_t, \Delta m_t$)	**0.2187**	0.4452	-0.1339	**0.5717**	0.6982	0.3126
Corr($\Delta p_t, \Delta m_{t-1}$)	**0.3299**	0.4579	-0.0460	**0.5703**	0.6142	0.4920
Corr($\Delta p_t, \Delta m_{t+1}$)	**0.1006**	0.2117	-0.3086	0.5447	**0.4836**	0.3824
Corr($\Delta p_t, \Delta a_t$)	**-0.0956**	-0.0471	-0.1470	**0.0714**	-0.1007	0.1373
Corr($\Delta p_t, \Delta a_{t-1}$)	-0.0552	**-0.0727**	-0.1343	-0.0087	**0.0030**	0.1930
Corr($\Delta a_t, \Delta a_{t+1}$)	**-0.0737**	-0.0523	-0.1421	**0.0864**	0.0991	-0.0212
total abs. dev.	2.0855	2.5673		1.7138	1.8664	
(without auto-corr.)	1.3399	1.8585		0.8802	1.1589	

Table 2: Second moments of inflation as predicted by models and in data (model moments closer to empirical observation in bold).

In the other countries, models perform worse than in the US. This is especially pronounced for Canada, where the total absolute deviations between model and empirical moments are four times as high as in the US. The UK and Germany show similar deviations of roughly twice the US magnitude.

Comparing the two models across countries, sticky information comes closer to the empirical observation for 34 out of the 48 selected moments (bold values). The total absolute deviation from model predictions to observed moments is smaller for sticky information in all six countries. In all countries, most of the total absolute deviation is due to the autocorrelation of inflation. Both models overpredict this moment systemically throughout countries. Model simulations generate autocorrelations close to one, whereas, in the data, it is between 0.15 and 0.7. The last line for each country reports the total absolute deviation without the auto-correlation.

Comparison using simulated likelihoods. Whereas the previous analysis has build on the comparison between observed values and model predictions from just one simulation, we now evaluate the models' empirical performance using simulated likelihoods. As described in Section 3.3, we generate a distribution of model moments by simulation. We then evaluate the densities of the actually observed moments within this distribution. Figure 3 displays the distribution of moments from our simulations for the US. The solid curve represents the sticky-information model and the dashed curve stands for the sticky-price model. The dashed vertical line marks the value observed in the data.

The simulations show that both models are able to produce the correlation between inflation and nominal income growth (with lead and lag) and the correlation between inflation and the lagged productivity growth. For this moments, the empirical observation lies well within the distribution of model-predicted moments. In our view, this finding justifies the statement that model moments and empirical moments are close. However, such statement can not be put forward for the standard deviation and the auto-correlation of inflation.

Table 3 summarizes the results of the model comparison using simulated

	United States		United Kingdom	
	S.I.	S.P.	S.I.	S.P.
S.D.(Δp_t)	1.00	1.00	1.00	1.00
Corr($\Delta p_t, \Delta p_{t-1}$)	0.00	0.00	0.00	0.00
Corr($\Delta p_t, \Delta m_t$)	1.00	0.96	0.00	0.00
Corr($\Delta p_t, \Delta m_{t-1}$)	0.99	0.00	0.00	0.00
Corr($\Delta p_t, \Delta m_{t+1}$)	0.98	0.95	0.02	0.02
Corr($\Delta p_t, \Delta a_t$)	0.00	0.00	0.01	0.01
Corr($\Delta p_t, \Delta a_{t-1}$)	0.00	0.33	0.62	1.00
Corr($\Delta a_t, \Delta a_{t+1}$)	0.00	0.00	0.16	0.15
$\prod_{x\in X} f(x)$	$< 1 \cdot 10^{-323}$	$3.04 \cdot 10^{-20}$	$< 1 \cdot 10^{-323}$	$1.48 \cdot 10^{-187}$

