Publishing the central bank's interest rate projections and learning*

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Abstract

We study the effects of publishing the central bank's interest rate projection when private agents form forecasts by using recursive learning algorithms based on past data. We investigate analytically the macroeconomic implications of this aspect of transparency under Taylor-type monetary policy rules. We find that when the central bank announces the path of expected short term interest rates obtained without taking into account that private agents are not perfectly rational, the properties of the equilibrium in terms of determinacy, learnability and speed of convergence may drastically change with respect to the case of no announcement. Revealing the path is counterproductive, as it makes conditions for determinacy and E-stability more stringent and the learning process slower. We also show that publishing output gap and inflation projections has opposite implications.

1 Introduction

As argued, among others, by Blinder (2000) and Svensson (2006) the current view of monetary policy making is based on the crucial observation that most of the effects of monetary policy on the economy are obtained through the impact on expectations (Woodford, 2005). According to this view, the increasing effort to enhance transparency of monetary policy allows, by moving expected future rates, to achieve the policy objectives in a more efficient way – smaller changes in the policy instrument are needed in order to reach the same objective. This is clearly a huge

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change with respect to the way monetary policy was thought to affect the economy some decades ago. It is not surprising that, in more recent years, the debate on the usefulness of the publication of any type of forward looking information by the central bank has been vibrant. The debate has been particularly fierce with respect to the publication of the central bank's future policy intentions, one of the aspects of transparency that presumably has the largest impact on expectations and, in turn, on the effectiveness of monetary policy decisions.

We can distinguish two approaches that central banks may adopt to communicate their view about the future developments of the policy instrument[1]. The first one is to reveal any type of information related to the implementation of monetary policy: final goals, economic data, models, the strategy and the operational implementation of policy decisions. This approach, even though theoretically appealing, is obviously not feasible in practice. An alternative is to provide information directly on the future policy intentions. This approach is characterized by different degrees of precision, depending on whether the central bank decides to reveal its expectation in an indirect manner - for example, the ECB and the Fed provide an implicit guidance to the market through the use of "key words"-, or with a direct and explicit revelation of the expected interest rate path - the Bank of Sweden, the Bank of Norway, the Bank of Island, the Czech National Bank and the Reserve Bank of New Zealand publish their own quantitative projections of the short-term interest rates.

The literature and policy discussions highlight a series of advantages and drawbacks associated with a direct signalling of the evolution of short term interest rates. In particular, this communication strategy might be beneficial because (*i*) it affects private expectations about the future evolution of monetary policy and, in turn, it allows a more efficient pricing of financial assets (Archer, 2005; Kahn, 2007; Svensson, 2004), (*ii*) it helps to enforce the optimal policy under commitment (Archer, 2005; Khan, 2007; Mishkin, 2004; Svensson, 2006; Woodford, 2005), (*iii*) it increases the accountability of the central bank and the incentives for producing good forecasts (Archer, 2005; Mishkin, 2004) and, finally, (*iv*) it also fosters the discussion within the monetary policy committee on policy objectives and on the appropriate models to be used in assessing the evolution of the economy (Archer, 2005). On the contrary, on top of the general observation that the provision of public information is not necessarily beneficial (Morris and Shin, 2002), it has been argued that the costs of an explicit announcement of policy intentions might outweigh the benefits if

[1]Here we assume that the short term interest rate is the policy instrument.

the central bank is already very transparent, in particular when the public does not understand its conditional nature. In such a situation, monetary policy decisions not in line with the announced policy intentions mayundermine the credibility of the central bank (Mishkin, 2004; Khan, 2007; Woodford, 2005).

So far the theoretical literature has found that when the central bank has an information advantage with respect to the private sector, the benefits of the publication of the interest rate path overcome the costs. For example, Eusepi and Preston (2007) find that when private agents do not have a model of how aggregate state variables are determined, communicating the entire policy decision process (i.e. the relevant conditioning variables and policy coefficients) anchors expectations and promotes macroeconomic stability. Similarly, Rudebusch and Williams (2006) find that when everything is known by the public but the parameters of the policy rule or the time-varying inflation target, publishing interest rate projections, in general, helps to align the public's and the central bank's expectations and, therefore, it reduces the magnitude of fluctuations in output and the inflation gap[2]. The analytical framework developed by Rudebusch and Williams (2006), however, also allows to evaluate the consequences of a situation in which the public misconstrues on the actual precision of the signal of the central bank. In this case the benefits of central bank communication are muted. Publishing interest rate projections can be counterproductive until private agents realize their misperception of the accuracy of central bank signals. This result, providing an example where the alignment of central bank and public expectations is not necessarily welfare improving, underlines the need for a well-developed communication strategy that mitigates such problems by highlighting both the conditionality and uncertainty regarding interest rate projections.

The unresolved debate among researchers – and central bankers – about the value of direct signalling the evolution of short term interest rates provides the key motivation for our analysis. In this work we focus on the effects of publishing the central bank's interest rate projection when (*i*) private agents form forecasts by using recursive learning algorithms based on past data, (*ii*) the central bank sets the interest rate as a reaction function to output and inflation and (*iii*) the policy path that the central bank publishes is the conditional interest rate obtained under the assumption of rational expectations.

[2]In their model the central bank communicates to the market a noisy signal of its one-step ahead expected interest rate. The variance of the information transmission error is an (inverse) measure of the degree of transparency.

Concerning the first hypothesis, there is now a large literature that assumes that private agents must "*obtain their quantitative knowledge of the dynamic properties of the system* [in which they operate] *on the basis of data generated by the economy itself; they are not given such knowledge by magic or divine revelation. Thus this knowledge must be based on some learning process that depends upon past observations of variables of the system*" (McCallum, 2009).

Concerning the second hypothesis, we restrict our attention to Taylor type rules, where the policy rule takes the form of a functional relationship between the interest rate - the dependent variable - and the inflation and the output gap.

Finally, the hypothesis that central banks projections are obtained under the assumption of rational expectations reflects the fact that those institutions that announce their policy path, in general, obtain their projections from large general equilibrium models where interest rates are endogenously determined and the expectations are assumed to be perfectly rational.[3]

Based on these three hypotheses we first ask whether the announcement of the interest rate path would enlarge or restrict the set of policy rules associated with uniqueness of the rational expectations equilibrium (REE). Subsequently, we ask whether, in a world where agents form forecasts by using recursive learning algorithms, this aspect of monetary policy transparency enlarges or restricts the set of policy rules under which agents end up learning the REE. Grounded on the set of policies that imply a unique and learnable REE, we look at speed of learning as a criterion to characterize further the effects of the publication of future policy intentions; here the exercise consists in taking a given policy rule that implies a determinate and learnable REE and ask whether the publication of the path helps agents to learn faster.

Since all central banks that are actually publishing their interest rate projections

[3]The Reserve Bank of New Zealand has at the core of its Forecasting and Policy System (FPS) a general equilibrium macro model "with around 30 key behavioral variables", where expectations are modeled "as some weighted combination of the model-consistent forecast and some other function of the recent data". Learning is not taken explicitly into account ("a valuable next step would be to specify how agents learn about the new policy rules, although as yet there is no generally-accepted theory of learning in macroeconomics"). The Norges Bank produces the forecasts using a core macroeconomic DSGE model with "rational agents reacting to exogenous disturbances". The Swedish Risksbank uses a macroeconomic general equilibrium model called RAMSES where individuals expectations are assumed to be rational ("RAMSES can be formulated mathematically as a system of non-linear differential equations with rational expectations"). The Central Bank of Island uses a model, called QMM "to assist with analyzing the current economic situation, making economic projections, assessing the effects of alternative policies and shocks" and where expectations "are assumed to be rational, i.e. consistent with the QMM model structure (model consistent expectations)".

were already publishing the output gap and inflation expectations, we repeat our analysis under the assumption that the policy maker publishes also its projections about these variables.

