Central Bank Transparency Under Model Uncertainty*

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Abstract

This paper explores the effects of central bank transparency on the performance of optimal inflation targeting rules. I assume that both the central bank and the private sector face uncertainty about the 'correct' model of the economy and have to learn. A transparent central bank can reduce one source of uncertinty for the private agents by communicating its policy rule to the public.

The paper shows that central bank transparency plays a crucial role in stabilizing the agents' learning process and expectations. On the contrary, lack of transparency can lead to expectations-driven fluctuations with destabilizing effects on the economy, even in the case where the central bank adopts optimal policies.
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> A central Bank that is inscrutable gives the markets little or no way to ground these perceptions [about monetary policy] in any underlying reality – thereby opening the door to expectational bubbles that can make the effects of its policies hard to predict. (Blinder, 1998)

1 Introduction

Since the 1990's, central banking practice has shifted from secrecy and opaqueness toward more transparency about monetary policy strategy and objectives. At the same time, an increasing number of central banks have adopted an inflation targeting framework for monetary policy. One important aspect of inflation targeting is the anchoring of expectations,[1] with its stabilizing effect on macroeconomic activity. Failing to anchor expectations might result in undesired fluctuations and economic instability. Given the role of expectations, central banks' credibility and transparency is a crucial ingredient of inflation targeting.

Most of the academic research on inflation targeting assumes rational expectations and perfect information on the part of market participants and the monetary authority. Under this assumption, it is not possible to evaluate the benefits of central bank transparency because the market participants can perfectly predict the central bank's decisions.

In contrast, this paper proposes an evaluation of central bank transparency assuming that both the private agents and the monetary authority face imperfect knowledge about the economic environment and have to learn about the correct model of the economy. I distinguish between uncertainty about the model of the economy and uncertainty about monetary policy. Uncertainty about monetary policy concerns the behavior of the short-term nominal interest rate, which depends on the central bank's objectives, its monetary policy strategy and its information about the economy. A transparent central bank reduces this source of uncertainty by providing information about its policy. Uncertainty about the model concerns the laws of motion for inflation and output, which are affected by other factors of the model beyond monetary policy decisions. In fact, even perfect knowledge about the central bank policy framework does not guarantee that market participants fully understand the 'true' model of the economy, especially in the case of decentralized markets, where agents face uncertainty about each others' tastes, production possibilities and about the effects of aggregate shocks.

A modeling approach with imperfect knowledge allows me to address two critical questions. First, to what extent a transparent implementation of inflation targeting affects the private agents' learning process and thus the dynamic properties of the economy under learning. In the words of governor Bernanke:[2]

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[1] See Levin, Natalucci and Piger (2003).

[2] Bernanke (2004), p.3.

> A skeptic might argue that noise and other sources of pricing inefficiency pervade the financial markets, so that improving the predictability of monetary policy is of limited importance in practice [...]. To the contrary, there is good reason to believe that informational asymmetries between the central bank and the financial markets may matter a great deal for economic welfare.

Second, what are the effects of central bank transparency on the monetary policy transmission mechanism and thus monetary policy effectiveness. Again, from Bernanke

> The fact that market expectations of future settings of the federal funds rate are at least as important as the current value of the funds rate [...] suggests a potentially important role for central bank communication: If effective communication can help financial markets develop more accurate expectations of the likely future course of the funds rate, policy will be more effective [...][3]

In this paper I study a simple sticky price model where optimal policy takes the form of an inflation targeting rule and I evaluate its properties under rational expectations and when agents learn over time. In the latter case, I consider both a transparent and a non-transparent central bank, where central bank transparency is measured by the public's knowledge of the policy rule.

The criterion used to evaluate the performance of the optimal policy rule is whether it induces stability under learning. I define the optimal policy rule as 'robust' to expectational mistakes if it gives a satisfactory performance also when expectations are out of the (rational expectations) equilibrium because of structural change. In particular, I consider two aspects of stability. First, I consider whether agents' expectations converge to the rational expectations equilibrium. Second, in the case of convergence, I explore how the agents' learning process might alter the transmission mechanism and, ultimately, the effectivenness of monetary policy.

I show that lack of transparency has profound consequences on the performance of optimal monetary policy under imperfect information and learning. While under the hypothesis of rational expectations, the optimal policy rule succeeds in stabilizing expectations - the REE is unique, i.e. no expectaions-driven equilibria exist under the optimal targeting rule-, under the hypothesis of imperfect information lack of transparency leads to economic instability and self-fulfilling expectations. The central bank's response to changes in expectations and macroeconomic disturbances induces undesired fluctuations in agents' beliefs which in turn affect the economy.

On the other hand, a transparent implementation of optimal policy stabilizes the economy even if economic agents face imperfect information about the economic system. As a result, the simple model shows that lack of transparency alone can be the source of expectation-driven fluctuations.

In the existing literature,[4] the benefits of central bank transparency are discussed in models with rational expectations assumptions. The central bank faces a time-inconsistency problem that

[3] Bernanke (2004), p. 2.

[4] Extensively surveyed in Geraats (2002).

results in inflation bias. The public is assumed to know the monetary policy rule and faces parametric uncertainty about central bank's preferences and about economic shocks. In this framework transparency is a reputation building device. If the central bank discloses its goals and incentives, the optimal policy prescribes low inflation because an expansionary policy would not succeed in increasing employment and would just increase inflation expectations. But, as shown in Faust and Svensson (2000), eliminating the inflation bias makes it optimal for the central bank to be opaque about its preferences.

In contrast, this paper offers a formal argument for central bank transparency that does not rely on the existence of a time-inconsistency problem or informational asymmetries about economic data. Even in the case where the central bank intends to maximize the representative consumer's welfare and its not subject to inflation bias, lack of transparency is destabilizing. This occurs because in this model the agents face different sources of uncertainty and because lack of transparency implies that the agents do not know the policy rule.

Recent contributes[5] emphasize the effects of learning on policy design but do not make an explicit distinction between uncertainty about the economic environment and uncertainty about the policy strategy. Thus, they do not study the effects of improved central bank communication under imperfect knowledge.

The paper is organized as follows. The first section introduces the model and the second describes the properties of the rational expectations equilibrium. In the third section I introduce the agents' learning process and discuss the main results. In the last section I consider two extensions: how central bank transparency affects history dependence of the optimal policy rule and what are the effects of introducing a cost channel of monetary policy.

2 The Model

I consider a simple microfunded general equilibrium model with nominal rigidities. The economy is populated by a continuum of identical consumers and producers and by a monetary authority. Households consume differentiated goods, accumulate assets and supply labor. Each firm produces a differentiated good that is sold in monopolistically competitive markets. The central bank sets the nominal interest rate with the objective to maximize households' welfare. The next sub-sections describe the agents' problems.

2.1 Consumers

Each household i seeks to maximize the value of the sum of future expected utilities of the form

[5] See for example Evans and Honkapohja (2002, 2003), Bullard and Mitra (2002), Howitt (1992) and Orphanides and Williams (2003a,b).

$$\hat{E}_{t-1}^{i}\sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{\Gamma_{s}\left(C_{s}^{i}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(h_{s}^{i}\right)^{1+\chi}}{1+\chi}\right]$$

where C_t denotes the consumption good, h_t^i denotes the amount of hours worked and Γ_s is a preference shock. The consumption good is an aggregate of a continuum of differentiated goods $C_{j,t}$ each produced by firm j, defined as

$$C_{t}^{i}=\left[\int_{0}^{1}\left(C_{j,t}^{i}\right)^{\frac{\eta_{t}}{1-\eta_{t}}}dj\right]^{\frac{\eta_{t}}{1-\eta_{t}}}$$

where $\eta_t > 1$ denotes the elasticity of substitution among the goods. It is assumed to vary stochastically around a constant value $\bar{\eta}$.

The expectation operator $\hat{E}_{t-1}^{i}$ denotes the subjective beliefs of agent j about the probability distribution of the model's state variables. I assume that agents take decisions for time t consumption, on the basis of $t-1$ information.[6]

Financial markets are incomplete, and the only non-monetary asset that is possible to trade is a one period riskless bond. The agents' flow budget constraint is

$$B_{t}^{i}\leq R_{t-1}B_{t-1}^{i}+W_{t}h_{t}^{i}+P_{t}\phi_{s}^{i}-P_{t}C_{t}^{i}$$

where B_t^i denotes the riskless bond, R_t denotes the gross interest paid on the bond and ϕ_t^i is the household share of the firms' profits.

The consumer intertemporal problem is then to choose a sequence for C_t^i, B_t^i, and h_t^i to maximize the intertemporal utility and satisfy the flow budget constraint and the transversality condition

$$\lim_{s\to\infty}\prod_{k=1}^{s}\frac{1}{R_{t+k}}B_{t+s+1}=0$$

where, taking as given R_t, W_t, ϕ_t^i and P_t and given an initial zero wealth, i.e. $B_0 = 0$.

2.2 Firms

There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the production function

$$Y_{t}=h_{jt}^{1-\theta}$$

where for simplicity I assume no technology shocks. I also assume that the firm has to pay a fraction of the wage bill before selling output. The cash in advance constraint is

$$\frac{M_{t}}{P_{t}}=\nu\frac{W_{t}}{P_{t}}h_{jt}$$

[6] This can be interpreted in two ways: either the agents plan their consumption in advance or they act on the basis of old information.

Each firm is assumed to maximize its expected profits:

$$\hat{E}_{t-1} \sum_{s=t}^{\infty} Q_{t,s} P_s \phi_{js}$$

where $Q_{t,s}$ denotes the stochastic discount factor, and

$$\phi_{jt} = \frac{P_{jt}}{P_t} C_{jt} - w_t h_{jt} \left[1 + \nu \left(1 - R_t^{-1}\right)\right] - \frac{\psi}{2} \left(\frac{P_{jt}}{P_{jt-1}} - 1\right)^2 - T_t$$

denotes the profits flow, where $\nu \left(1 - R_s^{-1}\right)$ denotes the opportunity cost of holding money,[7] the quadratic term denotes the cost of adjusting prices[8] as in Rotemberg (1983) and T_t denotes lump sum taxes. The intratemporal consumer problem gives the following aggregate demand for each differentiated good is

$$C_{jt} = \left(\frac{P_{j,t}}{P_t}\right)^{-\eta_t} C_t$$

where

$$P_t = \left[\int_0^1 \left(P_{j,t}\right)^{1-\eta_t} dj\right]^{\frac{1}{1-\eta_t}}$$

is the price index.

Summing up, the firm chooses a sequence for P_{jt}, h_{jt} and M_t so to maximize profits, given the constraint that demand should be satisfied at the posted price and the cash in advance constraint, taking as given P_t, C_t, R_t, T_t and W_t.

2.3 The Policy-Maker

The fiscal authority does not play a fundamental role in the model: assuming a zero net supply of government bonds, the government budget constraint becomes

$$T_t = M_{t-1} - M_t.$$

The description of the monetary policy authority follows Woodford (2003) and Svensson and Woodford (2003). The monetary authority sets its policy instrument, the nominal interest rate, in order to maximize the representative consumer's welfare. The optimal policy is implemented adopting a targeting rule.

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[7] For the derivation of the profit function, with a different price rigidity see Schmitt-Grohe' and Uribe (2004).

[8] Most of the results in the paper are based on the linearized REE solution of the model. Those results are exactly the same in the case of a Calvo pricing model.

3 The Rational Expectations Equilibrium

3.1 Private Sector

Log-linearization of the model's solution leads to the following equations describing the aggregate behavior of the economy close to the non-stochastic steady state. The 'demand side' of the economy derived from the consumes' Euler equation is described by

$$x_t = \sigma^{-1}\left(\beta^{-1} - 1\right) - \sigma^{-1} E^*_{t-1}\left(i_t - \pi_{t+1}\right) + E^*_{t-1} x_{t+1} + \sigma^{-1} \hat{r}^n_t \tag{1}$$

where the operator E^*_{t-1} denotes rational expectations and.x_t denotes the output gap, defined as the difference between output and the *efficient* level of output, that is the the equilibrium level of output in absence of price rigidity and mark-up shocks. The efficient rate of interest[9] $\hat{r}^n_t$, consistent with price flexibility, is expressed in deviations from the equilibrium real interest rate in absence of shocks $\left(\beta^{-1} - 1\right)$. It is assumed to be exogenous and AR(1). The most important feature of this equation is the role of expectations about the nominal interest rate. Predetermined consumption decisions imply that aggregate demand depends only on *expected* monetary policy decisions.