	Germany		France	
	S.I.	S.P.	S.I.	S.P.
S.D.(Δp_t)	0.12	0.34	0.00	0.10
Corr($\Delta p_t, \Delta p_{t-1}$)	0.00	0.00	0.00	0.00
Corr($\Delta p_t, \Delta m_t$)	0.78	0.26	0.69	0.02
Corr($\Delta p_t, \Delta m_{t-1}$)	0.86	0.00	0.48	0.00
Corr($\Delta p_t, \Delta m_{t+1}$)	0.89	0.68	0.66	0.02
Corr($\Delta p_t, \Delta a_t$)	0.35	0.33	0.66	0.09
Corr($\Delta p_t, \Delta a_{t-1}$)	0.98	1.00	0.55	0.21
Corr($\Delta a_t, \Delta a_{t+1}$)	0.43	0.41	0.69	0.27
$\prod_{x\in X} f(x)$	$3.02 \cdot 10^{-40}$	$2.55 \cdot 10^{-15}$	$< 1 \cdot 10^{-323}$	$1.04 \cdot 10^{-52}$

	Canada		Japan	
	S.I.	S.P.	S.I.	S.P.
S.D.(Δp_t)	1.00	1.00	0.01	0.10
Corr($\Delta p_t, \Delta p_{t-1}$)	0.00	0.00	0.00	0.00
Corr($\Delta p_t, \Delta m_t$)	0.13	0.00	0.40	0.07
Corr($\Delta p_t, \Delta m_{t-1}$)	0.06	0.00	0.63	0.01
Corr($\Delta p_t, \Delta m_{t+1}$)	0.04	0.00	0.61	0.20
Corr($\Delta p_t, \Delta a_t$)	0.02	0.00	0.25	0.28
Corr($\Delta p_t, \Delta a_{t-1}$)	0.02	0.05	0.46	0.95
Corr($\Delta a_t, \Delta a_{t+1}$)	0.05	0.01	0.02	0.01
$\prod_{x\in X} f(x)$	$< 1 \cdot 10^{-323}$	$3.98 \cdot 10^{-54}$	$< 1 \cdot 10^{-323}$	$6.29 \cdot 10^{-100}$

Table 3: Model comparison using simulated likelihoods (cumulated density of each observation in the distribution of generated model moments and the product of densities for eight moments).

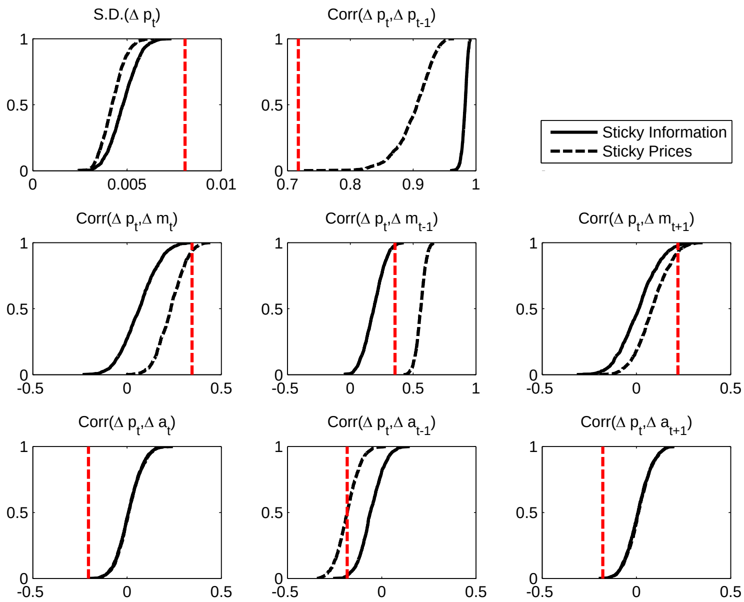

Figure 3: Distribution of moments in simulations of the sticky information (solid curves) and of the sticky price (dashed curves) model as well as observed moments (dashed vertical lines) for the US.

likelihoods for all countries. For each moment x, country c and model m, the entry in the table reports the cumulated density of the empirical observation within the distribution of generated model moments, $f_x^{c,z}\left(x^{c,data}|\alpha, \lambda_c, \Omega_c\right)$. A value strictly between zero and one implies that the empirical moment lies in the interior of the distribution of simulated model moments. In our view, such result justifies the statement that model moments and empirical observation are close.

Based on this criterion, the sticky-information model is able to match 32 out of 48 moments, whereas 16 moments are not reachable independent of the realization of the shock processes. For the sticky-price model, 26 moments lie in the interior of the distribution of simulated model moments. The overprediction of the auto-correlation occurs as a robust finding in this analysis and cannot be matched by any model for any country.