Our approach differs from the one of Rudebusch and Williams (2006), as asymmetric information in our model concerns all the parameters of the model, while in their analysis asymmetry concerns only one policy aspect (alternatively the relative weights to output gap in the policy maker loss function, or the medium-run inflation target). Differently from Eusepi and Preston (2007), we assume that the central bank announces its expected REE, while in in their model the policy maker announces the coefficients of the policy rule and/or the expected inflation and output gap.

In an environment where agents use the information about the policy path in order to form expectations about future inflation and output gap, revealing only the path of the interest rate makes conditions for determinacy and E-stability more stringent – the set of policies that implies a determinate and E-stable REE when the central bank communicates the path is a subset of the one that implies a determinate and E-stable REE under no announcement – and the learning process slower. In fact, by publishing the interest rate projection consistent with the REE, the central bank is not taking into account the fact that agents are learning and, therefore, is not taking into account the mistakes in private agents expectations. As a result the learning process is slower and the economy becomes more vulnerable to self-fulfilling expectations.

Publishing output gap and inflation projections has opposite implications . While the information about the policy rate (the instrument variable of the model) is implicitly used by private agents in order to form expectations about future inflation and output (the control variables of the model), information about output gap and inflation is used directly to predict those variables. Therefore, this type of announcement makes conditions for determinacy and E-stability less stringent and helps agents to learn faster than under no communication.

Finally, a central bank that has already been publishing the expected output gap and inflation and decides to publish also the interest rate path will increase the region of indeterminacy and instability under learning and slow down the learning process. In particular, if the agents pay – or are induced to pay – too much attention to the announcement of the interest rate path with respect to the one they dedicate to expected inflation and output gap, the region of instability under learning may result larger and the speed of learning lower than under no announcements.

The paper is organized as follows. In Section 2 we develop the baseline model; in section 3 we analyze the effect of publishing the projections about the policy instrument in terms of uniqueness of the REE, stability of the equilibrium under learning and speed of convergence; in section 4 we analyze the alternative scenario where the central bank also publishes its expectations about the control variables of the model – that is the output gap and inflation; in section 5 we consider some extensions; section 6 concludes.

2 The model

We consider a standard New-Keynesian model modified to allow for asymmetric information sets for private agents and the central bank. In particular, we assume the central bank has an informational advantage over the public. While the central bank has a perfect knowledge of the economy (and of his decisions), private agents know the states of the economy, but do not know exactly how these determine the actual value of the control variables of the model (inflation and output gap).

For our analysis, we use a standard log-linearized New Keynesian model. We write the system as

$$x_t = E_t^* x_{t+1} - \varphi \left(i_t - E_t^* \pi_{t+1} \right) + g_t \tag{2.1}$$

$$\pi_t = \alpha x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{2.2}$$

where x_t is the output gap, measured as the log deviation of actual output (y_t) from potential output (z_t) (i.e., the level of output that would arise if wages and prices were perfectly competitive and flexible), π_t is actual inflation at time t, $E_t^* \pi_{t+1}$ is the level of inflation expected by private agents for period $t+1$, given the information at time t. Similarly $E_t^* x_{t+1}$ is the level of the output gap that private agents expect for period $t+1$, given the information at time t. I write E_t^* to indicate that expectations need not be rational (E_t without $*$ denotes the rational expectation); i_t is the short-term nominal interest rate and is taken to be the instrument for monetary policy; g_t is a demand shock and u_t a cost-push shock

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt} \quad \text{and} \quad u_t = \rho_u u_{t-1} + \varepsilon_{ut}, \tag{2.3}$$

where ε_{gt} and ε_{ut} are mutually orthogonal white noises with variances σ_g^2, σ_u^2.

We supplement equations (2.1)-(2.3) with a policy rule. We consider the nominal interest rate as the policy instrument and model it by means of a reaction function.

Thus, a policy rule is just a functional relationship between a dependent variable – the interest rate – and some endogenous variables – the inflation and output gap

$$i_t = \gamma + \gamma_x x_t + \gamma_\pi \pi_t. \tag{2.4}$$

Under this type of policies the economy evolves according to the following stochastic dynamic system[4]:

$$y_t = Q + F \times E_t^* y_{t+1} + S w_t, \tag{2.5}$$

with

$$\begin{aligned} y_t' &= \begin{bmatrix} \pi_t & x_t \end{bmatrix} \\ w_t &= \Psi w_{t-1} + \varepsilon_t, \end{aligned}$$

Following the literature on monetary policy and learning we focus on the minimum state variable solution (MSV) of the model, which is of the form

$$y_t = A + B w_t \tag{2.6}$$

where A is a $(2X1)$ vector and B a $(2X2)$ matrix.

It is well known that the necessary and sufficient condition for the rational expectation equilibrium (REE) to be unique is that all eigenvalues of the F matrix lie inside the unit circle. As shown in Bullard and Mitra (2002) this condition reduces to have

$$\gamma_\pi > 1 - \frac{(1-\beta)}{\alpha}\gamma_x. \tag{2.7}$$

Let assume now that agents do not initially have rational expectations, but instead they form forecasts by using recursive learning algorithms. In particular, we assume that agents know the functional form of the MSV solution (2.6), but not the value of the A and B matrices. We follow Marcet and Sargent (1989) and Evans and Honkapohja (2001) to model the learning process. Private agents update their forecasts through recursive least squares algorithms. This procedure is an example of adaptive real-time learning, which basic idea is that agents follow a standard statistical or econometric procedure for estimating the perceived law of motion of

[4]See appendix 1, for a complete derivation of the system and solution.

the endogenous variables.

Let

$$\beta_t = \begin{pmatrix} A_t' \\ B_t' \end{pmatrix} \text{ and } z_t' = (1, u_t, g_t).$$

then, under recursive least squares learning, agents face the following perceived law of motion (PLM)

$$y_t = z_t'\beta_t \tag{2.8}$$

where β_t's are the statistics inferred recursively from past data according to

$$\beta_t = \beta_{t-1} + t^{-1}R_t^{-1}z_{t-1}\left(y_{t-1}' - z_{t-1}'\beta_{t-1}\right) \tag{2.9}$$

where

$$R_t = R_{t-1} + t^{-1}\left(z_{t-1}z_{t-1}' - R_{t-1}\right) \tag{2.10}$$

Therefore, forecasts are updated by a term that depends on the last prediction errors. The forecasts are then computed from the estimated law of motion[5]

$$E_t y_{t+1} = A_t + B_t\Psi w_t. \tag{2.11}$$

In order to study whether the recursive least-squares estimates, β_t, converge to the REE, $\overline{\beta}$, we refer to the concept of expectation stability (E-stability) described in Evans and Honkapohja (2001). It is known for learning problems of the type described here that, under fairly general assumptions, convergence to REE obtains if and only if E-stability conditions are satisfied.

The issue of stability under learning of a particular equilibrium is addressed by studying the mapping from the estimated parameters - the perceived law of motion, PLM - to the true data generating process - the actual law of motion, ALM. When expectations in system (2.5) evolve according to expression (2.11), the ALM can be written as

$$y_t = Q + FA_t + (FB_t\Psi + S)\, w_t. \tag{2.12}$$

We define the mapping from the PLM to the ALM

$$T(A_t, B_t) = (Q + FA_t, FB_t\Psi + S). \tag{2.13}$$

[5]Here we assume that private agents know the Ψ. This assumption is commonly used in the learning literature and does not affect our results in terms of determinacy, E-stability and speed of convergence.