Firms' pricing decisions result in the Phillips curve

$$\pi_t = E^*_{t-1} \beta \pi_{t+1} + \xi E^*_{t-1}\left(\kappa x_t + \zeta \hat{\imath}_t\right) + u_t \tag{2}$$

where $\kappa = \left(\frac{\theta+\chi}{(1-\theta)} + \sigma\right)$, $\zeta = \frac{\beta\nu}{[1+(1-\beta)\nu]}$ and u_t is a cost-push shock dependent on time-varying mark-ups, assumed to be AR(1). Provided $\nu \neq 0$, the model displays a cost channel of monetary policy. In this case firms' marginal cost depends on both aggregate output and the nominal interest rate. As firms set their prices in advance, inflation depends on both expected aggregate demand and monetary policy actions.

Summing up, the model displays a demand and a supply channels by which the monetary authority affects the economy. To emphasize the role of central bank transparency and keep the analysis as simple as possible, I assume that the monetary policy affects output and inflation only influencing expectations. In fact, it could be argued that expectations about future policy actions are the most important channel for monetary policy.[10]

3.2 Optimal Monetary Policy Under Discretion

The central bank sets the interest rate in order to minimize its loss function, given the aggregate behavior of output-gap and inflation. In this section I assume optimal policy under discretion, in

[9] The efficient rate of interest and the efficient level of output are described in the Appendix.

[10] As in Woodford (2003): "Not only expectations about policy matter, but, at least under current conditions, very little else matters".

order to keep the analysis as simple as possible. In the last section I consider optimal policy under the timeless perspective, as proposed in Woodford (2003).

Optimal policy requires the minimization of the following intertemporal loss function

$$L = \frac{1}{2}\hat{E}_{t-1}^{CB}\sum_{s=0}^{\infty}\left[(\pi_{t+s})^2 + \lambda_x x_{t+s}^2\right]$$

where the parameter λ_x defines the preferences of the current central bank for output-gap stabilization. The coefficient λ_x is derived from a second order approximation of the consumer's utility function, as in Woodford (2003). The central bank constraints are represented by the aggregate equations (1) and (2). First order conditions give the following targeting rule

$$E_{t-1}^{*}\pi_t + \frac{\lambda_x}{\xi\left(\kappa - \sigma\zeta\right)}E_{t-1}^{*}x_t = 0 \tag{3}$$

where $\lambda_x = \kappa\zeta/\bar{\eta}$ and $(\kappa - \sigma\zeta) > 0$. The rule prescribes the restriction that the central bank forecasts must satisfy in order to maximize the loss function. The monetary policy authority implements (3) by using a simple instrument rule, dictating that the nominal interest rate should be set as

$$i_t = \left(\beta^{-1} - 1\right) + \phi\left(E_{t-1}^{*}\pi_t + \bar{\lambda}_x E_{t-1}^{*}x_t\right) + \epsilon_t \tag{4}$$

where $\bar{\lambda}_x = \frac{\lambda_x}{\xi(\kappa-\sigma\zeta)}$ and ϵ_t is a control error, assumed i.i.d. Finally, ϕ determines the out-of-equilibrium response of the central bank. The central bank's reaction function is 'operative' in the sense that the actual output-gap and inflation are not available when setting the interest rate. Therefore, the monetary authority needs to produce a forecast for output-gap and inflation one period in advance. Svensson and Woodford (2003) show that (4) does not coincide exactly with the optimal policy rule, but the simple form of (4) is easy to communicate and to be verified by the private agents, especially under the assumption of imperfect information about the economic environment.

3.3 Properties of the Rational Expectations Equilibrium

The model is described by (1), (2) and (4). The Rational Expectations Equilibrium (REE) solution can be expressed in the form[11]

$$V_t = \Omega^* X_t \tag{5}$$

where $V_t = [x_t, \pi_t, i_t]'$, $X_t = [1, \hat{r}_t^n, u_t]'$ and Ω^* describes the matrix coefficients. The exogenous shock processes evolve as $X_t = HX_{t-1} + \zeta_t$, where H is a diagonal matrix containing the autocorrelation coefficients ρ_r and ρ_u of the efficient interest rate and cost-push shock and where ζ_t is a vector of i.i.d. shocks. The following Proposition shows the restrictions on ϕ which are sufficient for (5) to be the unique rational expectations equilibrium.

[11] This is also the mean styate variables solution.

Proposition 1 *A sufficient condition for a 'globally' unique REE is*

$$0 < \frac{\bar{\eta}}{1 + \bar{\eta} - \theta^{-1}} < \phi < \frac{1}{\xi} \tag{6}$$

Proof. See Appendix. ■

Proposition (1) shows that provided (17) is satisfied, no epectations-driven equilibria exist under rational expectations, including endogenous cycles and sunspots.[12] That is, under the hypothesis of rational expectations (and thus complete symmetry between the central bank and the private agents), the bank is successfully maximizing consumers welfare. Given that $0 < \theta < 1$, a necessary condition for determinacy is that the so called Taylor Principle holds, that is if $\phi > 1$. This preliminary result (the intuition is briefly discussed below) has the important implication of isolating the role of learning and (lack of) central bank communication in generating expectetions-driven 'equilibria' under learning, as discussed in the next sections.

4 Learning and Central Bank Transparency

4.1 Forecasting Output and Inflation

I model the agents' learning process using the methodology of Evans and Honkapohja (2001). Equations (1) and (2) describe the evolution of output and inflation, which depend on aggregate expectations. Agents do not know the true structure of the economic model and, instead, behave as econometricians. They are endowed with a parametric model of the economy that they use for prediction. The agents observe the main economic variables each period and then re-estimate the coefficients in their parametric model. The agent's model is appropriately specified, being consistent with the REE reduced-form solution (5). Convergence to the REE occurs if the agents are eventually able to estimate the correct coefficients Ω^*.

Two important aspects of the learning process need emphasis. First, convergence is not guaranteed because the agents' learning process feeds-back to the observed variables, making the economy a self-referential system. The agents do not take this into account and, as a result, their model is mis-specified during the learning process. This aspect is further discussed below.

Second, as noted in Preston (2002), (1) and (2) are optimal decision rules, given the assumed beliefs and microfundations, only under the assumption of rational expectations. Consumers and producers are therefore assumed to take suboptimal decisions during the learning process. In fact, optimal decision rules depend on infinite horizon forecasts. Nevertheless, the decision rules of the agents *converge asymptotically to the optimal decision rule*, under the assumption that their initial wealth is zero.

[12] For examples where those equilibria exists, see Benhabib, Schmitt-Grote' and Uribe (2001a,b) and Benhabib and Eusepi (2004).

Both the private agents and the central bank use the following model to forecast output gap and inflation[13]

$$x_t = \hat{\omega}_0^x + \hat{\omega}_{r^n}^x \hat{r}_t^n + \hat{\omega}_u^x \hat{u}_t \tag{7}$$

$$\pi_t = \hat{\omega}_0^\pi + \hat{\omega}_{r^n}^\pi \hat{r}_t^n + \hat{\omega}_u^\pi \hat{u}_t \tag{8}$$

where I assume that both the central bank and the private sector observe the economic shocks and a correct measure of the output gap. While they share the same model for prediction, I allow some disagreement between the central bank and the private sector's forecast. This is captured by potentially different estimates of the model's coefficients, reflecting different beliefs about the economic model.[14] The model's equations are estimated recursively using the Recursive Least Squares (RLS) estimator. Expressing the coefficients in matrix notation, each period, the model coefficients are updated according to the following algorithm

$$\hat{\Omega}_t^j = \hat{\Omega}_{t-1}^j + \delta_t R_{t-1}^{-1} X_t \left[(x_t, \pi_t)' - X_t' \hat{\Omega}_{t-1}^j + o_t \right]'$$

$$R_t = R_{t-1} + \delta_t \left(X_t X_t' - R_{t-1} \right)$$

where $\hat{\Omega}_t^j$ denotes the parameters' estimates, and $j = PS, CB$, R_t is the precision matrix and δ_t is a decreasing sequence of gains.[15] The updating equation includes an observational i.i.d. error, o_t that makes the learning process non trivial.

Given that the agents estimate a reduced-form model, its coefficients change as the economy undergoes structural changes, including preference, technology and policy changes. Any structural change requires the agents to adjust their model, irrespective to their knowledge of how monetary policy is conducted. No matter how precise it is the knowledge about the policy rule, *the agents are still uncertain about the economic environment* and thus they cannot properly calculate the effects of the monetary policy on the main economic variables such as output, inflation and the interest rate.[16] Still, once the learning process has converged the model delivers the same forecasts as the true model.

[13] Eusepi (2003) also considers the case where the agents have a more 'structural model' which includes current endogenous variables. In a similar model environment the result are not affected by this choice, while the analysis becomes more complex.

[14] Notice that different beliefs do not reflect asymmetry in the economic data available. Also, in this simplified version of the model, the central bank cannot use its knowledge about the policy strategy for a better prediction of output and inflation.

[15] See Evans and Honkapohja (2001).

[16] This is because the information available to the agents is not enough to recover all the policy-invariant parameters that define the economy. In other words, the model that they estimate is still subject to the Lucas critique, since the parameters change with the monetary policy rule.

In conclusion, knowledge of the policy rule does not eliminate the problem of stability under learning. It is then possible to evaluate whether transparency has effects on the stability under learning of a given policy rule. The next section discusses how the private agents forecast the interest rate and central bank transparency affects the learning process.

4.2 Central Bank Transparency

A general definition of transparency is the "absence of asymmetric information between the monetary policy-makers and other economic agents" (Geraats, 2002).

There are many aspects of transparency,[17] but the paper focuses on the communication of the central bank's policy rule. The degree of transparency is measured by the degree to which central bank's optimal policy rule can be inferred by market participants. As mentioned above, the only source of informational asymmetry concerns the central bank's objective function, its 'forecasting model' and therefore the forecasts used to implement the policy strategy. In this simple model, this reduces to the policy reaction function (4), which requires an adequate understanding of how the central bank formulates its forecasts.

4.2.1 A Non-Transparent Central Bank

Consider first the case of a central bank that is not transparent (or opaque) about its policy strategy. The public is not given enough reliable information about the central bank's policy goals and the strategy to achieve them. Therefore, the only clear and publicly available information about the central bank's intentions is the observed interest rate. In terms of the model, the agents do not know the policy rule (4), i.e. the private agents face uncertainty about; 1) the *form* of the policy rule, 2) what economic variables appear in the reaction function, 3) what variables are in the objective function and with what weights, 4) what is the central bank forecasting model.

Since prior information is not available, the best option for forecasting purposes is to use a reduced-form model. I further assume that the agents' model is (asymptotically) consistent with the rational expectation forecast of the interest rate. Therefore

$$i_t = \psi_0 + \psi_{r^n} \hat{r}_t^n + \psi_u u_t + e_t \tag{9}$$

where $\psi_0, \psi_{r^n}, \psi_u$ are the model coefficients that the agents are estimating.

Although my choice is in some sense arbitrary, it can be defended as follows. The reduced-form model (9) is consistent with many different policy rules. For example, it is consistent with a REE induced by policy rules responding to inflation, expected future inflation, output gap, output and their forecasts. Given the uncertainty interest rate determination, the model (9) is the only one consistent with each of those rules.[18]

[17] For different classifications of transparency see Geraats (2002).

[18] Notice that the agents might want to experiment whether the policy responds to some endogenous variables. But,

4.2.2 Transparency

In the case of a transparent central bank, the agents have valuable information monetary policy decisions. Nevertheless, it would be implausible and to some extent not even desirable to assume that the agents fully understand every aspect of the policy decision process.