The final row for a country reports the product of moment densities

$\prod_{x \in X} f_x^{c,z} \left(x^{c,data} | \alpha, \lambda_c, \Omega_c \right)$ for both models. Considering this measure of accuracy, sticky prices perform better than sticky information in all countries. Even though the previous analysis showed that the absolute deviation between model moments and their empirical counterparts is smaller for the sticky-information model, the density of the observed moments within a simulated distribution of model moments is higher for the sticky-price model. Thus the likelihood of the observed second moments is higher under this model. The relative difference of likelihoods is severe and ranges from factor 10^{25} to over factor 10^{300}.

The analysis furthermore reveals that the sticky-price model is capable of matching second moments of US and German inflation reasonably well, whereas no model performs well for UK inflation dynamics. For France, Canada, and Japan, model accuracy is mediocre for sticky prices but sticky information does not match inflation dynamics at all.

4.4 Estimation of Real Rigidities and Second-Moment Performance under Estimated Parametrization

In this section, we estimate the Phillips curve parameter α, which describes the degree of real rigidity, by matching our two models to the observed second moments of inflation. A higher α means less real rigidity. For each country c and model z, we determine the value for α which maximizes the joint density of observed second moments in the distribution of model moments, as described in Section 3.3.

Table 4 reports the results of our estimations of the parameters $\alpha_{c,z}$ as well as the resulting product of densities under the parametrization $\alpha_{c,z}$,λ_c, Ω_c. As a comparison, the table also repeats the product of densities under the baseline parametrization used in section 4.3 where $\alpha = 0.11$.

The estimated values $\alpha_{c,z}$ differ substantially across countries as well as across models. For the sticky-price model, the degrees of real rigidity we estimate lie close to the values discussed in the literature, which range from 0.11 (Reis (2006)) to 0.17 (Chari et al. (2000)), in four countries, the US, Germany, France, and Canada.[11] For the sticky-information model, much less real rigidities are estimated. This is also true for the sticky-price case for the UK and Japan where model accuracy is anyway low.

Not surprisingly, using the estimated values for α improves model accuracy. Improvements are smaller for sticky prices where accuracy was better under the baseline parametrization. Under the estimated parametrizations, the two models perform similarly. Sticky information produces a higher likelihood of the observed moments in four cases. Sticky prices performs better in two cases. However, in order to produce this adequate fit, the sticky-information model has to be quantified in an unusual – and possibly unrealistic – way.

The results show that for the United States, Germany, France, and Canada the estimated α is not far away from the value of $\alpha = 0.11$ that used in the literature. Even so α could take any value between zero and two.

The impact of parameter variations on the distribution of specific moments are illustrated in Figures 4 and 5.[12] In Figure 4, the dashed vertical lines again represent empirical observations. The thin curves are distributions of the sticky-information model predictions under different values for the parameter α. The lighter the line, the higher the value for α. Figure 5 is the counterpart of Figure 4 for the sticky-price model.

The figures shows for which moments model predictions are sensitive to changes in the calibration. In general, correlations increase with falling rigidities, α. However, increasing α lowers the predicted auto-correlation of inflation and thus helps matching this moment. For the sticky-information model, the relative large estimated value, $\alpha_{US,SI} = 1.06$, thus reflects the relatively well-matched auto-correlation under this parametrization. By contrast, in the sticky-price case, the estimated value, $\alpha_{US,SP} = 0.14$, is much lower since the correlation of inflation and lagged nominal income growth is well-matched

[11] In the Mankiw and Reis (2002) version of the two Phillips curves we use α is a combination of the mark-up power of monopolistic firms θ, the labor-supply elasticity of real wages ψ, and the income elasticity of real wages σ, $\alpha = \frac{\sigma+\psi}{1+\theta\nu}$. Chari, Kehoe and McGrattan 2000 offer a quantification of these structural parameters which results in the stated value $\alpha = 0.17$.