As shown in Marcet and Sargent (1989) and Evans and Honkapohja (2001), it turns out that the dynamic system described by equations (2.11) and (2.12) can be studied directly in terms of the associated *ordinary differential equations* (ODE)[6]

$$\frac{\partial}{\partial \tau}(A) = T_A(A) - A \tag{2.14}$$

$$\frac{\partial}{\partial \tau}(B) = T_B(B) - B \tag{2.15}$$

where τ denotes "notional" or "artificial" time and $T_A(A) = Q + FA$ and $T_B(B) = FB\Psi + S$.

The E-stability criterion governs stability under such adaptive learning. Under E-stability the economy converges in the long run to MSV solution. A particular MSV solution $\left(\overline{A}, \overline{B}\right)$ is E-stable if the MSV fixed point of the differential equations (2.14) and (2.15) are locally asymptotically stable at that point (see Evans and Honkapohja, 2001). E-stability conditions are readily obtained by computing the derivative of the ODE's

$$\frac{d\left(T_A(A) - A\right)}{dA}$$

and

$$\frac{d\left(T_B(B) - B\right)}{dB}$$

and checking whether all the eigenvalues have real part less than zero.

As shown in Bullard and Mitra (2002) expression (2.7) provides also necessary and sufficient condition for E-stability.

3 Central Bank interest rate path communication

The aim of this section is to to analyze the effect of publishing the interest rate path in terms of determinacy and stability under learning of the REE and in terms of the speed of convergence. While agents expectations evolves according to the learning procedure described in the previous section, for the central bank we retain the assumption of perfect rationality. This assumption reflects the fact that in practice those institutions that announce their policy path obtain their projections - to a

[6]In general, a REE is locally stable under real time learning if the relevant real time learning algorithm converges to it. The REE is E-stable if it is locally asymptotically stable under the associated ODE. In our example, since regularities conditions described in chapter 6 of Evans and Honkapohja (2001) are satisfied, E-stability implies stability under real time learning, and therefore local stability of the REE under learning can be studied directly in terms of the ODE.

large extent - from (DSGE) macroeconomic models solved under the rational expectation hypothesis. Therefore, our approach (i.e. the public and the central bank have asymmetric information about all parameters describing the model economy) differs from the one considered in Rudebusch and Williams (2006) where asymmetric information concerns only specific aspects of the policy rule. The main difference is that, while in Rudebusch and Williams (2006) the public, knowing all the parameters in the economy except one, needs only to improve its ability in forecasting future policy actions in order to learn the REE, in our model the public's learning process involves all the parameters of the model and in order to learn the REE has to use all information derived from the IS, the Phillips curve and the interest rate rule.

In order to study the macroeconomic effects of the announcement we write the IS and the Phillips curve $T-1$ periods ahead and substitute them in the (2.1) and (2.2) expressions to obtain

$$x_t = E_t^* x_{t+T} - E_t^* \sum_{j=0}^{T-1} \left(\varphi i_{t+j} - \varphi \pi_{t+j+1} - g_{t+j}\right) \tag{3.1}$$

and

$$\pi_t = \beta^T E_t^* \pi_{t+T} + E_t^* \sum_{j=0}^{T-1} \beta^j \left(\alpha x_{t+j} + u_{t+j}\right) \tag{3.2}$$

It is worth to notice that in order to obtain equations (3.1) and (3.2) we are using the law of iterated expectations hypothesis, that holds both under RE and least square learning. [7] This formulation makes evident the central role not only of actual real interest rate, but also of expected future short term real interest rates in determining today output and inflation.

3.1 Determinacy and E-Stability of the REE

For simplicity we consider the case where the central bank announces only the next period expected interest rate. In this case we can write (3.1) and (3.2) for $T = 2$, as

$$\pi_t = \beta^2 E_t^* \pi_{t+2} + \alpha x_t + u_t + \beta\alpha E_t^* x_{t+1} + \beta E_t^* u_{t+1} \tag{3.3}$$

$$x_t = E_t^* x_{t+2} - \varphi \left(i_t - E_t^* \pi_{t+1} + E_t^* i_{t+1} - E_t^* \pi_{t+2}\right) + g_t + E_t^* g_{t+1}. \tag{3.4}$$

[7] See Evans, Honkapohja and Mitra (2003).

Ferrero and Secchi (2008) study the case of the Reserve Bank of New Zealand, that publishes its interest rate projections since 1999, and show that market expectations on short term interest rates respond in a significant and consistent way to the unexpected component of the published path, even though adjustment is not complete. That is, private agents move their expectations only partially in the direction of the announcement. Therefore, we assume that private agents expectations about the expected interest rate depend on both central bank's announcement and their own expectation. [8] Let $0 \leq (1-\lambda_1) \leq 1$ be the weight that agents give to the central bank's announcement, than equation (3.4) can be written as

$$x_t = E_t^* x_{t+2} - \varphi \left(i_t - E_t^* \pi_{t+1} + (1-\lambda_1) E_t^{CB} i_{t+1} + \lambda_1 E_t^* i_{t+1} - E_t^* \pi_{t+2} \right) + g_t + E_t^* g_{t+1}.$$

where $E_t^{CB} i_{t+1}$ is obtained from the MSV solution (2.6) for time t inflation and output gap,

$$E_t^{CB} i_{t+1} = a_i + \rho_u b_{u,i} u_t + \rho_g b_{g,i} g_t \tag{3.5}$$

and a_i, $b_{u,i}$ and $b_{g,i}$ are easily obtained by substituting (2.6) in the contemporaneous reaction function (2.4).

Under these hypotheses the economy evolves according to the following expression:

$$\widehat{y}_t = \widehat{Q} + \widehat{F} \times E_t^* \widehat{y}_{t+1} + \widehat{S} w_t, \tag{3.6}$$

where $\widehat{Q}$, $\widehat{F}$ and $\widehat{S}$ are derived in appendix 2 and

$$\widehat{y}_t' = \left[\begin{array}{cccc} \pi_t & x_t & E_t^* \pi_{t+1} & E_t^* x_{t+1} \end{array} \right]$$

We provide a characterization of the necessary and sufficient condition for determinacy in the following proposition.

Proposition 1. *Let $\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1 \neq 0$. In an economy that (i) evolves according to the system of equations (3.6), where (ii) at time* t *the central bank publishes the time* t+1 *interest rate projection consistent with the REE and (iii) private agents give weight $0 \leq (1-\lambda_1) \leq 1$ to these projections, revealing the interest rate path makes conditions for determinacy more stringent. In particular, the necessary and*

[8] Alternatively, we may assume that a fraction λ_1 of private agents in the economy use their own prediction and a fraction $(1-\lambda_1)$ fully internalize the central bank's projection.

sufficient condition for uniqueness of the REE is

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{1-\beta}{\alpha}\gamma_x. \tag{3.7}$$

Proof. See appendix 2. □

In order to understand the implications of Proposition 1 it is useful to look at the case where the policy maker does not announce future policy intentions. In that case the necessary and sufficient condition for determinacy has a straightforward interpretation (Woodford, 2003). The Phillips curve (2.2) being an equilibrium condition implies that each percentage point of permanently higher inflation determines a permanently higher output gap of $(1-\beta)/\alpha$ percentage points. Therefore, expression (2.8) states that necessary and sufficient condition for determinacy is that the long-run increase in the nominal interest rate prescribed by policy rules with contemporaneous endogenous variables (2.4) should be larger than the permanent increase in the inflation rate.