I consider the case where market participants receive sufficient information about the *form* of the policy rule and the relevant variables to which the central bank respond, but not the exact value of the parameters ϕ, $\bar{\lambda}_x$ and the constant in the policy rule. Furthermore, the private agents have noisy observations of the central bank's forecasts, i.e. $\tilde{E}_{t-1}^{CB}\pi_t = \hat{E}_{t-1}^{CB}\pi_t + e_{\pi,t-1}$ and $\tilde{E}_{t-1}^{CB}x_t = \hat{E}_{t-1}^{CB}x_t + e_{x,t-1}$, where $\tilde{E}_{t-1}^{CB}$ denotes the private agent's signal and the shocks are *i.i.d.* The above assumptions are meant to capture different aspects of central bank communications. First, uncertainty about parameters and forecasts can be interpreted as a constraint on the communication ability of the central bank. This reflects the fact that the policy decision is the outcome of a complex process, the details of which are often too costly to communicate.[19] Second, the central bank might face credibility issues, so that the private sector intends to verify the announced policies. Third, complete announcement might not be the optimal choice for the central bank, given the agent's learning process.[20] The private agents' model of the central bank's behavior becomes

$$i_t = \phi_0 + \phi_\pi \tilde{E}_{t-1}^{CB}\pi_t + \phi_x \tilde{E}_{t-1}^{CB}x_t + e_t \tag{10}$$

where the private agents estimate over time the coefficients ϕ_0, ϕ_π andϕ_x. Notice that the 'true' value of the parameters is $\phi_0^* = \beta^{-1} - 1$, $\phi_\pi^* = \phi$, $\phi_x^* = \phi\bar{\lambda}_x$. The private agents can thus identify the policy parameters and learn the policy strategy. Nevertheless, for forecasting purposes they only need their estimate of (10).

5 Transparency and Optimal Policy Under Learning

The model of the economy is given by equations (1), (2) and the policy rule (4), where I substitute the rational expectation operator E^* with the, possibly non-rational expectations of the private sector and the central bank. The expectations formation mechanism depends on the degree of central bank transparency and thus on the information available to the public on the policy rule.

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unless, the agents by sheer luck start off with the right model, any other specification would lead to a misspecified model, thus preventing convergence.

[19] See Mishkin (2004).

[20] A discussion of the optimal policy under learning is left for further research.

5.1 A Non-Transparent Central Bank

The agents use the model (7), (8) and (9) to form their expectations, given the estimated coefficients.[21] Inserting the agents' forecasts into the IS, Phillips and policy rule equation I obtain the following the Actual Law of Motion (ALM) of the economic system[22]

$$V_t = T'\left(\hat{\Omega}^{PS}_{t-1}, \hat{\Omega}^{CB}_{t-1}, \psi_{t-1}\right) X_t.$$

Inspection of the ALM shows that during the learning process the agents' model is misspecified. The ALM implies a model with time-varying coefficients. The PLM is a correctly specified model of the economy only asymptotically, if the learning process converges to the REE. The temporary misspecification is the source of self-fulfilling expectations where the learning process does not converge to the REE. The stability condition requires that the mapping between PLM and ALM to be locally stable at the REE, where $T(\Omega^*) = \Omega^*$. From Evans and Honkapohja (2001), the stability under learning of the REE depends on the local stability of the following ODE, describing the agents beliefs in notional time

$$\begin{bmatrix} \dot{\hat{\Omega}}^{PS} \\ \dot{\hat{\Omega}}^{CB} \\ \dot{\psi} \end{bmatrix} = T\left(\hat{\Omega}^{PS}, \hat{\Omega}^{CB}, \psi\right) - \begin{bmatrix} \hat{\Omega}^{PS} \\ \hat{\Omega}^{CB} \\ \psi \end{bmatrix}. \tag{11}$$

The following proposition describes the stability results for the case of the non-transparent central bank. In order to simplify the analysis, I first consider the case where the cost channel of monetary policy is absent from the model. In the last section of the paper I discuss the implications of including of including monetary frictions.[23]

Proposition 2 *Lack of Transparency. Assume that ϕ satisfies (17) and $\nu = 0$.*
(a) The REE is unstable under learning if and only if

$$\xi > \frac{1}{\bar{\eta}\kappa} + \epsilon_\beta \tag{12}$$

where $\epsilon_\beta \to 0$ as $\beta \to 1$.

[21] For example, the private sector forecasts for the inflation rate are,

$$\begin{aligned} \hat{E}^{PS}_{t-1}\pi_t &= \hat{\omega}^{\pi}_{0,t-1} + \hat{\omega}^{\pi}_{r^n,t-1}\hat{r}^n_t + \hat{\omega}^{\pi}_{u,t-1}\hat{u}_t \\ \hat{E}^{PS}_{t-1}\pi_{t+1} &= \hat{\omega}^{\pi}_{0,t-1} + \hat{\omega}^{\pi}_{r^n,t-1}\rho_r\hat{r}^n_t + \hat{\omega}^{\pi}_{u,t-1}\rho_u\hat{u}_t. \end{aligned}$$

See also the Appendix.

[22] Notice that, in order to form expectations at time t, the agents are assumed to use $t-1$ estimates of the coefficients. This is to avoid simulateneity problems.

[23] Eusepi (2003) considers different modeling assumptions of moneay demand and obtains the same results.

(b) If (a) holds, achieving stability under learning requires an excessive weight to output-gap stabilization with respect to the representative agent's loss function;

(c) There exist policy-induced "learning equilibria" where inflation and output fluctuate around the inflation target.

Proof. See Appendix. ■

Consider a sudden increase in inflation expectations. The direct effect is a rise in output, followed by an increase in expected output and expected inflation. In the case of perfect information, the monetary authority responds with a rise in the interest rate which is fully anticipated by the private agents. As a result, inflation and output expectations revert back to their equilibrium values. If ϕ satisfies (17) and initial expectations are not self-fulfilling,[24] excluding expectations-driven equilibria, as shown in Proposition (1).

If agents have imperfect information and the central bank is not transparent, the initial rise in expectations becomes self-fulfilling because, a) the central bank responds to higher output and inflation with a delay while agents expectations keep increasing and, b) due to lack of transparency, the private agents fail to anticipate changes in the interest rate and, as a consequence, monetary policy affects the economy with a *further* delay. With inflation expectations further away from equilibrium values, the required increase in the interest rate is higher than in the case of perfect information. As the bank keeps increasing the interest rate, expectations start declining but eventually overshoot the equilibrium values. This stimulates excessively lower interest rates, thus creating the conditions for a future overheating of the economy. As a result, inflation, output and the interest rate (actual and expected) display an oscillatory behavior. Whether the economy eventually reaches the equilibrium or becomes unstable depends on its structural parameters and the policy rule.

In order to gain some further intuition of the result and especially of condition (19), I consider a simplified version of the model with no shocks and homogeneous beliefs. As shown in the Appendix, the stability conditions of this simple model determine the stability conditions of the more general model discussed above. The agents learn about the model' s steady state values, i.e. the constant terms[25] in equations (7), (8) and (9). The asymptotic behavior of the agents' *beliefs* is approximated by the ODE (11). Setting $\beta \approx 1$, gives

$$\begin{aligned} \dot{x}^e &= -\sigma^{-1}\left(i^e - \pi^e\right) \\ \dot{\pi}^e &= \kappa\xi x^e \\ \dot{i}^e &= \phi\left(\pi^e + \frac{1}{\bar{\eta}}x^e\right) - i^e. \end{aligned} \tag{13}$$

[24] A similar result, in a different model environment is obtained in Preston (2003).

[25] To simplify the exposition, I define $x^e = \hat{\omega}_0^x$, $\pi^e = \hat{\omega}_0^\pi$, $i^e = \psi_0$.

Inspection of (13) shows that expected inflation responds gradually to changes in expected output and expected interest rate responds gradually to changes in expected inflation. The speed of adjustment of the expected interest rate determines the convergence of the learning process. In fact, (13) shows that a prompt response of interest expectations stabilizes output expectations and subsequently inflation expectations, leading to convergence. The dynamics of expected interest rate can be re-written as

$$\dot{i}^e = \phi\left(\pi^e + \Xi\dot{\pi}^e\right) - i^e$$

where $\Xi = \frac{1}{\kappa\xi\bar{\eta}}$. For $\Xi > 1$, the expected interest rate reacts substantially to *changes* in inflation expectations, thus anticipating future expected inflation. As a result expected interest rate adjusts at higher speed, stabilizing output and inducing convergence back the equilibrium. The value of Ξ depends on two factors. First, to what extent the policy rule responds to the expected output-gap *relatively* to expected inflation, which is captured by the parameter $\bar{\eta}$. Second, the effect of output-gap expectations on the change in inflation expectations, which depends[26] on $\xi\kappa$. The result indicates that the more competitive and flexible the economy, the more likely lack of transparency is destabilizing.

As stated in part (b) of Proposition (2), the central bank can affect the learning process by altering its response to the output-gap. In fact, we can re-write $\Xi = \frac{\lambda_x^*}{\xi\kappa}$: an higher Ξ implies an higher weight on output stabilization than consistent with society's preferences, for any given degree of competition and price rigidity. The modified targeting rule becomes

$$E_{t-1}^*\pi_t + \frac{\lambda_x^S}{\xi\kappa}E_{t-1}^*x_t = 0$$

where the output gap coefficient that induces local stability is $\lambda_x^S = \rho_S\lambda_x^*$ and $\rho_S \geq 1$, with strict inequality if (19) is not satisfied.[27]

In order to asses the empirical plausibility of the result, I calibrate the parameters following the literature. I assume $\beta = 0.99$, $\chi = 0.1, \bar{\eta} = 9$, $\theta = 0.3, \phi = 1.5$ and a value of ψ that implies $\xi = 0.06$. Given the uncertainty about σ, Figure (1) shows how λ_x^* and λ_x^S change for $\sigma \in [0.2, 3]$.

For plausible values of σ, commonly used in the literature, the optimal policy rule is not robust to expectational errors. The learning process fails to converge to the rational expectation equilibrium.

[26] Notice further that the stability condition (19) does not depend on the on a specific value for ϕ. This shows that a more aggresive policy stance does not affect beliefs and therefore have any effect on the stability of the learning process. On the contrary, an excessive response to inflation, *relative* to the output gap is more likely to lead to instability. This contrast somewhat with the findings of Orphanides and Williams (2003), arguing that a more aggressive response to inflation is stabilizing. They key difference in the results is that in other models the agents do not have to forecast future policy actions and therefore react instantaneously to any change in the interest rate.

[27] A policy rule that reacts more aggressively to the output-gap, is also more prone to instability, given the available inaccurate measures of the output-gap.