[12] For reasons of brevity, we report the results only for the US. The results for the other countries are similar and available on request.

	sticky information		sticky prices	
	α	$\Pi_{x\in X}F(X)$	α	$\Pi_{x\in X}F(X)$
United States	1.0639	$2.78 \cdot 10^{-12}$	0.1438	$3.86 \cdot 10^{-17}$
	0.1100	$< 1 \cdot 10^{-323}$	0.1100	$3.04 \cdot 10^{-20}$
United Kingdom	0.4268	$1.26 \cdot 10^{-130}$	1.9154	$3.69 \cdot 10^{-135}$
	0.1100	$< 1 \cdot 10^{-323}$	0.1100	$1.48 \cdot 10^{-187}$
Germany	0.4308	$5.34 \cdot 10^{-3}$	0.2459	$1.10 \cdot 10^{-13}$
	0.1100	$3.02 \cdot 10^{-40}$	0.1100	$2.55 \cdot 10^{-15}$
France	3.8879	$1.23 \cdot 10^{-37}$	0.1024	$6.29 \cdot 10^{-48}$
	0.1100	$< 1 \cdot 10^{-323}$	0.1100	$1.04 \cdot 10^{-52}$
Canada	1.4100	$1.87 \cdot 10^{-57}$	0.0789	$1.71 \cdot 10^{-52}$
	0.1100	$< 1 \cdot 10^{-323}$	0.1100	$3.98 \cdot 10^{-54}$
Japan	1.7227	$9.18 \cdot 10^{-111}$	1.7070	$3.17 \cdot 10^{-38}$
	0.1100	$< 1 \cdot 10^{-323}$	0.1100	$6.29 \cdot 10^{-100}$

Table 4: Estimated values for α and likelihood of observation under estimated and baseline parametrization.

with low real rigidities.

5 Conclusion

The analysis in this paper has provided an empirical comparison of the sticky-price and sticky-information Phillips curves on the basis of second moments of inflation. It built on the work of Reis (2006) who has shown that the sticky-information Phillips curve performs reasonably well even in a very simple model.

The quantitative analysis of Reis has been augmented in several respects in this paper. First, we considered five more countries, the UK, Germany, France, Canada, and Japan. Second, we also studied a model with a sticky-price Phillips curve and compared the models' empirical performances. We third extended the analysis methodologically by comparing not only absolute deviations between model moments and empirical observations but also evaluating the likelihood of the observed moments given the two models. We finally used the models to estimate the degrees of real rigidity in several coun-

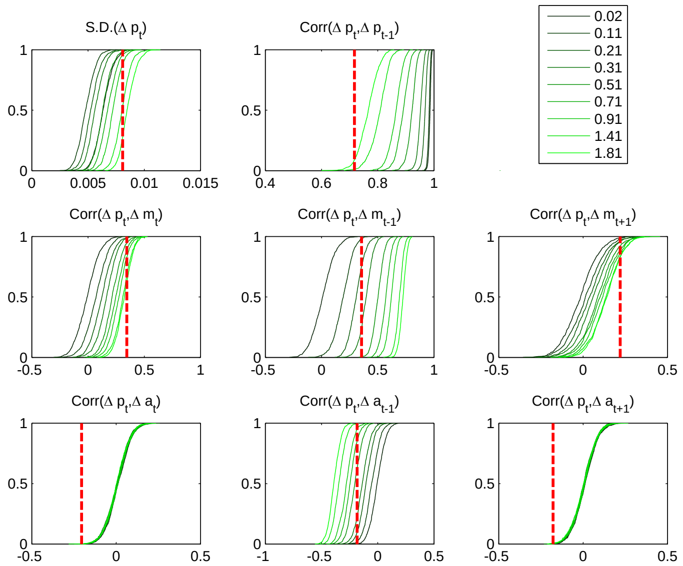

Figure 4: Distribution of moments in simulations of the sticky information model with varying α for the United States.

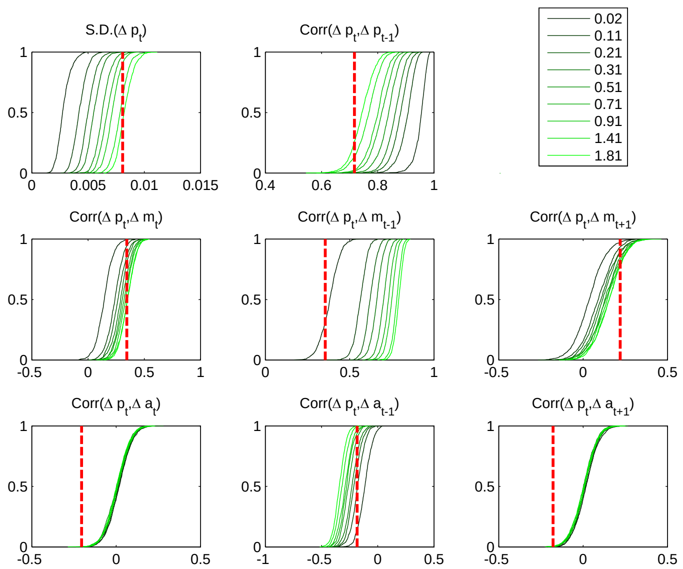

Figure 5: Distribution of moments in simulations of the sticky-price model with varying α for the United States.

tries on the basis of second moments of inflation using a Maximum-Likelihood approach.