Applying a similar reasoning to the case where the central bank announces the next period expected interest rate, we have that the Phillips curve still implies that each percentage point of permanently higher inflation determines a permanently higher output gap of $(1-\beta)/\alpha$ percentage points. However now, expression (3.7) states that necessary and sufficient condition for determinacy is that the long-run increase in the nominal interest rate should be at least $2/(1+\lambda_1)$ times as big as the permanent increase in the inflation rate. For $0 \leq (1-\lambda_1) \leq 1$, this implies a larger response than under no announcement. Notice in particular, that depending on the value of λ_1, the classical Taylor rule with $\gamma_x = 0.5$ and $\gamma_\pi = 1.5$ may imply an undetermined REE - depending on the weigth λ_1. The reason why conditions for determinacy are more stringent under the announcement of the expected interest rate, is related to the well known fact that when the interest rate is set as a function only of exogenous shocks, small deviations from the equilibrium cannot be offset. By publishing the interest rate projection consistent with the REE, the central bank is announcing a policy rate that behaves like a "fundamental" reaction function and is, therefore, unable (or less able) to offset initial deviations from the equilibrium.

Let assume now that agents do not initially have rational expectations, but instead they form forecasts by using the recursive learning algorithms described in the previous section. To study stability of the REE under learning we rewrite the system as

$$y_t = \widetilde{Q} + \widetilde{F} \times E_t^* Y_{t+1} + \widetilde{V} \times E_t^* Y_{t+2} + \widetilde{S} w_t, \tag{3.8}$$

where $\widetilde{Q}$, $\widetilde{F}$, $\widetilde{V}$ and $\widetilde{S}$ are derived in appendix 3 and

$$y_t' = \begin{bmatrix} \pi_t & x_t \end{bmatrix}$$

As the REE is still given by equation (2.6), the private agents forecasts under recursive learning are computed from the estimated PLM

$$y_t = A + Bw_t$$

from which we compute the expectations

$$\begin{aligned} E_t y_{t+1} &= A + B\Psi w_t \\ E_t y_{t+2} &= A + B\Psi'\Psi w_t \end{aligned}$$

In this case the actual law of motion (ALM) of y_t is

$$y_t = \left(\widetilde{Q} + \widetilde{F}A + \widetilde{V}A\right) + \left(\widetilde{F}B\Psi + \widetilde{V}B\Psi'\Psi + \widetilde{S}\right) w_t,$$

the mapping from the PLM to the ALM is

$$T(A, B) = (\widetilde{Q} + \left(\widetilde{F} + \widetilde{V}\right) A, \widetilde{F}B\Psi + \widetilde{V}B\Psi'\Psi + \widetilde{S}) \tag{3.10}$$

and the associated *ordinary differential equations* are

$$\frac{\partial}{\partial \tau}(A) = \widetilde{Q} + \left(\widetilde{F} + \widetilde{V}\right) A - A \tag{3.11}$$

$$\frac{\partial}{\partial \tau}(B) = \widetilde{F}B\Psi + \widetilde{V}B\Psi'\Psi + \widetilde{S} - B \tag{3.12}$$

In the following proposition we state the conditions under which the REE is E-stable.

Proposition 2. *Let $\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1 \neq 0$. In an economy that (i) evolves according to the system of equations (3.8), where (ii) at time* t *the central bank publishes the time* t+1 *interest rate projection consistent with the REE and (iii) private agents give weight $0 \leq (1 - \lambda_1) \leq 1$ to these projections, revealing the path makes condition for E-stability more stringent. In particular, the necessary and sufficient condition* *for stability under learning (E-stability) of the equilibrium (2.6) is*

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{1-\beta}{\alpha}\gamma_x. \tag{3.13}$$

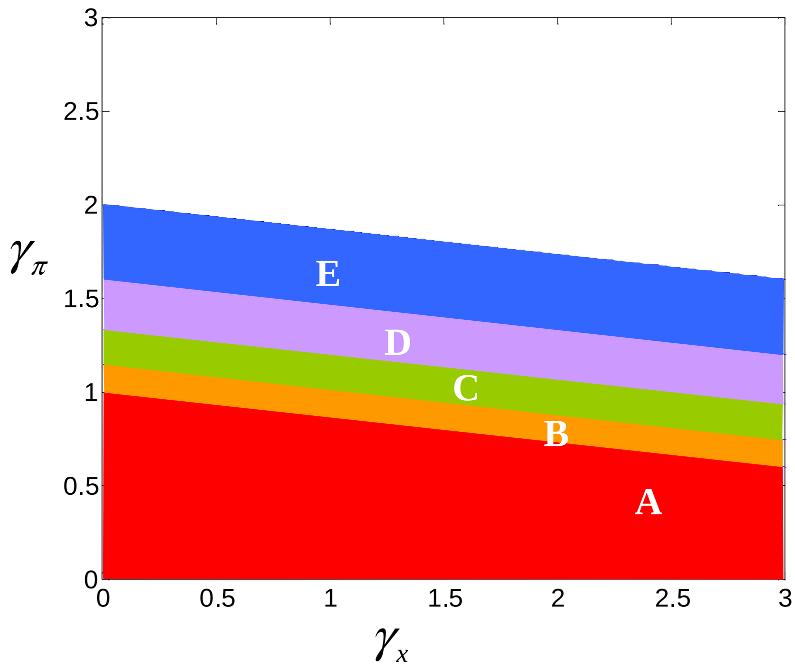
Proof. See appendix 3. □

Comparing conditions (2.7) and (3.13) we immediately observe that revealing the path makes conditions for E-stability more stringent.[9] By publishing the interest rate projection consistent with the REE, the central bank - that is not taking into account the fact that agents are learning - is acting as if the expected interest rate is a "fundamental" reaction function and therefore is actually increasing the initial bias of the learning agents, and makes it more difficult for the agents to learn. To better understand, let consider a model where private agents have an initial positive bias in expected inflation. This positive bias will lead to higher inflation both directly through the Phillips curve ($\pi_t = \alpha x_t + \beta E_t^* \pi_{t+1} + u_t$) and indirectly through the real interest rate that affects the output gap in the IS curve ($x_t = E_t^* x_{t+1} - \varphi\left(i_t - E_t^* \pi_{t+1}\right) + g_t$) and therefore inflation (in the Phillips curve). A policy rule that reacts directly to inflation or expected inflation would help to offset the bias, while a policy that reacts directly to the fundamentals is not. In that case the policy is not able to contrast the cumulative movement away from REE. In understanding our results two points are crucial: (i) the hypothesis that, in a world where private agents are learning from past data – and along their learning process they produce biased predictions of the main macro variables – the central bank publishes its projection obtained under rational expectations; (ii) publishing the interest rate projection consistent with the REE, the central bank is acting as if the expected interest rate is determined by a "fundamental" policy rule and therefore is actually increasing the bias of the learning agents. To offset this effect, the response of the policy to inflation has to be larger than in the case of no announcement.

Figure 1 compares in the (γ_x, γ_π) space the region where we have a unique and E-stable REE for different weights $0 \leq (1-\lambda_1) \leq 1$. Region A describes all the policies under which the REE is not unique and not E-stable, under no announcement; when the central bank announces its policy path and private agents give weight 0.25 to it, the region of instability and indeterminacy enlarges and includes also the region B. When the weight increases to 0.5, the region of instability includes also the subspace C; for a weight of 0.75 it includes also region D and, finally when agents

[9] Even though in this case conditions for determinacy and E-stability coincide, this has not to be the case in general (see Evans and Honkapohja, 2003).

Figure 1: Determinacy and E-stability and policy path announcement



fully internalize the announcement, the region of indeterminacy and instability under learning is given by A+B+C+D+E. Therefore when the policy maker reveals the path, the smaller the weight the agents give to the announcement, the smaller is the region of indeterminacy and instability under learning. In particular, when the central bank publishes its expected interest rate and the weight that private agents give to the projection is 0.5, the classical Taylor rule with $\gamma_x = 0.5$ and $\gamma_\pi = 1.5$ would imply a unique and E-stable REE; however for larger weights, the Taylor rule will fall in the region of indeterminacy and E-instability.