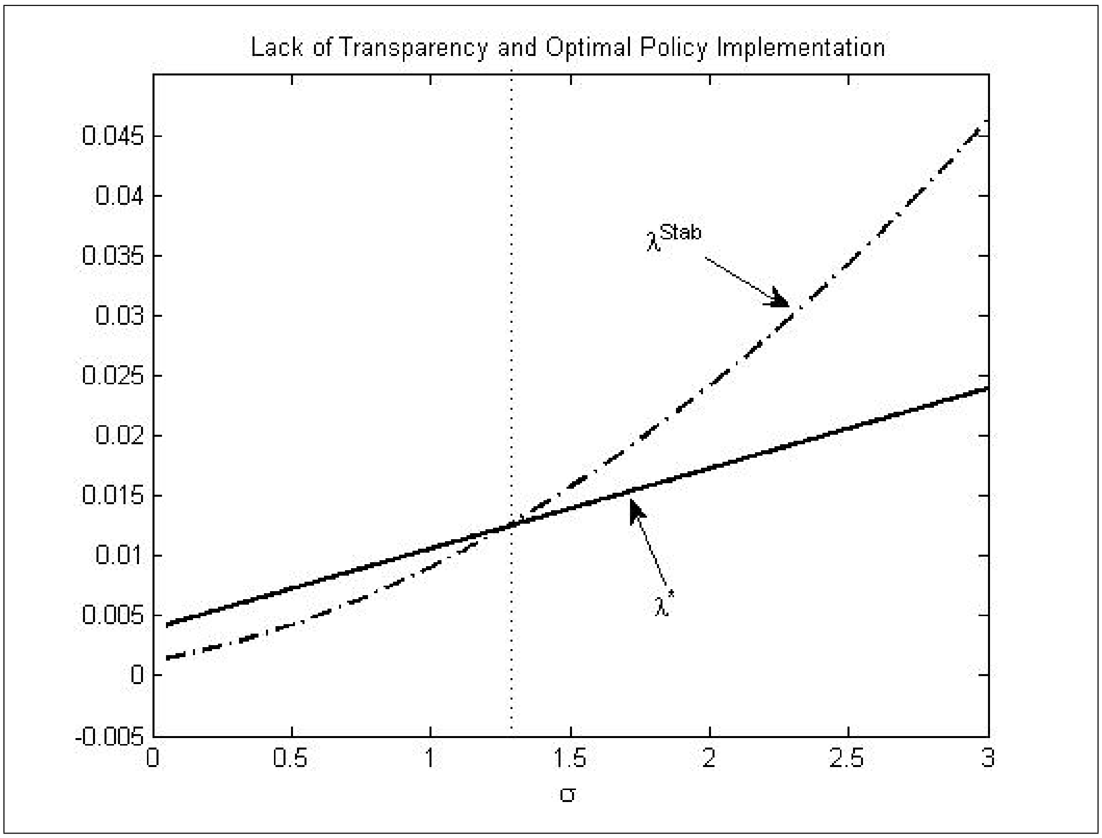
Lack of Transparency and Optimal Policy Implementation
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
0
0.5
1
1.5
2
2.5
3
σ
λ^Stab
λ*

Figure 1:

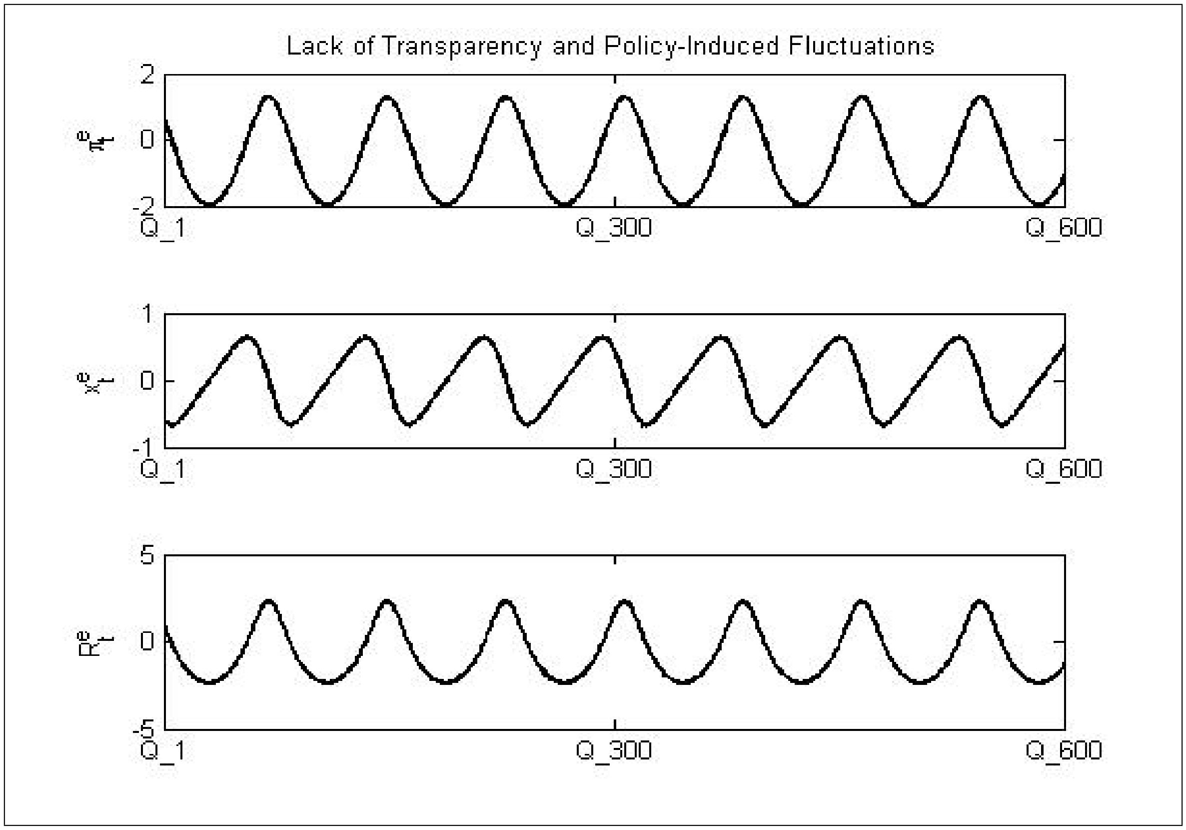

Figure 2:

The linearized version of the model implies that the self-fulfilling expectations eventually lead to an explosive path for beliefs and actual variables.[28]

Nevertheless, part (*c*) in Proposition (2) suggest what might happen in a non-linear version of the model. Figure (2) describes the 'learning equilibrium'[29] where output, inflation and the interest rate fluctuate around the optimal REE. This is obtained by simulating the nonlinear solution of the model under learning,[30] by assuming $\sigma \simeq 1.9$ and by assuming a fixed gain[31] $\delta_t = \delta = 0.5$. This

[28] In this case, we should expect the private agents and the central bank to change their behavior well before this happens. A treatment of such problem is outside the scope of the paper.

[29] This is an equilibrium in the sense that given the agents decision rules and beliefs the economic system converges to a cycle. Nevertheless, on this 'equilibrium'; a) the agent decision rules do not converge to the optimal decision rule and b) the agents make systematic mistakes. Thus it might be argued that at some point in time consumers and producers might change theri decision rules, inducing a change in the dynamic properties of the economy.

[30] For simplicity I assume a deterministic environment, where there are no shocks. Given the non linearity of the system, I assume that agents have point expectations. For a more detailed analysis of nonlinear models see Benhabib et a. (2003) and Benhabib and Eusepi (2004).

[31] The choice of the gain does not affect the existence of the cycle. I choose this values because it gives somewhat

particular example is meant to describe how policy-induced fluctuations might emerge as a result of imperfect information.

Notice that the non-linear solution of the model under perfect foresight has a globally unique equilibrium, as shown in Proposition (1). The agents' learning process is the only source of instability in the model.

5.2 A Transparent Central Bank

The results in the section above indicate that learning and lack of transparency might lead to instability, but the result does not clarify the *relative* importance of central bank transparency in expectations stabilization. In other words, given the uncertainty about output and inflation behavior faced by the private agents and the central bank, even a transparent implementation of the optimal policy rule might be insufficient to stabilize the learning process.

In the case of transparency, the agents forecast the interest rate by using (10). At the end of each period the agents update their estimate of the policy rule according to the Recursive Instrumental Variable estimator[32]

$$\phi_t = \phi_{t-1} + \delta_t R_{\psi,t-1}^{-1} X_{t-1} \left(i_t - \phi'_{t-1} Z_{t-1} \right)$$

$$R_{\psi,t} = R_{\psi,t-1} + \delta_t \left(X_{t-1} Z'_{t-1} - R_{\psi,t-1} \right)$$

where

$$\phi_t = \begin{bmatrix} \phi_{0,t-1} & \phi_{x,t} & \phi_{\pi,t} \end{bmatrix}'; \quad Z_{t-1} = \begin{bmatrix} 1 & \tilde{E}_{t-1}^{CB} x_t & \tilde{E}_{t-1}^{CB} \pi_t \end{bmatrix}'$$

and the agents use the vector of shocks as X_t as instruments in the regression. The actual law of motion in the case of transparency becomes[33]

$$V_t = T' \left(\hat{\Omega}_{t-1}^{PS}, \hat{\Omega}_{t-1}^{CB}, \phi_{t-1} \right) X_t$$

and the stability analysis depnds on the stability of the associated ODE, as in the previous section. The following proposition describes the stability results for the case of transparency.

Proposition 3 *Transparent Central Bank. Consider the model with $\nu = 0$ and assume that ϕ satisfies (17).*

(a) The rational expectations equilibrium is stable under learning for every parameter value;

(b) there exist no 'learning equilibria' .

higher frequency cycles. A 'correct' choice of δ would depend on the properties of the shocks, which are absent in this version of the model.

[32] The RIV is required by potential i.i.d. observation errors in the central bank's forecasts.

[33] The details are in the Appendix.

Proof. See Appendix. ■

I order to understand the result, the above simplified version of the model implies the following evolution of agents' beliefs

$$\begin{aligned} \dot{x}^e &= -\sigma^{-1}\phi_0 - \sigma^{-1}(\phi_\pi - 1)\pi^e - \sigma^{-1}\phi_x x^e \\ \dot{\pi}^e &= \xi\kappa x^e \end{aligned} \tag{14}$$

where the private agents use their information about the central bank strategy in order to forecast the interest rate. Provided the private sector's estimates of the policy parameters are sufficiently close to the true parameters local stability is guaranteed. In fact, consider the example described in the previous section. As the agents expect higher inflation, they also expect that consistently with its policy strategy, the central bank will rise the interest rate, the exact amount being determined by the coefficients ϕ_π, ϕ_x. But this has the effect of decreasing *immediately* the expected output gap and therefore inflation expectations, inducing convergence to the equilibrium. As shown in the last section, this does not guarantee that the other equations coefficients converge.

Summing up, Proposition (3) shows that the *only* source of the instability and 'learning equilibria' in this model is the lack of transparency. Implementation of the optimal policy rule is possible also under the assumption that the central bank and the private agents face uncertainty about the economic model and (possibly) disagree on the 'correct' model, provided the central bank is transparent about its policy rule.

5.3 Transparency and Effectiveness of Monetary Policy

I now consider how central bank communication affects the monetary transmission mechanism, by comparing the effect of a change in inflation expectations under different assumptions about transparency. I consider parameter values that imply stability under learning independently of transparency and compare the dynamics of beliefs. Figure (3) describes the response of *expected* output-gap and inflation and *actual* interest rate to a 1% increase in inflation expectations. I consider $\sigma = 0.8$, which implies learnability of the optimal rule but is not close to the instability area. The variables' simulated paths are obtained from the ODE describing the approximate behavior of the agents' beliefs.

Two features are worth noticing. First, inflation and output expectations fluctuate around their steady state values in the case of a non-transparent central bank, while they converge back much more rapidly in the case of a transparent central bank. Second, the interest rate is less volatile in the case of transparency: because monetary policy directly affect expectations, a small increase in the actual interest rate is more effective in stabilizing output and inflation. Instead, lack of transparency reduces the effectiveness of monetary policy and therefore requires more frequent and prolonged changes in the interest rate. In order to stabilize expectations, the central bank must