Our results confirmed the relative closeness of the sticky-information model to empirical second moments. In an extended US sample, we found results that are similar to those of Reis (2006). Comparing the two models put the empirical success of the sticky-information model into perspective. The absolute deviation of model moments to data moments was lower for sticky information in all countries. However, the joint density of the observed moments was higher for sticky prices, also in all countries.

Considering specific countries, the sticky-price model was capable of matching second moments of US and German inflation reasonably well, sticky information performed reasonably in these countries. No model performed well for UK inflation dynamics. French, Canadian, and Japanese inflation dynamics had been matched poorly by the sticky-price model, whereas sticky information performed even worse.

Considering specific moments, it was a robust finding across countries that both model were especially successful in generating cross-correlations of inflation to productivity and demand that were close to empirical observations. However, our analysis suggested that both models had difficulties to generate an autocorrelation of inflation in the magnitude found in the data. Both models overpredict the autocorrelation substantially in all countries.

By calculating the degree of real rigidities using the sticky-information model we estimated rather large values for α (low rigidity), especially for France. By contrast, using the sticky-price model we, in general, estimated values in the magnitude discussed in the literature.

References

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. *The Quarterly Journal of Economics 118*(1), 269–298.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2000). Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem? *Econometrica 68*(5), 1151–1180.

Döpke, J., J. Dovern, U. Fritsche, and J. Slacalek (2008a). The dynamics of european inflation expectations. *Topics in Macroeconomics 8*(1), 1–21.

Döpke, J., J. Dovern, U. Fritsche, and J. Slacalek (2008b). Sticky information phillips curves: European evidence. *Journal of Money, Credit and Banking 40*(7), 1513–1520.

Gali, J., M. Gertler, and J. David Lopez-Salido (2005). Robustness of the estimates of the hybrid new keynesian phillips curve. *Journal of Monetary Economics 52*(6), 1107–1118.

Khan, H. and Z. Zhu (2002). Estimates of the sticky-information phillips curve for the united states, canada, and the united kingdom. Bank of Canada Working Paper 02-19.

Kiley, M. T. (2007). A quantitative comparison of sticky-price and sticky-information models of price setting. *Journal of Money, Credit and Banking 39*(s1), 101–125.

Korenok, O. (2005). Empirical comparison of sticky price and sticky information models. Macroeconomics, EconWPA.

Mankiw, N. G. and R. Reis (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve. *The Quarterly Journal of Economics 117*(4), 1295–1328.

Mankiw, N. G. and R. Reis (2010). Imperfect information and aggregate supply. In B. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*. Elsevier-North Holland. Forthcoming.

Paustian, M. and E. Pytlarczyk (2006). Sticky contracts or sticky information? Evidence from an estimated euro area DSGE model. *mimeo.*

Reis, R. (2006). Inattentive producers. *Review of Economic Studies 73*(3), 793–821.

Sbordone, A. M. (2005). Do expected future marginal costs drive inflation dynamics? *Journal of Monetary Economics 52*(6), 1183–1197.

Trabandt, M. (2007). Sticky information vs. sticky prices: A horse race in a DSGE framework. Working Paper Series 209, Sveriges Riksbank (Central Bank of Sweden).