3.2 Speed of convergence

In the previous sections we have analyzed the effect of announcing the expected interest rate in terms of uniqueness of the REE and of its asymptotic properties under learning. Combinations of (γ_x, γ_π) that imply a determinate and E-stable REE are usually defined in the literature as "good" policies. The concept of speed of convergence can be used in order to refine further the set of "good" policies (see Ferrero, 2007). If convergence is rapid, then we may think to focus on asymptotic behavior, because the economy would typically be close to the REE. Conversely, if convergence is slow, then the economy would be far from its REE, and, hence, its behavior would be dominated by the transitional dynamics.

In the literature, the problem of the speed of convergence of recursive least square learning algorithms in stochastic models has been analyzed mainly through numerical procedures and simulations. The few analytical results on the transition to the rational expectations equilibrium environment are obtained by using a theorem of Benveniste, Metiver and Priouret (1990) that relates the speed of convergence of the learning process to the derivative of the associated ODE at the fixed point. In the present case, the ODE's to be analyzed are those described in expression (3.11) and (3.12).

We define

$$S_1 = \left\{ \gamma_\pi, \gamma_x : \gamma_\pi > \max \left[\begin{array}{c} \frac{\left(\beta^2+2\alpha\varphi\right)}{\alpha\varphi(1+\lambda_1+\beta)} - \frac{\left(1+\lambda_1-\beta^2\right)}{\alpha(1+\lambda_1+\beta)}\gamma_x \\ \frac{4(2\beta+1)\alpha\varphi-\left(2\beta^2-1\right)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi} - \frac{\left(1-2\beta^2\right)}{(1+2\beta)\alpha}\gamma_x \end{array} \right] \right\}$$

the set of policies (i.e. combinations of γ_π and γ_x) under which all the eigenvalues of $F+V$ matrix have real part smaller 1/2.

The following proposition, adapting arguments from Marcet and Sargent (1995), shows that by choosing the γ_π and γ_x, the policy-maker not only determines the level of inflation and output gap in the long run, but also the speed at which the the economy converges to the REE, i.e. the speed at which agents learn

Proposition 3. *In an economy that (i) evolves according to the system of equations (3.8), where (ii) private agents give weight* $0 \leq (1-\lambda_1) \leq 1$ *to the central bank's announcement, and (iii) the central bank chooses a policy* $(\gamma_\pi, \gamma_x) \in S_1$*, then*

$$\sqrt{t}\left(\beta_t - \overline{\beta}\right) \xrightarrow{D} N\left(0, \Omega_\beta\right)$$

where the matrix Ω_β *satisfies*

$$\left[\frac{I}{2}(F-I)\right]\Omega_\beta + \Omega_\beta\left[\frac{I}{2}(F-I)\right]' + E\left[T(\overline{\beta}') - \overline{\beta}'\right)\right]\left[T(\overline{\beta}') - \overline{\beta}')\right]' = 0 \qquad (3.14)$$

Proof. see appendix 4. □

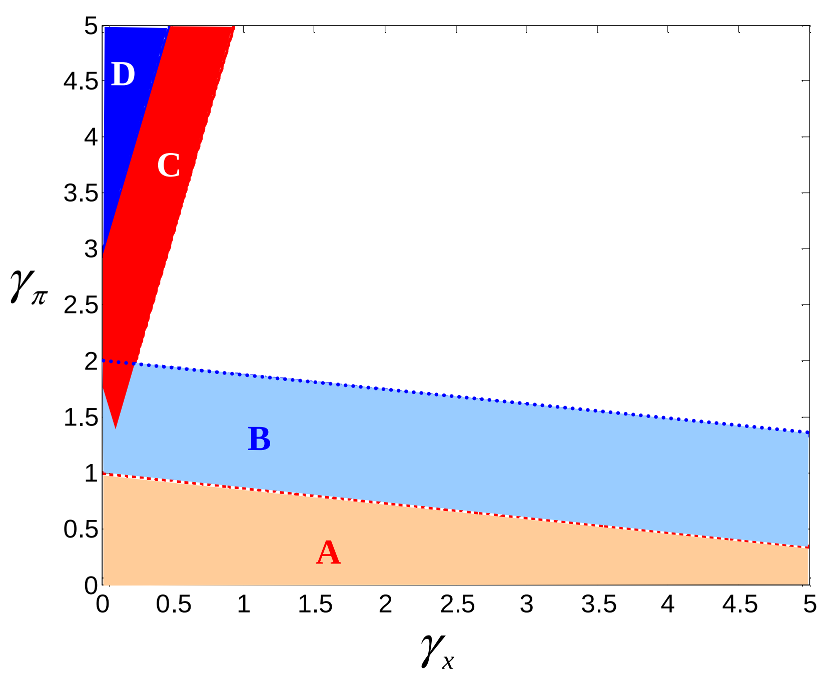
If the conditions of Proposition 3 are satisfied, the estimated β_t converges to the REE, $\overline{\beta}$, at root-t speed. Root-t is the speed at which, in classical econometrics, the mean of the distribution of the least square estimates converges to the true value of the parameters estimated. Note that the formula for the variance of the estimator β_t is modified with respect to the classical case. In particular, if $(\gamma_\pi, \gamma_x) \in S_1$, the higher the eigenvalues of F, the larger the asymptotic variance of the limiting distribution (Marcet and Sargent, 1995). In this case, convergence is slower in the sense that the probability that a shock will drive the estimates far away from the REE is higher and the period of time that agents will need in order to learn it back is larger.

Proposition 4. *In an economy that (i) evolves according to teh system of equations (3.8), where (ii) private agents give weight* $0 \leq (1 - \lambda_1) \leq 1$ *to the central bank's announcement, and (iii) the central bank chooses a policy* $(\gamma_\pi, \gamma_x) \in S_1$*, the smaller the weight to the announcement, the smaller the set of policies under which private agents learn at root-t speed.*

Proof. see appendix 5. □

In Figure 2 we focus on the two extreme cases where there is no announcement ($\lambda_1 = 1$) and where private agents fully internalize the announcement ($\lambda_1 = 0$).

Figure 2: Determinacy, E-stability, root-t convergence (no communication)



By looking at Figure 2, it is clear that the set of combinations (γ_x, γ_π) resulting in root-t convergence is much smaller than the one under which E-stability holds. Moreover, the region of "fast" convergence (i.e. root-t convergence) is smaller when the central banks announces its policy (region D) than under no announcement (region C+D)

Let's define

$$S_2 = \left\{ (\gamma_\pi, \gamma_x) \in R_+^2 : \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x < \gamma_\pi < \max \begin{bmatrix} \frac{\left(\beta^2+2\alpha\varphi\right)}{\alpha\varphi(1+\lambda_1+\beta)} - \frac{\left(1+\lambda_1-\beta^2\right)}{\alpha(1+\lambda_1+\beta)}\gamma_x \\ \frac{4(2\beta+1)\alpha\varphi-\left(2\beta^2-1\right)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi} - \frac{\left(1-2\beta^2\right)}{(1+2\beta)\alpha}\gamma_x \end{bmatrix} \right\} \tag{3.16}$$

the set of policies (i.e. combinations of γ_π and γ_x) under which all the eigenvalues of F have real part less than one but not all have real part less than 0/5.