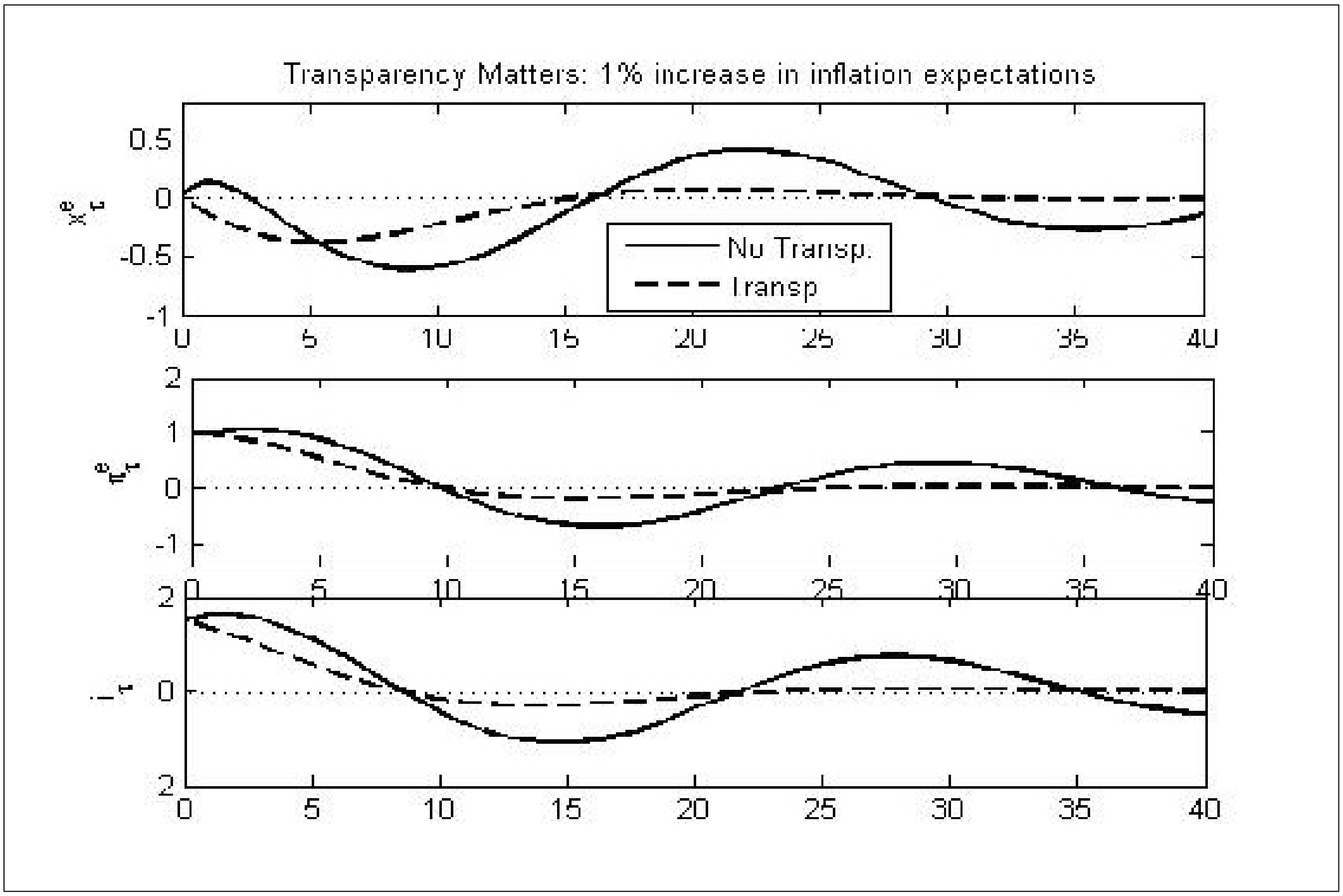
Transparency Matters: 1% increase in inflation expectations
No Transp.
Transp
0.5
0
-0.5
-1
0
5
10
15
20
25
30
35
40
2
1
0
-1
2
0
2

Figure 3:

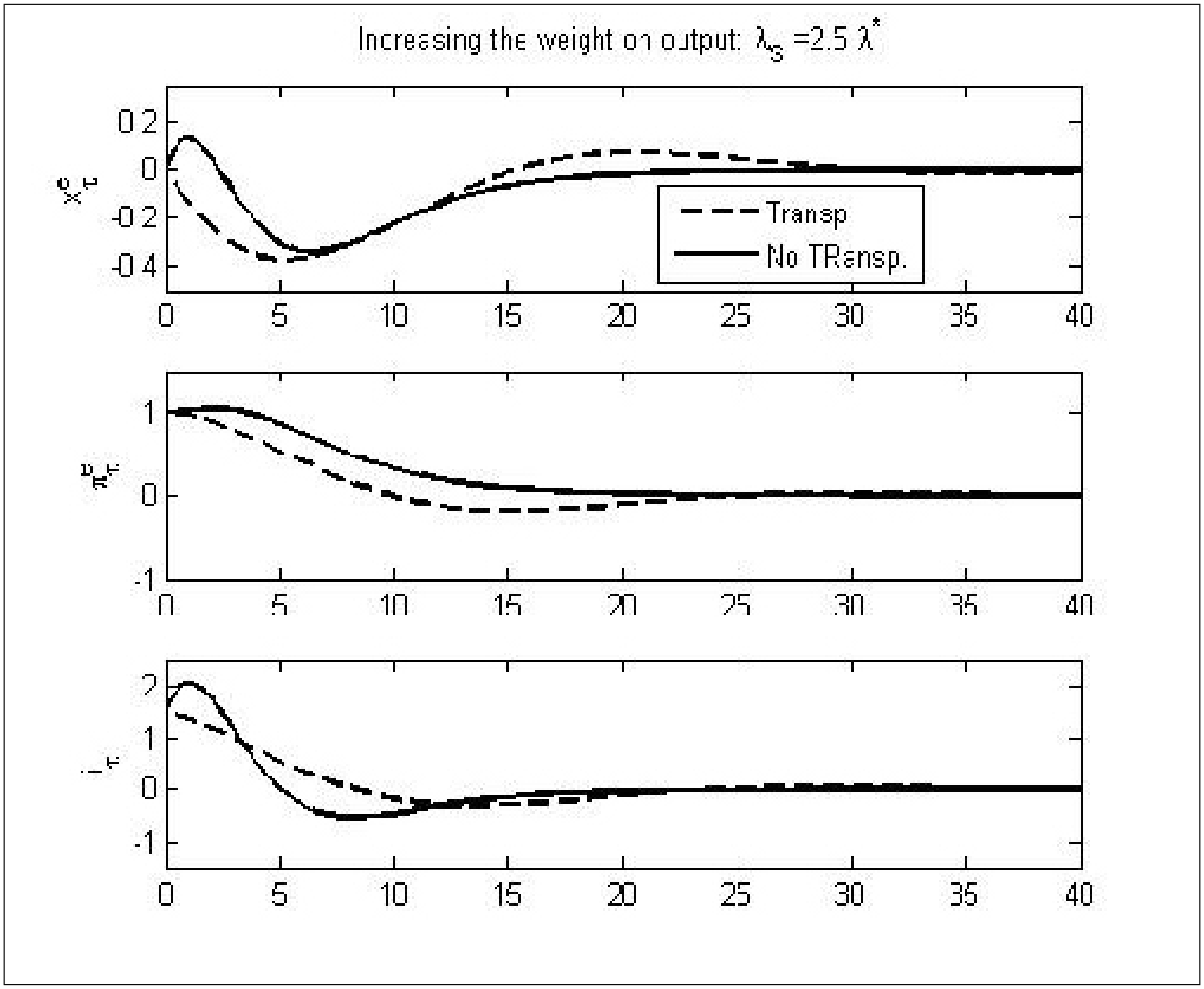

Figure 4:

put more weight on output gap than would be optimal. Figure (4) shows how the response of expectations changes when the weight on output increases.

Recent[34] literature shows how choosing a policy rule with $\phi < 1$ makes monetary policy less effective because of *rational*-expectations-driven fluctuations. This paper shows that a similar result can be obtained even in the case of optimal policy, if the central bank has a poor communication strategy.

[34] See Clarida et al. (1999) and Boivin and Giannoni (2003), for example.

5.4 Does the Central Bank Need to Publish its Forecasts?

The simple model might suggest that the central bank needs *only* to disclose information about its targeting rule, in order to achieve stability. It is straightforward to show that, even if the private agents do not observe the central bank's forecasts and use their own forecasts to predict the interest rate

$$\hat{E}_{t-1}^{PS} i_t = \phi_{0,t-1} + \phi_{\pi,t-1} \hat{E}_{t-1}^{PS} \pi_t + \phi_{x,t-1} \hat{E}_{t-1}^{PS} x_t$$

the REE is stable under learning. But this result depends on the simple structure of the model where the *determinants* of the private sector's and central bank's expectations are assumed to be the same, since both the private agents and the central bank include the same variables in their forecasting model.

But assume that the central bank includes judgment elements in its forecast. Adding an exogenous "judgment vector" $z_t = \rho_z z_{t-1} + \epsilon_{z,t}$, the central bank forecast becomes $\hat{E}_{t-1}^{CB} [\pi_t, x_t]' = \hat{\Omega}_{t-1}^{CB} X_t + \rho_z z_{t-1}$. If the central bank does not publish information about z_t and the private agents use their own forecasts to predict interest rate behavior, they will not be able to learn the correct policy rule and, as a consequence, the correct laws of motion for inflation and output.

Summing up, if the central bank and the private agents have forecasting models that include different variables, as it is likely the case in the real world, publication of economic forecasts and how they are obtained is a crucial element of central bank communication, allowing market participants to learn the monetary policy strategy.

6 Extensions

6.1 What History Dependence?

Woodford (2003) shows that even in a model with forward-looking components, the optimal policy displays history dependence. This means that past conditions matter for the determination of current policy decisions, even if the model is fully forward-looking. For simplicity I consider the model without monetary frictions. Following Woodford (2003), the targeting relation between output-gap and inflation becomes

$$E_{t-1}^{*} \pi_t + \frac{1}{\bar{\eta}} \left(E_{t-1}^{*} x_t - E_{t-2}^{*} x_{t-1} \right) = 0.$$

This condition requires the central bank to respond to *changes* in expected output-gap. It implies a more gradual response of the interest rate to economic shocks. In terms of the rule, this implies a more gradual response to deviation of output gap from equilibrium values. But, from the results in the previous section, this makes the economy *more likely* to experience self-fulfilling expectations, if the central bank is not transparent about its objectives.

I assume for simplicity that the central bank and the private agents share the same forecast. Under imperfect information, the targeting rule is again implemented by the following instrumental rule.

$$i_t = r + \sigma \hat{r}_t^n + \phi \left[\hat{E}_{t-1}\pi_t + \frac{1}{\bar{\eta}} \left(\hat{E}_{t-1}x_t - \hat{E}_{t-2}x_{t-1} \right) \right] + \epsilon_t$$

where $\phi > 1$. I consider a simpler version of the model where the natural rate of interest is i.i.d and the central bank completely off-sets it, *if* the economy is at the rational expectation equilibrium. I assume, as before, that the agents can observe the mark-up shock u_t. The actual law of motion of the economy can be described in matrix notation as

$$Y_t = A_0 + \sum_{j=1}^{2} A_j \hat{E}_{t-1} Y_{t-1+j} + A_3 \hat{E}_{t-2} Y_{t-1} + A_4 u_{t-1} + A_5 v_t$$

where Y_t is a vector including output gap, inflation and the interest rate and v_t is the vector of the natural interest, cost and monetary i.i.d shocks. Under the hypothesis that the central bank is not transparent, the agents PLM is

$$Y_t = \Omega_0 + \Omega_1 x_{t-1} + \Omega_2 u_t + \Omega_3 \hat{r}_t^n.$$

I assume that even if the central bank is non-transparent, the private sector understands that the evolution of the economic variable is history dependent. Thus, the agents' model is consistent with the REE. [35]

Because of technical complications, it is harder to get analytical stability conditions for this problem.[36] I consider the calibration described in the previous section and study how different values of σ affect the stability properties of the equilibrium. The simulation's results show that if the central bank lacks of transparency, instability under learning occurs for every value of σ ranging from as low as 0.2 to as high as 3.

On the contrary, under central bank transparency the REE is stable for each value of σ in the same range.[37] In fact, the simulations seem to suggest that $\phi > 1$ is a the necessary and sufficient condition for having stability under learning in the case of transparency.

These findings suggest that unless the central bank is transparent, the optimal policy should not be history dependent. But what about a *sub-optimal* policy? Consider a policy rule that includes interest smoothing component

$$i_t = \rho i_{t-1} + i_t^*$$

[35] The learning problem of the agents becomes more complicated in this setting. The main reason is that the PLM does not coincide with the ALM *during the learning process*. On the other hand, it can be showed that during the learning process the ALM is an ARMA(2,1) process, while PLM is an AR(1) process. Nevertheless, at the REE the actual law of motion is actually overparametrized: it can be written as an AR(1) process, which is consistent with the agents' model. In other words, the PLM misspecification vanishes as the learning process converges. See the Appendix for details.

[36] The Matlab codes are available on request.

[37] I also experimented choosing different values for $\rho_u \in [0.1, 0.9]$ and $\phi \in [1.1, 2.5]$, but the results were not changed.

where i_t^* is the interest rate set according to the optimal policy rule under discretion. In accordance to what Bullard and Mitra (2001) found in a different model environment, simulations show that increasing ρ the system becomes more stable under learning. Hence, the model predicts that less transparent central banks are more likely to behave according to interest smoothing policy rules, because of their stabilizing effects on expectations. Interest smoothing makes the interest rate more predictable, but involves a suboptimal policy rule at the REE.