A Appendix

A.1 Model Solution

A.1.1 Sticky Information

We start from the Sticky-information Phillips curve (2). In this appendix, we solve for the coefficients on Δm_t, the solution for the coefficients on Δa_t is equivalent except for the opposite sign. We solve for coefficients on Δm_t using the method of undetermined coefficients. For convenience, we assume $\Delta a_{t+i} = a_{t+i} = 0 \forall i$. Our guessed solution for inflation (5) then simplifies to

$$\pi_t = \sum_{i=0}^{\infty} \gamma_i^{SI} \varepsilon_{t-i}^m. \tag{11}$$

Plugging the solution for inflation in (2) yields:

$$\sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m = \left[\frac{\alpha\lambda}{1-\lambda}\right] y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}\left(\sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m + \alpha \Delta y_t\right)$$

Thus expressions for the log output gap and the log output gap growth are needed. Using the definition of the output gap, the MA representation of nominal income growth (4),

$$\Delta m_t = \sum_{i=0}^{\infty} \rho_i \varepsilon_{t-i}^m,$$

and the assumption of $\Delta a_{t+i} = a_{t+i} = 0 \forall i$ gives an expression for the log output gap growth:

$$\begin{aligned} \Delta y_t &= \Delta m_t - \Delta p_t \\ &= \sum_{i=0}^{\infty} \rho_i \epsilon_{t-i}^m - \sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m \end{aligned} \tag{12}$$

The log output y_t can be described by using equation (12) as:

$$\begin{aligned} y_t &= \sum_{i=0}^{\infty} \rho_i \epsilon_{t-i}^m - \sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m + y_{t-1} \\ &= \sum_{i=0}^{\infty} \rho_i \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m \right] - \sum_{i=0}^{\infty} \gamma_i^{SI} \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m \right] \end{aligned} \tag{13}$$

Substituting (12) and (13) into the Philips curve (2):

$$\begin{aligned} \sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m &= \left[\frac{\alpha\lambda}{1-\lambda} \right] \left\{ \sum_{i=0}^{\infty} \rho_i \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m \right] - \sum_{i=0}^{\infty} \gamma_i^{SI} \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m \right] \right\} \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} \left\{ \sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m + \alpha \left[\sum_{i=0}^{\infty} \rho_i \epsilon_{t-i}^m - \sum_{i=0}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m \right] \right\} \\ &= \left[\frac{\alpha\lambda}{1-\lambda} \right] \left\{ \sum_{i=0}^{\infty} \rho_i \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m \right] - \sum_{i=0}^{\infty} \gamma_i^{SI} \left[\sum_{k=0}^{\infty} \epsilon_{t-k-i}^m \right] \right\} \\ &\quad + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \left\{ (1-\alpha) \sum_{i=j+1}^{\infty} \gamma_i^{SI} \epsilon_{t-i}^m + \alpha \sum_{i=j+1}^{\infty} \rho_i \epsilon_{t-i}^m \right\} \end{aligned} \tag{14}$$

Because (14) must hold for all possible realizations of ε_{t-j-k}^m, we can use $\varepsilon_t^m = 1$, $\varepsilon_{t-u}^m = 0 \forall u > 0$ to determine the coefficient γ_j^{SI}. Under this realization, equation (14) simplifies to:

$$\begin{aligned}
\gamma_0^{SI} &= \left[\frac{\alpha\lambda}{1-\lambda}\right]\left\{\rho_0 - \gamma_0^{SI}\right\} \\
&= \left[\frac{\alpha\lambda}{1-\lambda}\right]\left\{1 - \gamma_0^{SI}\right\} \\
&\Leftrightarrow \;\gamma_0^{SI} = \left[\frac{\alpha\lambda}{1-\lambda+\alpha\lambda}\right]
\end{aligned}$$

For a general v, we use the realization $\varepsilon_{t-k}^m = 1$, $\varepsilon_{t-u}^m = 0 \forall u \neq k$ for which (14) becomes:

$$\begin{aligned}
\gamma_k^{SI} &= \left[\frac{\alpha\lambda}{1-\lambda}\right]\left\{\sum_{i=0}^{k}\rho_i - \sum_{i=0}^{k}\gamma_i^{SI}\right\} + \lambda\sum_{i=0}^{k-1}(1-\lambda)^j\left\{(1-\alpha)\gamma_k^{SI} + \alpha\rho_k\right\} \\
&= \left[\frac{\alpha\lambda}{1-\lambda}\right]\left\{\sum_{i=0}^{k}\rho_i - \sum_{i=0}^{k}\gamma_i^{SI}\right\} + \lambda\left\{(1-\alpha)\gamma_k^{SI} + \alpha\rho_k\right\}\cdot\sum_{i=0}^{k-1}(1-\lambda)^i \\
\gamma_k^{SI} &= \alpha\lambda\left(1-\lambda(1-\alpha)\sum_{i=0}^{k}(1-\lambda)^i\right)^{-1}\cdot\left[1-\sum_{i=0}^{k-1}\gamma_i^{SI} + \sum_{i=1}^{k}\rho_i + \rho_k\sum_{i=1}^{k}(1-\lambda)^i\right]
\end{aligned}$$