Propositions 3 and 4 do not apply when $(\gamma_\pi, \gamma_x) \in S_2$. However, it can be shown by Monte Carlo calculations that the estimates, β_t, converge to the REE, $\overline{\beta}$, at a speed different from root-t. In this case, the effects of initial conditions fail to die out at an exponential rate (as it is needed for *root-t convergence*) and agents' beliefs converge to rational expectations at a rate slower than root-t. In particular, also when $(\gamma_\pi, \gamma_x) \in S_2$, the link between the derivative of the ODE, the slope of the $T()$ mapping and the speed of convergence holds.

Marcet and Sargent (1995) suggest a numerical procedure to obtain an estimate of the rate of convergence when $(\gamma_\pi, \gamma_x) \in S_2$. In this case it is possible to define the rate of convergence, δ, for which

$$t^\delta \left(\beta_t - \overline{\beta}\right) \xrightarrow{D} F \tag{3.15}$$

for some non-degenerate well-defined distribution F with mean zero and variance Ω_F.

Expression (3.16) can be used to obtain an approximation of the rate of convergence[10] for large t. Since $E\left[t^\delta \left(\beta_t - \overline{\beta}\right)\left(\beta_t - \overline{\beta}\right)'\right] = \Omega_F$ as $t \to \infty$, we can write

$$\frac{E\left[t^{2\delta}\left(\beta_t - \overline{\beta}\right)\left(\beta_t - \overline{\beta}\right)'\right]}{E\left[(tz)^{2\delta}\left(\beta_{tz} - \overline{\beta}\right)\left(\beta_{tz} - \overline{\beta}\right)'\right]} \to 1 \quad \text{or} \quad \delta = \frac{1}{2\log z} \log \frac{E\left[\left(\beta_t - \overline{\beta}\right)\left(\beta_t - \overline{\beta}\right)'\right]}{E\left[\left(\beta_{tz} - \overline{\beta}\right)\left(\beta_{tz} - \overline{\beta}\right)'\right]}$$

The expectations can be approximated by simulating a large number of inde-

[10] The calculation of the rate of convergence is based on the assumption that such a δ exists.

pendent realizations of length t and tz, and calculating the mean square distance from the $\overline{\beta}$ across realizations for each coefficient. Table 1 reports the rate of convergence, δ, the real part of the largest eigenvalue of the $(F+V)$ matrix, k, the number of quarters needed in order to halve the initial expectation error[11], $T_{1/2}$, and the number of quarters needed in order to reduce to one third the initial error, $T_{1/3}$, for different values of $(\gamma_\pi, \gamma_x) \in S_2$. [12]

Table 1: Speed of convergence and simulations

		$\gamma_\pi = 1.5$		$\gamma_\pi = 2.5$		$\gamma_\pi = 3.5$	
		$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$
$\gamma_x = 0.25$	k	0.76	>1	0.19	0.86	0.07	0.62
	δ	0.24	*N.A.*	>0.5	0.14	>0.5	0.37
	$T_{1/2}$	40	*N.A.*	5	390	4	19
	$T_{1/3}$	220	*N.A.*	11	>400	7	70
$\gamma_x = 0.5$	k	0.85	>1	0.63	0.90	0.42	0.73
	δ	0.14	*N.A.*	0.36	0.10	>0.5	0.26
	$T_{1/2}$	235	*N.A.*	11	>400	5	40
	$T_{1/3}$	>400	*N.A.*	35	>400	11	192
$\gamma_x = 1$	k	0.91	>1	0.79	0.92	0.67	0.82
	δ	0.08	*N.A.*	0.21	0.07	0.32	0.17
	$T_{1/2}$	>400	*N.A.*	43	>400	14	144
	$T_{1/3}$	>400	*N.A.*	295	>400	48	>400

NOTE: In all simulations we compute the rate of convergence with 1000 independent realizations for t=9000 and tz=10000 periods. k is the real part of the largest eigenvalue of the $F+V$ matrix; $T_{1/2}$ indicates number of quarters needed in order to reduce the inflation forecast to one half of the initial bias; $T_{1/3}$ indicates number of quarters needed in order to reduce the inflation forecast to one third of the initial bias.

Calculations show that (i) the larger the response to output gap, γ_π, the higher the real part of the larger eigenvalue, the smaller δ and the lower the speed of convergence; (ii) the opposite relation holds for the response to inflation: the larger γ_x, the higher the speed of convergence; (iii) for a given (γ_π, γ_x) policy, the announcement of the policy path has a large impact on the speed of convergence. For the Taylor rule's parameter $(\gamma_\pi = 1.5, \gamma_x = 0.5)$, under no announcement we need more than 50 years in order to halve the initial expectation error, when the announcement is

[11]Initial expectation error is 10 per cent of the REE.

[12]Simulations are obtained under Clarida, Galí and Gertler (CGG, 2000) calibration: US data, $\varphi = 4$, $\alpha = 0.075$, $\beta = 0.99$; We use quarterly interest rates and we measure inflation as quarterly changes in the log of prices. Therefore our CGG calibration divides by 4 the α and multiplies by 4 the φ reported by CGG (see also Honkapohja and Mitra, 2004).

fully internalized agents never learn. Considering a stronger response to inflation we can speed up the learning process, but differences between announcing or not the interest rate path remain substantial: for $\gamma_\pi = 3.5$ and $\gamma_x = 0.5$ the initial error is halved in about one year under no announcement, but we still need 10 years under announcement.

In order to characterize all combinations of (γ_π, γ_x) in terms of the speed of convergence we define the speed of convergence isoquants that map elements of the set of *contemporaneous-data reaction functions* into a speed of convergence measure.[13].

Definition 1. *A speed of convergence isoquant-k is a curve in R^2 along which all points – combinations (γ_π, γ_x) of a monetary policy reaction function – imply that the largest eigenvalue of $F+V$ has real part equal to k. In an economy that evolves according to equations (2.1), (2.2), (2.3) and contemporaneous-data reaction function (2.4), the k-isoquant satisfies*

$$\gamma_\pi = \max \begin{bmatrix} \frac{\left(1-2k+\beta^2+2\alpha\varphi\right)}{(2k+\beta+\lambda_1)\alpha\varphi} - \frac{\left(-\beta^2+2k+\lambda_1\right)}{(2k+\beta+\lambda_1)\alpha}\gamma_x, \\ \frac{2}{(k+\lambda_1)} - \frac{\left(\beta^2-k\right)(1-k)}{\alpha\varphi(k+\beta)(k+\lambda_1)} - \frac{\left(k-\beta^2\right)}{\alpha(k+\beta)}\gamma_x \end{bmatrix}. \tag{3.16}$$