6.2 Is Transparency Enough?

This section briefly discusses the more general case where monetary frictions exist, i.e. $\nu > 0$. The discussion is meant to show that understanding the central bank policy framework might not guarantee a satisfactory performance of a given policy rule, because of the agents' learning process. The next Proposition includes the case of perfect transparency (and perfect credibility), where the private agents fully understand the policy rule but still have to learn about the economy.[38]

Proposition 4 *Consider the case where $\nu > 0$ and ϕ satisfies (17)*

(a) transparency increases the set of parameters for which the REE is stable under learning;

(b) Under a perfectly credible and transparent regime instability under learning occurs if

$$\xi > \frac{\sigma^{-1}\bar{\lambda}_x}{\zeta} + \epsilon_\beta \tag{15}$$

where $\epsilon_\beta \to 0$ as $\beta \to 1$.

Proof. See Appendix. ■

Even if *every agent* can fully predict the behavior of the nominal interest rate, a parameter configuration leading to (28) would lead to economic instability. Notice also that any sub-optimal rule that does *not* respond sufficiently to the output-gap is destabilizing, even if perfectly transparent. The evolution of beliefs in the model including a cost channel of monetary policy becomes

$$\begin{aligned} \dot{x}^e &= -\sigma^{-1}(\phi - 1)\pi^e - \sigma^{-1}\phi\bar{\lambda}_x x^e \\ \dot{\pi}^e &= \xi\left(\kappa + \phi\bar{\lambda}_x\right)x^e + \xi\zeta\phi\pi^e. \end{aligned} \tag{16}$$

Assume for simplicity that the economy is very close to perfect competition, i.e. $\bar{\eta} \to \infty$ and $\bar{\lambda}_x \to 0$. Then, from (16) we obtain the following relation between the change in expected inflation and change in expected output

$$\dot{\pi}^e = \xi\kappa x^e - \frac{\xi\zeta\phi}{\sigma^{-1}(\phi - 1)}\dot{x}^e$$

[38] Notice that in this case, the welfare-optimizing weght on the output gap is different, as shown in the Appendix.

which shows how expectations tend to go in the opposite direction, preventing the system to settle down to the equilibrium.[39] Start with an increase in output-gap expectations: from (16) this increases inflation expectations. Now two effects are in place. On one hand, the agents anticipate a reaction from the central bank and this has a moderating effect on output expectations. On the other end, the expected increase in the interest rate increases expected marginal costs for the firms which further stimulates inflation.

If $\bar{\lambda}_x$ is sufficiently large that the optimal policy prescribes non-negligible response to the output-gap (relatively to the response to inflation), the policy response to, say, an increase in inflation expectations is different. In fact, from (16) as output expectations start decreasing the bank sets a lower interest rate to accommodate an adjustment in output. This milder adjustment keeps the expected marginal cost for firms low enough to allow convergence back to the equilibrium.

Summing up, the example shows that uncertainty about the economic environment can affect the performance of policy rules also in the case where the central bank can perfectly communicate its policy intentions. Still the relative importance of central bank communication is confirmed.

7 Conclusion

The paper shows that central bank communication is an important ingredient in monetary policy design. The paper considers the performance of optimal targeting rules when the central bank and the market participants are uncertain about the model environment. In particular the paper distinguishes between uncertainty about the evolution of output and inflation and uncertainty about the central bank monetary policy strategy. I find that a sufficient degree of transparency by central banks make the optimal policy rule robust to expectational mistakes, even in the plausible case where the economic agents face other sources of uncertainty about the economic environment. On the other end, lack of transparency can lead to welfare-reducing outcome where self-fulfilling expectations destabilize the economic system. Instability is induced by the private agents' trying to predict monetary policy actions with very limited information about the central bank objectives and strategy.

Furthermore, the paper shows that a nontransparent central bank can stabilize expectations only by adopting policy rules that are sub-optimal. In fact, the policy rules inducing stability prescribe an excessive response to the output-gap and the wrong type of history dependence.

[39] Notice that for $\zeta = 0$, changes in expected inflation depend only on expected output-gap, guaranteeing convergence.

8 Technical Appendix 1

8.1 Model Solution

8.1.1 First-order conditions

The first order conditions for the consumer are

$$\left(\Gamma_t C_t^i\right)^{-\sigma} = \hat{E}_{t-1}^i \left[\frac{\beta R_t \left(\Gamma_{t+1} C_{t+1}^i\right)^{-\sigma}}{\pi_{t+1}}\right]$$

$$\left(h_t^i\right)^{\chi} \left(\Gamma_t C_t^i\right)^{\sigma} = \frac{W_t}{P_t}.$$

Profit maximization, gives the following expression for the real marginal cost

$$s_t = \frac{h_{jt}^{\theta}}{(1-\theta)} \frac{W_t}{P_t} \left(1 + \nu \frac{R_t - 1}{R_t}\right)$$

which is the real wage divided by the marginal productivity of labor times the opportunity cost of holding money. The firm pricing decision gives

$$\pi_t \left(\pi_t - 1\right) = \beta \hat{E}_{t-1}^i \left[\left(\frac{\Gamma_t C_t}{\Gamma_{t+1} C_{t+1}}\right)^{\sigma} \pi_{t+1} \left(\pi_{t+1} - 1\right)\right] + \hat{E}_{t-1}^i \left[\frac{C_t \eta_t}{\psi} \left(s_t - \frac{\eta_t - 1}{\eta_t}\right)\right]$$

8.1.2 Linearization and natural rates: Deriving equations (1) and (2)

Solving for the symmetric equilibrium and assuming market clearing, the linearized model becomes

$$\hat{y}_t = \sigma^{-1} \left(\beta^{-1} - 1\right) - \sigma^{-1} E_{t-1}^* \left(i_t - \pi_{t+1}\right) + E_{t-1}^* \hat{y}_{t+1} + \sigma^{-1} E_{t-1}^* \left(\gamma_{t+1} - \gamma_t\right)$$

$$\hat{s}_t = \left[\left(\frac{\theta + \chi}{(1-\theta)} + \sigma\right) \hat{y}_t - \hat{\gamma}_t + \frac{\beta \nu}{\left[1 + (1-\beta)\nu\right]} \hat{\imath}_t\right]$$

where $\hat{\gamma}_t$ denotes the log of the preference shock and $\hat{\imath}_t$ denotes deviation of the gross interest rate from the steady state level.

$$\pi_t = E_{t-1}^* \beta \pi_{t+1} + \xi E_{t-1}^* \left(\hat{s}_t - \frac{\hat{\eta}_t}{\bar{\eta} - 1}\right)$$

where $\xi = \bar{C} \bar{s} \frac{\bar{\eta}}{\psi}$ and where I define the cost-push shock as $u_t = -E_{t-1}^* \xi \frac{\hat{\eta}_t}{\bar{\eta} - 1}$.

Finally, I define the efficient level of output and the natural rate of interest. Following Woodford (2003), the equilibrium output under flexible prices is consistent with

$$s\left(y_t^n, \gamma_t, i_t\right) = \frac{\eta_t - 1}{\eta_t}$$

Log-linearization leads to

$$\hat{y}_t^n = \left(\frac{\theta+\chi}{(1-\theta)}+\sigma\right)^{-1}\left(\hat{\gamma}_t + \frac{1}{\bar{\eta}-1}\hat{\eta}_t - \frac{\beta\nu}{[1+(1-\beta)\,\nu]}\hat{\imath}_t\right)$$

where $\hat{y}_t^n$ is the natural level of output. The efficient output is independent from variations in mark-up. Thus

$$\hat{y}_t^e = \hat{y}_t^n - \left(\frac{\theta+\chi}{(1-\theta)}+\sigma\right)^{-1}\frac{1}{\bar{\eta}-1}\hat{\eta}_t.$$

Consequently, the natural rate of interest is defined as

$$r_t^n = E_{t-1}^*\left(y_{t+1}^e - y_t^e + \gamma_{t+1} - \gamma_t\right)$$

8.1.3 Second Order Approximation of the Consumer's Welfare

I can express the welfare function of the representative agent as

$$U(Y_t,\pi_t,\Gamma_t) = \frac{\left[\Gamma_t Y_t - \frac{\psi}{2}\pi_t^2\right]^{1-\sigma}}{1-\sigma} - \frac{Y_t^{\frac{\gamma}{1-\theta}}}{\gamma}$$

where I use the following

1. All agents and firms take the same consumption, pricing and production decisions in the symmetric equilibrium
2. $\frac{h_t^\gamma}{\gamma} = \frac{\left[f^{-1}(Y_t)\right]^\gamma}{\gamma}$
3. $\Pi_t - 1 = \pi_t$

Consider the second order approximation of the first component of the utility

$$U(Y_t,\pi_t,\Gamma_t) = \bar{u} + \bar{Y}^{-\sigma}\tilde{Y}_t + \bar{Y}^{-\sigma}\bar{Y}\tilde{\gamma}_t - \frac{1}{2}\sigma\bar{Y}^{-\sigma-1}\tilde{Y}_t^2 - \sigma\bar{Y}^{-\sigma-1}\tilde{Y}_t\tilde{\gamma}_t + \bar{Y}^{-\sigma}\frac{\psi}{2}\pi_t^2 + \frac{1}{2}\sigma\bar{Y}^{-\sigma-1}\bar{Y}\bar{Y}\tilde{\gamma}_t^2$$

where $\tilde{Y}_t = Y_t - \bar{Y}$. Now use

$$\frac{Y_t - \bar{Y}}{\bar{Y}} = \hat{y}_t + \frac{1}{2}\hat{y}_t^2 + rest$$

to get

$$\begin{aligned} U(Y_t,\pi_t,\Gamma_t) &= \bar{u} + \bar{Y}^{-\sigma}\bar{Y}\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right) + \bar{Y}^{-\sigma}\bar{Y}\tilde{\gamma}_t - \frac{1}{2}\sigma\bar{Y}^{-\sigma-1}\bar{Y}^2\hat{y}_t^2 - \sigma\bar{Y}^{-\sigma-1}\bar{Y}\bar{Y}\hat{y}_t\tilde{\gamma}_t + \frac{\psi}{2}\pi_t^2 + \frac{1}{2}\sigma\bar{Y}^{-\sigma-1}\bar{Y}\bar{Y}\tilde{\gamma}_t^2 \\ &= \bar{Y}^{1-\sigma}\left\{\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2 + \sigma\hat{y}_t\tilde{\gamma}_t + \frac{\psi}{2}\bar{Y}^{-1}\pi_t^2 + \frac{1}{2}\sigma\tilde{\gamma}_t^2\right\} \end{aligned}$$

The second component of the utility function is obtained as

$$\begin{aligned}
\frac{Y_t^{\frac{\gamma}{1-\theta}}}{\gamma} &= \frac{1}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-1}\tilde{Y}_t + \frac{\left(\frac{\gamma}{1-\theta}-1\right)}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-2}\tilde{Y}_t^2 \\
& \frac{1}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-1}\bar{Y}\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right) + \frac{1}{2}\frac{\left(\frac{\gamma}{1-\theta}-1\right)}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-2}\bar{Y}^2\hat{y}_t^2 \\
&= \frac{1}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-1}\bar{Y}\left\{\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right) + \frac{1}{2}\left(\frac{\gamma}{1-\theta} - \frac{1-\theta}{1-\theta}\right)\hat{y}_t^2\right\} \\
&= \frac{1}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-1}\bar{Y}\left\{\hat{y}_t + \frac{1}{2}\left(1 + \frac{\gamma-1}{1-\theta} + \frac{\theta}{1-\theta}\right)\hat{y}_t^2\right\}
\end{aligned}$$

Now, use the expression for the marginal cost in steady state

$$\begin{aligned}
\bar{s} &= \frac{\frac{1}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-1}}{\bar{Y}^{1-\sigma}} = \frac{(\bar{\eta}-1)}{\bar{\eta}} = (1-\Phi_y) \\
&\rightarrow \frac{1}{1-\theta}\bar{Y}^{\frac{\gamma}{1-\theta}-1} = (1-\Phi_y)\bar{Y}^{1-\sigma}
\end{aligned}$$