A.1.2 Sticky Prices

We start from the following representation of the Sticky-price Phillips curve,

$$p_t = \theta p_{t-1} + (1-\theta)^2\sum_{i=0}^{\infty}\theta^i E_t\left(m_{t+i} - a_{t+i}\right), \tag{15}$$

which is equation (A13) from Mankiw and Reis (2002) extended with a non-constant log productivity a_t. In this appendix, we solve for the coefficients on Δm_t, the solution for the coefficients on Δa_t is once again equivalent except for the opposite sign.

We solve for coefficients on Δm_t using the method of undetermined coefficients. For convenience, we assume $\Delta a_{t+i} = a_{t+i} = 0 \forall i$. Our guessed solution for inflation (5) then simplifies to

$$\pi_t = \sum_{i=0}^{\infty}\gamma_i^{SP}\varepsilon_{t-i}^m. \tag{16}$$

We also use the MA representation of nominal income growth (4). Getting rid of the difference operator by backward iteration yields

$$p_t = \sum_{j=0}^{\infty} \gamma_j^{SP} \sum_{k=0}^{\infty} \varepsilon_{t-j-k}^m \tag{17}$$

$$m_t = \sum_{j=0}^{\infty} \rho_j \sum_{k=0}^{\infty} \varepsilon_{t-j-k}^m \tag{18}$$

Plugging (17) and (18) into (15) gives

$$\sum_{j=0}^{\infty} \gamma_j^{SP} \sum_{k=0}^{\infty} \varepsilon_{t-j-k}^m = \theta \sum_{j=0}^{\infty} \gamma_j^{SP} \sum_{k=0}^{\infty} \varepsilon_{t-j-k-1}^m + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i E_t \sum_{j=0}^{\infty} \rho_j \sum_{k=0}^{\infty} \varepsilon_{t-j-k+i}^m$$

which can be simplified to

$$\sum_{j=0}^{\infty} \gamma_j^{SP} \sum_{k=0}^{\infty} \varepsilon_{t-j-k}^m = \theta \sum_{j=0}^{\infty} \gamma_j^{SP} \sum_{k=0}^{\infty} \varepsilon_{t-j-k-1}^m + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{\infty} \rho_j \sum_{k=\max(i-j,0)}^{\infty} \varepsilon_{t-j-k+i}^m. \tag{19}$$

Using matching coefficients as described in the preceding section (use the realization $\varepsilon_t^m = 1$, $\varepsilon_{t-u}^m = 0 \forall u > 0$ in (19)) yields for γ_0^{SP}:

$$\begin{aligned}
\gamma_0^{SP} &= 0 + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{i} \rho_j \\
&= (1-\theta)^2 \sum_{i=0}^{\infty} \rho_i \sum_{j=i}^{\infty} \theta^j \\
&= (1-\theta)^2 \sum_{i=0}^{\infty} \rho_i \left\{ \sum_{j=0}^{\infty} \theta^j - \sum_{j=0}^{i-1} \theta^j \right\} \\
&= (1-\theta)^2 \sum_{i=0}^{\infty} \rho_i \left\{ \frac{1}{1-\theta} - \frac{\theta^i - 1}{\theta - 1} \right\} \\
&= (1-\theta)^2 \sum_{i=0}^{\infty} \rho_i \cdot \frac{-\theta^i}{\theta - 1} \\
&= (1-\theta) \sum_{i=0}^{\infty} \theta^i \rho_i.
\end{aligned}$$

and for γ_j^{SP} (using $\varepsilon_{t-j}^m = 1$, $\varepsilon_{t-u}^m = 0 \forall u \neq j$ in (19))

$$\begin{aligned}
\sum_{j=0}^{v} \gamma_j^{SP} &= \theta \sum_{j=0}^{v-1} \gamma_j^{SP} + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{v+i} \rho_j \\
&\Leftrightarrow \gamma_v^{SP} + \sum_{j=0}^{v-1} \gamma_j^{SP} = \theta \sum_{j=0}^{v-1} \gamma_j^{SP} + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{v+i} \rho_j \\
&\Leftrightarrow \gamma_v^{SP} = (\theta - 1) \sum_{j=0}^{v-1} \gamma_j^{SP} + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{v+i} \rho_j \qquad (20)
\end{aligned}$$