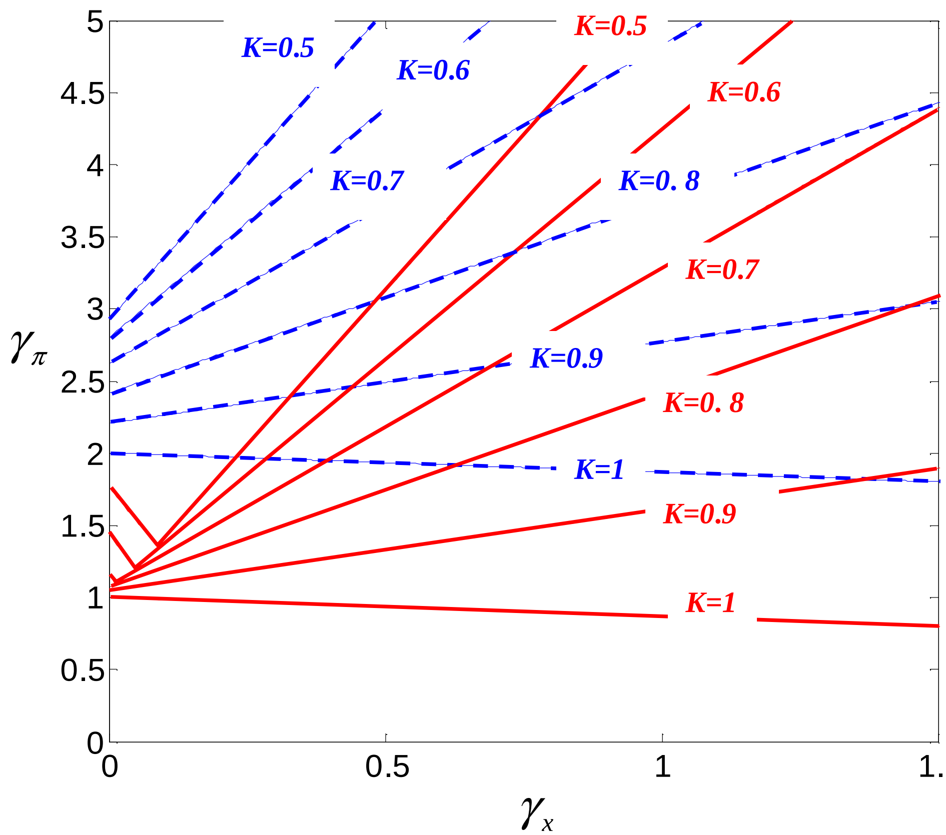
Figure 3 shows the speed of convergence isoquants in the two extreme cases where there is no announcement ($\lambda_1 = 1$) and where the agents fully internalize the announcement ($\lambda_1 = 0$).

We observe that, for a given λ_1, the lower the isoquant, the slower the convergence. In fact, from Marcet and Sargent (1995), the larger the real part of the largest eigenvalue of $F+V$, the slower the convergence and the lower the isoquant. Moreover, for a given policy, the speed at which agents learn is lower if the central banks announces its policy path. The next proposition formalizes these results.

Proposition 5. *In an economy that (i) evolves according to the system of equations (3.8), where (ii) private agents give weight $0 \leq (1-\lambda_1) \leq 1$ to the central bank's announcement, and (iii) the central bank chooses a policy $(\gamma_\pi, \gamma_x) \in S_2$, for a given γ_x, the larger is λ_1 – the smaller the weight that private agents give to the policy path projections – the smaller has to be γ_π in order to reach the same speed of convergence – the k-isoquant. Or in other terms, for a given combination of γ_x,γ_π, the larger is λ_1 the smaller is k and the faster the learning process.*

[13] In the definition we relate speed of convergence to the eigenvalues of the matrix $F+V$. In general, the speed of convergence is related to the eigenvalues of the derivatives of the mapping from PLM to ALM, $T(A)$. In this case, the derivative is equal to $F+V$ (see Ferrero, 2003).

Figure 3: The speed of learning isoquants for $\lambda_1 = 0$ (dotted line) and $\lambda_1 = 1$ (continue line)



Proof. see appendix 6. ☐

In a world where private agents are learning from past data, and therefore along their learning process they produce biased predictions of the main macro variables, the fact that the central bank publishes its projection obtained under rational expectations will lower the speed at which agents learn.

4 Announcing also expected inflation and output gap

In the previous sections we have seen that by announcing its expected short term interest rate, the central bank reduces the regions of stability under learning and lower the speed of convergence to the REE. The main reason is that by publishing the interest rate projection consistent with the REE, the central bank is not taking into account the fact that agents are learning and, therefore, is not offsetting the bias of the learning agents.

In this section we analyze the implications in terms of E-stability and speed of convergence when the central bank announces also its projections about inflation and output gap. We assume that also these projections are obtained by the central bank under the REE,

$$E_t^{CB} y_{t+1} = A + B\Psi w_t.$$

The main difference between the announcement of the policy path and the inflation and output gap paths is that, while former (the instrument variable of the model) is implicitly used by private agents in order to form expectations about future inflation and output (the control variables of the model), information about output gap and inflation are used directly to predict those variables. Let $0 \leq (1 - \lambda_2) \leq 1$ be the weight that private agents give to the central bank's announcement of inflation and output gap

$$E_t^P y_{t+1} = (1 - \lambda_2) E_t^{CB} y_{t+1} + \lambda_2 E_t^* y_{t+1}.$$

Under learning the economy evolves according to the system of equation[14]

$$Y_t = \tilde{\tilde{\tilde{Q}}} + \tilde{\tilde{\tilde{F}}} \times E_t^* Y_{t+1} + \tilde{\tilde{\tilde{V}}} \times E_t^* Y_{t+2} + \tilde{\tilde{\tilde{S}}} w_t. \tag{4.1}$$

4.1 Announcing only expected inflation and output gap

First we focus on the case where the central bank only announces the inflation and output gap projections and the private agents, at least, partly internalize this announcement to form their expectations, i.e. $\lambda_1 = 1$ and $0 < (1 - \lambda_2) \leq 1$.

Proposition 6. *In an economy that (i) evolves according to the system of equation (4.1), where (ii) the central bank publishes only the projection about inflation and output gap consistent with the REE ($\lambda_1 = 1$) and (iii) private agents give weight $0 < (1 - \lambda_2) \leq 1$ to those projections, revealing the path makes condition for E-stability less stringent than under no announcement. In particular, the necessary*

[14]See appendix 8 for a derivation of $\tilde{\tilde{\tilde{F}}}$ and $\tilde{\tilde{\tilde{V}}}$.

and sufficient condition for stability under learning of the REE is

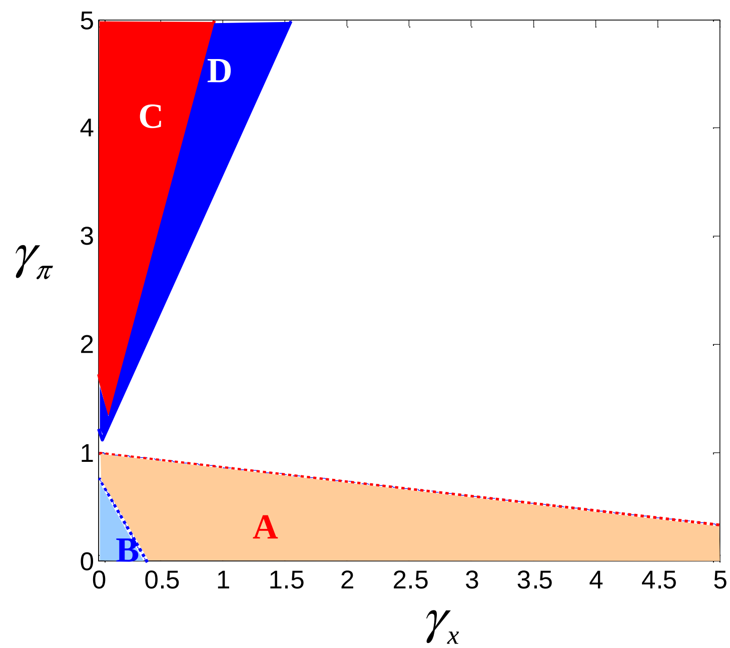
$$\gamma_\pi > \frac{2\lambda_2 (\beta\lambda_2 + 1)\alpha\varphi - (\beta^2\lambda_2 - 1)(\lambda_2 - 1)}{(\beta\lambda_2 + 1)(1 + \lambda_2)\alpha\varphi} - \frac{(1 - \beta^2\lambda_2)}{(\beta\lambda_2 + 1)\alpha}\gamma_x$$

Proof. See appendix 7. □

Therefore, announcing inflation and output gap projections has opposite implications than publishing the interest rate path. The former enlarges the region of policies that imply an E-stable equilibrium, while the latter enlarges the region of instability.