Putting the two parts together I get

$$\begin{aligned}
U &= \bar{Y}^{1-\sigma}\left\{\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2 + \sigma\hat{y}_t\tilde{\gamma}_t + \frac{\psi}{2}\bar{Y}^{-1}\pi_t^2 + \frac{1}{2}\sigma\tilde{\gamma}_t^2\right\} - \\
& +\bar{Y}^{1-\sigma}\left\{(1-\Phi_y)\hat{y}_t + \frac{1}{2}\left(1 + \frac{\gamma-1}{1-\theta} + \frac{\theta}{1-\theta}\right)\hat{y}_t^2\right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{\bar{Y}^{1-\sigma}}{2}\left\{2\Phi_y\hat{y}_t - \hat{y}_t^2 - \sigma\hat{y}_t\tilde{\gamma}_t + \sigma\tilde{\gamma}_t^2 - \psi\bar{Y}^{-1}\pi_t^2\right\} \\
&= -\frac{\bar{Y}^{1-\sigma}}{2}\left\{\left(\sigma + \frac{\gamma-1}{1-\theta} + \frac{\theta}{1-\theta}\right)(x-x^*)^2 + \psi\bar{Y}^{-1}\pi_t^2\right\}
\end{aligned}$$

$$\begin{aligned}
\bar{Y}\bar{s}\frac{\bar{\eta}}{\psi} &= \xi \\
&\rightarrow \psi = \frac{\bar{Y}\bar{\eta}(1-\Phi_y)}{\xi}
\end{aligned}$$

$$\begin{aligned}
L_t &= -\frac{\bar{Y}^{1-\sigma}}{2}\left\{\lambda_x(x_t - x^*)^2 + \pi_t^2\right\} \\
\lambda_x &= \xi\kappa/\bar{\eta} \\
x^* &= \left(\sigma + \frac{\gamma-1}{1-\theta} + \frac{\theta}{1-\theta}\right)\Phi_y
\end{aligned}$$

8.1.4 Learning in the Nonlinear Model

To keep the analysis as simple as possible, I consider a deterministic version of the model where the agents learn about the steady state variables. In order to have a well-defined nonlinear equation I assume that firm discount profits by using the time invariant discount rate β. Also, I assume that the agents have point expectations. The nonlinear equations become

$$Y_t = \left[\frac{\beta R^e_{t-1} \left(Y^e_{t-1} \right)^{-\sigma}}{\pi^e_{t-1}} \right]^{-\frac{1}{\sigma}}$$

$$\pi_t = \frac{1 + \sqrt{1 + 4\beta\pi^e_{t-1}(\pi^e_{t-1} - 1) + \bar{\eta}\psi Y^e_{t-1} \left(\frac{Y_t^{e\left(\frac{\theta+\chi}{1-\theta}+\sigma\right)}}{(1-\theta)} - \frac{\bar{\eta}-1}{\bar{\eta}} \right)}}{2}$$

$$R_t = \beta^{-1} + \left(\beta^{-1} - 1 \right) \left(\pi^e_t \right)^{\frac{\phi}{1-\beta}} \left(\frac{Y^e_t}{\bar{Y}} \right)^{\frac{\phi\bar{\lambda}^*}{1-\beta}}$$

where Z^e denotes the point expectation of the variable Z and $\bar{Y}$ denotes the steady state level of output. This differs from the efficient level of output, but the difference is assumed to be small. What is important is that in this deterministic version of the model, the difference between $\bar{Y}$ and the steady state efficient level of output is also constant. Notice that the policy rule satisfies the zero-bound for the nominal interest rate. Assuming that the agents estimate the steady state value with RLS, the evolution of agents' beliefs is described by the following system

$$V^e_t = V^e_{t-1} + \delta \left[T^{nl} \left(V^e_{t-1} \right) - V^e_{t-1} \right]$$

where $V^e_t = \begin{pmatrix} Y^e_t & \pi^e_t & R^e_t \end{pmatrix}'$ and $T^{nl}\left(Y^e_{t-1}, \pi^e_{t-1}, R^e_{t-1} \right)$ is now a non-linear function. Notice that I use a constant gain $\delta_t = \delta$.

8.1.5 Timeless Perspective

In order to study the stability properties of this REE, I follow Adam, Evans and Honkapohja (2003) and project the ARMA(2,1) actual law of motion onto the AR(1) parameter space. This because the agents estimate their model by using the AR(1) PLM.

I proceed in three steps. First, I define a vector $Z_t = (Y_t, x_{t-1}, u_{t-1}, \hat{r}^n_t)$, and rewrite the ALM as

$$Z_t = A + BZ_{t-1} + Cv_t$$

where the matrices A, B, C are function of the model's parameters and the agents' estimates.

Second, I calculate the variance-covariane matrix of Z_t

$$\Sigma_Z = mat\left[\left(I_{36} - B \otimes B\right)^{-1}\left(C\Sigma_v C'\right)\right]$$

Third and last step, I calculate the mapping between the PLM and ALM. The least square estimator for the agents' regression is

$$T_{\Omega_i}\left(\Omega_i\right) = \left(\Sigma_{Z_{4:6,4:6}}\right)^{-1}\left(\Sigma_{Z_{1:3,4:6}}\right)'$$

for $i = 1, 2, 3$. For the constant we obtain

$$T_{\Omega_0}\left(\Omega_0\right) = \left(I_3 - \Omega_1\right)\bar{Z}_{1:3}$$

where $\bar{Z}_{1:3}$ is the expected value of the the vector Z_t: $\bar{Z} = (I - B)^{-1} A$.

Local stability of the REE is obtained as above, by analyzing the local stability of the following ODE, evaluated at the fixed point Ω^*.

$$\dot{\Omega} = T(\Omega) - \Omega$$

9 Technical Appendix 2

9.1 Proofs

Proposition 1 *A sufficient condition for a globally unique equilibrium is*

$$0 < \frac{\bar{\eta}}{1 + \bar{\eta} - \theta^{-1}} < \phi < \frac{1}{\xi} \tag{17}$$

Proof. Local determinacy obtains if $\left(I_3 - A_1^{PS} + A_1^{CB}\right)^{-1} A_2^{PS}$ has all eigenvalues inside the unit circle. The characteristic equation can be factorized to give

$$P(\lambda) = -\lambda\left(\lambda^2 + a_1\lambda + a_0\right)$$

where

$$a_0 = \frac{\beta}{1 + \phi\xi\left[\frac{\sigma^{-1}(\theta+\chi)}{(1-\theta)} + \frac{\kappa\sigma^{-1}}{\xi(\kappa-\sigma\zeta)\bar{\eta}} + 1 - \frac{\beta\nu}{[1+(1-\beta)\nu]}\right]} > 0$$

and

$$a_1 = -\frac{\left(\xi\zeta\phi\bar{\lambda}_x + \kappa\xi + \beta\phi\bar{\lambda}_x\right)\sigma^{-1} - \xi\zeta\phi + 1 + \beta}{1 + \phi\xi\left[\frac{\sigma^{-1}(\theta+\chi)}{(1-\theta)} + \frac{\kappa\sigma^{-1}}{\xi(\kappa-\sigma\zeta)\bar{\eta}} + 1 - \frac{\beta\nu}{[1+(1-\beta)\nu]}\right]}$$

The conditions for determinacy are

$$|a_0| < 1, \;\; |a_1| < 1 + a_0$$

The first condition is verified, given that $0 < \nu < 1$. Also, provided $\phi < 1/\xi$ we have that $a_1 < 0$, so that the condition for determinacy becomes

$$1 + a_0 + a_1 > 0. \tag{18}$$

Simple algebraic manipulation shows that if $\phi > \frac{\bar{\eta}}{1+\bar{\eta}-\theta^{-1}}$. Notice that this is a sufficient condition.

Consider the issue of global unicity, in the sense of Benhabib, Schmitt-Grothe' and Uribe (2001). Since local indeterminacy occurs as $-a_1 = 1 + a_0$, the eigenvalues are real at the bifurcation point. Hence, no Hopf bifurcation exist provided (17) is satisfied. Moreover, from (18) we know that as we choose a value of ϕ that gives indeterminacy, the maximum eigenvalue crosses the unit circle at $+1$ so that we can exclude a flip bifurcation. Finally, no saddle-node bifurcation exist because the model has a unique steady state for each value of ϕ. ■

Proposition 2 *Lack of Transparency. Assume that ϕ satisfies (17) and $\nu = 0$.*

(a) The REE is unstable under learning if and only if

$$\xi > \frac{1}{\bar{\eta}\kappa} + \epsilon_\beta \tag{19}$$

where $\epsilon_\beta \to 0$ as $\beta \to 1$.

(b) If (a) holds, achieving stability under learning requires an excessive weight to output-gap stabilization with respect to the representative agent's loss function;

(c) There exist policy-induced "learning equilibria" where inflation and output fluctuate around the inflation target.

Proof. (a) The model can be written in matrix form as

$$V_t = A_0 + A_1^{PS} E_{t-1}^{PS} V_t + A_1^{CB} E_{t-1}^{CB} V_t + A_2 E_{t-1}^{PS} V_{t+1} + A_3 X_t + \epsilon_t.$$

where the A's matrices depend on the structural parameters of the model.

Inserting the PLMs I get

$$V_t = T\left(\hat{\Omega}_0^{PS}, \hat{\Omega}_0^{CB}, \hat{\Omega}_1^{PS}, \hat{\Omega}_1^{CB}\right) X_{t-1} + \epsilon_t$$

where $\hat{\Omega}_0^j$ denotes the vector of constants $(\hat{\omega}_0^x, \hat{\omega}_0^\pi, \psi_0)'$ and $\hat{\Omega}_1^j$ is the matrix of shocks' coefficients. The mapping between the PLM and ALM is described by the following ODE

$$\begin{bmatrix} \dot{\hat{\Omega}}_0^{PS} \\ \dot{\hat{\Omega}}_0^{CB} \end{bmatrix} = F_1 \begin{bmatrix} \hat{\Omega}_0^{PS} \\ \hat{\Omega}_0^{CB} \end{bmatrix}$$

$$\left[\begin{array}{c} vec\dot{\hat{\Omega}}_1^{PS} \\ vec\dot{\hat{\Omega}}_1^{CB} \end{array}\right] = F_2 \left[\begin{array}{c} vec\hat{\Omega}_1^{PS} \\ vec\hat{\Omega}_1^{CB} \end{array}\right]$$

where

$$F_1 = \left(\begin{array}{ll} A_1^{PS} - I_3 + A_2 & A_1^{CB} \\ A_1^{PS} + A_2 & A_1^{CB} - I_3 \end{array}\right)$$

$$F_2 = \left(\begin{array}{ll} I_2 \otimes A_1^{PS} + H \otimes A_2 - I_6 & I_2 \otimes A_1^{CB} \\ I_2 \otimes A_1^{PS} + H \otimes A_2 & I_2 \otimes A_1^{CB} - I_6 \end{array}\right)$$

In order to extract the stability conditions I follow Honkapohja and Mitra (2002). Stability under learning is obtained if the eigenvalues of F_1 and F_2 have negative real parts. The characteristic equations of associated to the two matrices can be simplified to

$$\begin{aligned} |F_1 - \lambda I_6| &= \left|\begin{array}{ll} A_1^{PS} - I_3(1+\lambda) + A_2 & A_1^{CB} \\ A_1^{PS} + A_2 & A_1^{CB} - I_3(1+\lambda) \end{array}\right| \\ &= (-(1+\lambda))^2 \left|A_1^{PS} + A_1^{CB} + A_2 - I_3(1+\lambda)\right| \end{aligned}$$

and

$$\begin{aligned} |F_2 - \lambda I_{12}| &= \left|\begin{array}{ll} -(1+\lambda) I_6 & (1+\lambda) I_6 \\ I_2 \otimes A_1^{PS} + H \otimes A_2 & I_2 \otimes A_1^{CB} - I_6(1+\lambda) \end{array}\right| \\ &= (-(1+\lambda))^6 \left|I_2 \otimes A_1^{PS} + H \otimes A_2 + I_2 \otimes A_1^{CB} - (1+\lambda) I_6\right| \end{aligned}$$

So, determining stability boils down to determinate whether the eigenvalues of the following matrices have negative real part

$$\tilde{A}_1 = A_1^{PS} + A_1^{CB} + A_2 - I_3 = \left(\begin{array}{lll} 0 & \sigma^{-1} & -\sigma^{-1} \\ \kappa\xi & \beta - 1 & \xi\zeta \\ \phi\bar{\lambda}_x & \phi & -1 \end{array}\right) \tag{20}$$

and

$$\tilde{A}_2 = A_1^{PS} + A_1^{CB} + \rho_i A_2 \tag{21}$$

for $i = r, u$.