The double sum $\sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{v+i} \rho_j$ at the right hand side of (20) can be expressed as follows:

$$\begin{aligned}
\sum_{i=0}^{\infty} \theta^i \sum_{j=0}^{v+i} \rho_j &= \sum_{i=0}^{\infty} \rho_i \sum_{j=\max(0,i-v)}^{\infty} \theta^j \\
&= \sum_{i=0}^{\infty} \rho_i \left\{ \sum_{j=0}^{\infty} \theta^j - \sum_{j=0}^{i-v-1} \theta^j \right\} \\
&= \sum_{i=0}^{\infty} \rho_i \left\{ \frac{1}{1-\theta} - \max\left(\frac{\theta^{i-v} - 1}{\theta - 1}, 0 \right) \right\} \\
&= \frac{1}{1-\theta} \sum_{i=0}^{\infty} \rho_i - \sum_{i=v}^{\infty} \rho_i \left(\frac{\theta^{i-v} - 1}{\theta - 1} \right) \\
&= \frac{1}{1-\theta} \left[\sum_{i=0}^{\infty} \rho_i - \sum_{i=v}^{\infty} \rho_i \left(1 - \theta^{i-v}\right) \right] \\
&= \frac{1}{1-\theta} \left[\sum_{i=0}^{\infty} \rho_i - \sum_{i=v}^{\infty} \rho_i + \sum_{i=v}^{\infty} \rho_i \theta^{i-v} \right] \\
&= \frac{1}{1-\theta} \left[\sum_{i=0}^{v-1} \rho_i + \sum_{i=v}^{\infty} \rho_i \theta^{i-v} \right]
\end{aligned}$$

Using this, (20) becomes

$$\begin{aligned}\gamma_v^{SP} &= (\theta-1)\sum_{j=0}^{v-1}\gamma_j^{SP} - (\theta-1)\left[\sum_{i=0}^{v-1}\rho_i + \sum_{i=v}^{\infty}\rho_i\theta^{i-v}\right] \\ &\Leftrightarrow \ \gamma_v^{SP} = (\theta-1)\left\{\sum_{j=0}^{v-1}\gamma_j^{SP} - \sum_{i=0}^{v-1}\rho_i - \sum_{i=v}^{\infty}\rho_i\theta^{i-v}\right\}.\end{aligned}$$

A.2 Data Appendix

For each country, we use data on nominal GDP, the price level, and productivity. Table 5 provides sources and details on the data used.

Country	Nominal GDP	CPI	Productivity	Sample period
US	Bureau of Economic Analysis; Table 1.1.5. Gross Domestic Product [Billions of dollars]; Seasonally adjusted at annual rates	Bureau of Labor Statistics; Series Id: CUUR0000SA0; Not Seasonally Adjusted Area: U.S. city average Item: All items; Base Period: 1982-84=100	Bureau of Labor Statistics; Output per Person; Nonfarming Sector; 1992=100	1947 to 2008
UK	Office for National Statistics UK; ABMI; Gross Domestic Product; Chained volume measures; Seasonally adjusted; Constant 2003 prices	OECD; Index 2005=100	Office for National Statistics UK; A4YM; Output per Worker; Whole Economy SA; Index 2003=100; Seasonally adjusted	1959 to 2008
Germany	Bundesamt für Statistik; before 1990 West Germany; linear extrapolation of growth rate in 1990Q1	OECD; Index 2005=100	Bundesbank; Productivity per hour; Seasonally adjusted; Index 1995=100	1970 to 2008
France	INSEE National Institute of Statistics and Economic Studies	OECD; Index 2005=100	National Institute of Statistics and Economic Studies; GDP per employed person	1978 to 2008
Canada	Datastream	OECD; Index 2005=100	Cansim; Labour productivity; Total economy	1981 to 2008
Japan	DSI Data Service; Seasonally adjusted	OECD; Index 2005=100	Datastream; Labour productivity; Total economy	1970 to 2008

Table 5: Sample periods, data sources, and details.