Let us consider first (20). According to the Routh's Theorem, the number of roots of (20) with positive real parts is equal to the number of variations of sign in the following scheme

$$-1 \quad \text{Trace }(\tilde{A}_1) \quad -B_1 + \frac{\text{Det}(\tilde{A}_1)}{\text{Trace }(\tilde{A}_1)} \quad \text{Det}(\tilde{A}_1)$$

where

$$\text{Trace }(\tilde{A}_1) = \beta - 2 < 0$$

$$\begin{aligned}\text{Det}(\tilde{A}_1) &= -\left[\kappa\xi\left(\phi-1\right)+\left(1-\beta\right)\phi\bar{\lambda}_x\right]+\xi\zeta\phi\bar{\lambda}_x < 0 \\ &\quad \text{provided (17) holds}\end{aligned}$$

$$B_1 = -\kappa\sigma + \sigma\phi\bar{\lambda}_x + (1-\beta) - \xi\zeta\phi$$

where B is the sum of all second order principle minors of $\tilde{A}_1$. A pattern of $----$ corresponds to all eigenvalues having negative real part. In order to obtain that we need

$$-B_1 \cdot \text{Trace }(\tilde{A}_1) + \text{Det}(\tilde{A}_1) > 0 \tag{22}$$

Algebraic manipulations show that (22) is positive if

$$\bar{\lambda}_x > \frac{\sigma^{-1}\kappa\xi\left[\phi\left(\frac{\sigma^{-1}+\frac{\zeta}{\kappa}(2-\beta)}{\sigma^{-1}}\right)-1\right]+\left[\kappa\xi\sigma^{-1}-(1-\beta)\right](2-\beta)}{\phi\left[1+\xi\zeta\right]\sigma^{-1}}. \tag{23}$$

If $\nu = 0$, then $\zeta = 0$. In this case $\bar{\lambda}_x = 1/\overline{\eta}$. By letting $\beta \to 1$ I get condition (19).

Consider the matrix $\tilde{A}_2$. It is easy to show that

$$\text{Trace }(\tilde{A}_2) = \rho_i\left(1+\beta\right) - 3 < 0$$

$$\begin{aligned}\text{Det}(\tilde{A}_2) &= -\sigma^{-1}\left[\kappa\xi\left(\phi-\rho_i\right)+\phi\bar{\lambda}_x\left(1-\beta\rho_i\right)\right]+ \\ -\kappa\xi\left(1-\rho_i\right)\left(\sigma^{-1}-\frac{\zeta}{\kappa}\right)-\left(1-\rho_i\right)\left(2-\beta\rho_i\right) &< 0 \\ &\quad \text{provided (17) holds}\end{aligned} \tag{24}$$

where $i = r^n, u$,

$$\begin{aligned}B_2 &= -\kappa\xi\rho_i\sigma^{-1} + \sigma\phi\bar{\lambda}_x + 2 - \beta\left(1+\rho_i\right) + \\ &\quad \left(1-\rho_i\right)\left(1-\beta\rho_i\right) - \xi\zeta\phi\end{aligned}$$

Assume (determinacy) holds. Since$B_2 \geq B_1$, Det$(\tilde{A}_2)\leq$Det$(\tilde{A}_1)$ and Trace $(\tilde{A}_2)\leq$Trace $(\tilde{A}_1)$, $-B_2\cdot$Trace $(\tilde{A}_2)$+Det$(\tilde{A}_2)>0$, if (22) is satisfied. So that provided that (23) holds, the REE is stable under learning.

(b) If condition (19) is violated, we have

$$\lambda^* < \left(\xi\kappa\right)^{-2}.$$

In order to achieve stability the central bank needs to put an higher weight on the output gap, i.e. $\lambda_x^S > (\xi\kappa)^{-2} > \lambda^*$.

(c) Notice that if (22) is not verified the sign pattern becomes $- - +-$, which indicates two eigenvalues with positive real parts. Since, the determinant of $\tilde{A}_1$ does not vanish at as (23) is violated, we know that the eigenvalues are complex. Hence, an Hopf bifurcation occurs, provided additional technical conditions are verified. If the bifurcation is *supercritical*, the learning process converges to another 'equilibrium', as described in Figure (2). A detailed analysis of the nonlinear model and the Hopf bifurcation is outside the scope of the paper. The interested reader should consult Benhabib, Schmitt-Grothe' and Uribe (2003) and Benhabib and Eusepi (2004). ■

Proposition 3 *Transparent Central Bank. Consider the model with* $\nu = 0$ *and assume that* ϕ *satisfies (17).*

(a) The rational expectations equilibrium is stable under learning for every parameter value;

(b) there no exist 'learning equilibria' .

Proof. (a) The ALM can be written in matrix notation as

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = B_1\phi'_{t-1}\begin{bmatrix} 1 \\ \tilde{E}^{CB}_{t-1}x_t \\ \tilde{E}^{CB}_{t-1}\pi_t \end{bmatrix} + B_2\begin{bmatrix} 1 \\ \hat{E}^{PS}_{t-1}x_t \\ \hat{E}^{PS}_{t-1}\pi_t \end{bmatrix} + B_3\begin{bmatrix} 1 \\ \hat{E}^{PS}_{t-1}x_{t+1} \\ \hat{E}^{PS}_{t-1}\pi_{t+1} \end{bmatrix} + B_4X_t$$

and the optimal policy rule

$$i_t = \beta - 1 + \phi\left(\hat{E}^{CB}_{t-1}\pi_t + \bar{\lambda}_x\hat{E}^{CB}_{t-1}x_t\right) + \epsilon_t$$

where B_1 is a 2 by 1 vector, and B_2, B_3, B_4 are 2 by 3 matrices, depending on the structural parameters. Consider first the convergence properties of the policy rule estimates. Substituting for the nominal interest rate we get

$$\phi_t = \phi_{t-1} + \delta_t R^{-1}_{\psi,t-1}X_{t-1}\left[\bar{\phi}'\begin{pmatrix} 1 \\ \hat{\Omega}^{CB}_{t-1}X_{t-1}\end{pmatrix} + \epsilon_t - \phi'_{t-1}\begin{pmatrix} 1 \\ \hat{\Omega}^{CB}_{t-1}X_{t-1}\end{pmatrix} - \phi'_{t-1}\begin{pmatrix} 0 \\ e_t\end{pmatrix}\right] \tag{25}$$

$$R_{\psi,t} = R_{\psi,t-1} + \delta_t\left[X_{t-1}\begin{pmatrix} 1 \\ \hat{\Omega}^{CB}_{t-1}X_{t-1}\end{pmatrix}' - R_{\psi,t-1} + X_{t-1}\begin{pmatrix} 0 \\ e_t\end{pmatrix}\right]$$

where $\bar{\phi} = \left(\beta - 1,\ \phi\bar{\lambda}_x,\ \phi\right)'$. The expression for ϕ_t can be rearranged to yield

$$\phi_t = \phi_{t-1} + \delta_t R^{-1}_{\psi,t-1}X_{t-1}\begin{pmatrix} 1 \\ X_{t-1}\hat{\Omega}^{CB}_{t-1}\end{pmatrix}'\left(\bar{\phi} - \phi_{t-1}\right) + \delta_t R^{-1}_{\psi,t-1}X_{t-1}\left[\epsilon_t - \phi'_{t-1}\begin{pmatrix} 0 \\ e_t\end{pmatrix}\right].$$

Taking the asymptotic mean, the dynamics of beliefs is described by the following ODE

$$\dot{\phi} = R_\psi^{-1}\tilde{M}_X\left(\bar{\phi} - \phi\right)$$

$$\dot{R}_{\psi} = \left(\left[\tilde{M}_X - R_{\psi}\right]\right)$$

where $\tilde{M}_X = E_{t\to\infty}\begin{pmatrix} 1 & \\ 0_{2\times 1} & M_X\hat{\Omega}^{CB\prime}\end{pmatrix}$ is a 3 by 3 matrix. Hence, we have that $R_{\psi} \to \tilde{M}_X$. Substituting in the above we obtain $\phi \to \bar{\phi}$. The estimates of the parameters converge to the true estimates, for every parameter values, provided the estimate of the other coefficients stays bounded. Following the same steps as for Proposition (2), the associated ODE can be calculated as

$$\dot{\hat{\Omega}}' = \left[\tilde{T}\left(\hat{\Omega}', \bar{\phi}\right) - \hat{\Omega}'\right]$$

where I use the fact that, a) $R \to M_X$; b) $\psi \to \bar{\phi}$. It is straightforward to show that stability under learning depends on the eigenvalues of the following matrix

$$\tilde{B}_1 + \tilde{B}_2 - I_2 = \begin{bmatrix} -\phi\bar{\lambda}_x\sigma^{-1} & \sigma^{-1}(1-\phi) \\ \kappa\xi\left(1+\frac{\zeta}{\kappa}\phi\bar{\lambda}_x\right) & \beta+\xi\zeta\phi-1 \end{bmatrix} \tag{26}$$

and

$$\tilde{B}_1 + \rho_i\tilde{B}_2 - I_2 \tag{27}$$

where

$$\tilde{B}_1 = B_1\bar{\phi}' + B_2;\ \tilde{B}_2 = B_3$$

In order to have negative eigenvalues, I need both the trace and the determinant to be negative.

Consider the case $\nu = 0$. It is straightforward to show that the eigenvalues of the matrix (26) are negative provided (17) holds. Also it is possible to show that if the matrix (27) satisfies this property, then also matrix (26) will satisfy it.

(b) The determinant of $\tilde{B}_1 + \tilde{B}_2 - (1+\lambda) I_2$ vanishes if the Taylor condition holds with equality. Therefore, the eigenvalues are *real* and no Hopf bifurcation occurs. The only 'equilibrium' for the learning process is the REE. ■

Proposition 4 *Consider the model under perfect transparency with $\nu > 0$ and ϕ satisfying (17)*

(a) transparency induces stability under learning for a larger parameter space with respect to lack of transparency;

(b) Under a transparent regime instability under learning occurs if

$$\xi > \frac{\sigma^{-1}\bar{\lambda}_x}{\zeta} + \epsilon_{\beta} \tag{28}$$

where $\epsilon_{\beta} \to 0$ as $\beta \to 1$.

Proof. Consider the case where $\nu > 0$. The trace of $\tilde{B}_1 + \tilde{B}_2 - (1+\lambda) I_2$ is negative if $\phi\bar{\lambda}_x\sigma^{-1} + 1 - \beta - \xi\zeta\phi > 0$. This implies that also in the case of full transparency a policy rule that does not react to the output gap is destabilizing. Nevertheless transparency increases the set of rules that are robust to expectational mistakes. In order to show this, notice that from (22), the stability condition for the case of no-transparency is instead

$$\phi\bar{\lambda}_x\sigma^{-1} + 1 - \beta - \xi\zeta\phi > \sigma^{-1}\frac{\kappa\xi + \kappa\xi\left(\phi - 1\right) + \phi\bar{\lambda}_x\left(1-\beta\right) - \xi\zeta\phi\bar{\lambda}_x}{2-\beta} > 0$$

which is a more stringent condition for stability than in the case of full transparency. Combining the stability condition for the case of transparency and the restriction on ϕ, I obtain the condition

$$\xi > \frac{\sigma^{-1}\bar{\lambda}_x}{\zeta} + \epsilon_\beta$$

where $\epsilon_\beta = \frac{(1-\beta)}{\phi\zeta}$. ■

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