Figure 4 show the region of E-stability and root-t convergence when the central bank announces only the inflation and output gap and the private sector gives weight 0.25 to this announcement.

Figure 4: E-stability and root-t convergence when the central bank announces only the expected inflation and output gap, $(1 - \lambda_2) = 0.25$

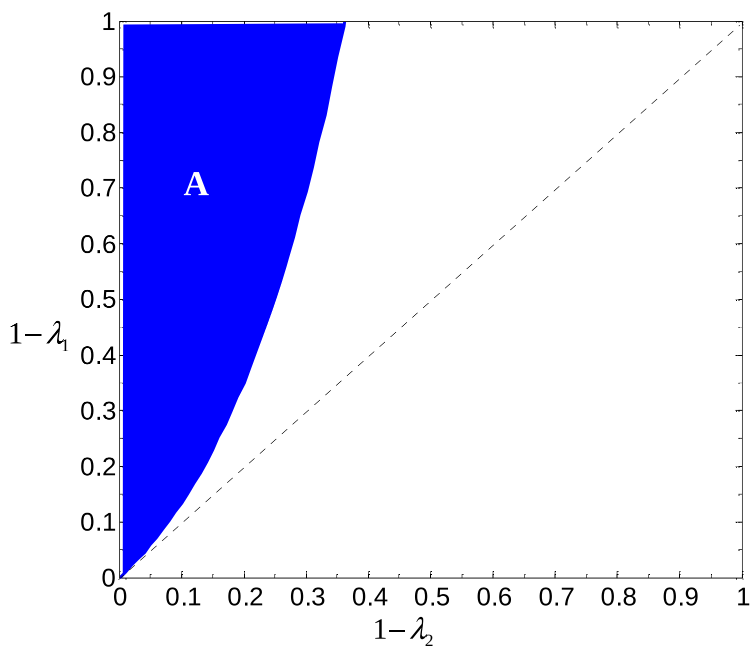


It is enough that private agents give a little weight to inflation and output gap projections in order to shrink substantially the region of instability under learning (from region $A + B$ to region B) and to increase the speed of learning (the region of root-t convergence enlarges from C to $C + D$).

4.2 Announcing expected interest rate, inflation and output gap

In practice, all central banks that are publishing their interest rate projections were previously - and also subsequently - publishing their inflation and output gap expectations. Therefore, we also focus on the case where the central bank announces both the interest rate path and the inflation and output gap projections and the private agents at least partly internalize these announcements to form their expectations.

Figure 5: Weights to the projections and E-stability announcing the interest rate, inflation and the output gap path



Proposition 7. *In an economy that (i) evolves according to the system of equation (4.1), where (ii) the central bank publishes both the interest rate path and the inflation and output gap projections, consistent with the REE and (iii) private agents give weight* $0 < (1 - \lambda_1) \leq 1$ *to the former and* $0 < (1 - \lambda_2) \leq 1$ *to the latter, in order to have condition for E-stability less stringent under no announcement than under*

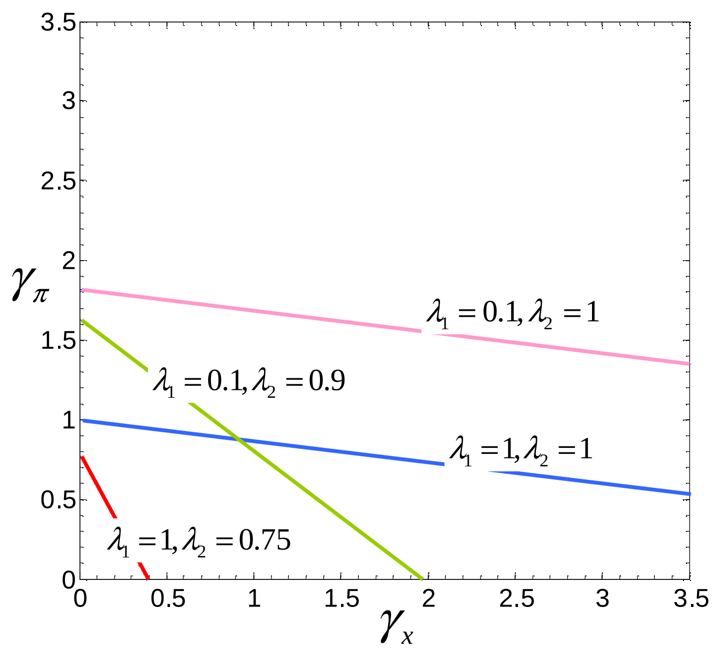
announcement the necessary and sufficient condition is that

$$0 \leq \lambda_1 < 2 - \frac{(1-\lambda_2)(1-\beta^2\lambda_2)}{\lambda_2 \alpha\varphi (1+\beta\lambda_2)} - \frac{1}{\lambda_2}$$

Proof. See appendix 8. □

In Figure 5, region A shows the combination of weights to the two projections under which the announcement of the paths reduces the region of stability under learning. In particular when private agents give relatively high weight to the interest rate path and low weight to the inflation and output gap projections, transparency increases the region of instability under learning.

Figure 6: E-stability and announcement of the (i) interest path, (ii) inflation and output paths, (iii) interest rate, inflation and output gap paths, (iv) no announcement.



Finally, Figure 6 describes the region of stability under learning under the different communication strategies analyzed: when the central bank announces (i) only the interest rate path, (ii) only the inflation and output gap path, (iii) both the interest and the inflation-output gap paths (iv) and when it doesn't announces anything.

The overall picture is the following: announcing the interest rate path increases the region of instability; announcing inflation and output gap reduces the region of instability; these two effects goes in the opposite direction when the central bank announces both and depending on the weights that private agents give to them we may have that a no announcement strategy is preferable.

5 Extensions: Publication of a longer path

One interesting extension considers the case where the central bank announces a T-period path. In this case equation (2.1) becomes

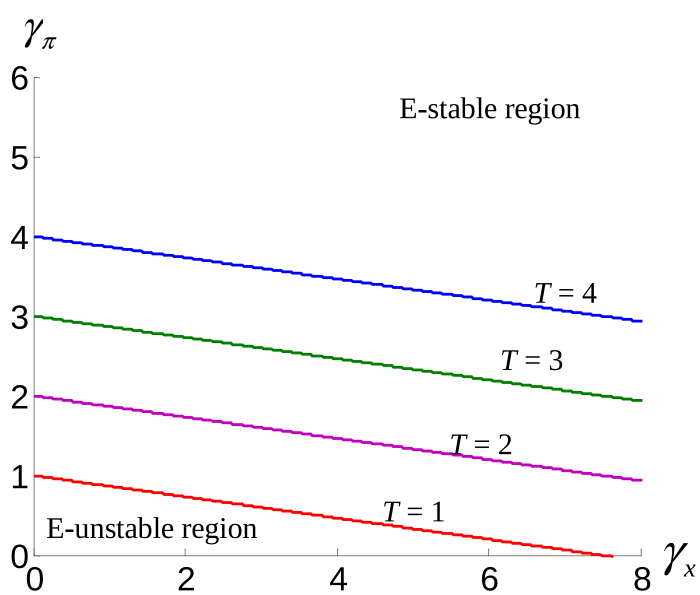
$$x_t = E_t^* x_{t+T} - E_t^* \sum_{j=0}^{T-1} \left(\lambda \varphi i_{t+j} - \varphi \pi_{t+j+1} - g_{t+j}\right) - (1-\lambda)\, E_t^{CB} \sum_{j=0}^{T-1} \left(\varphi i_{t+j}\right)$$

where

$$E_t^{CB} i_{t+j} = a_i + \rho_u^j b_{u,i} u_t + \rho_g^j b_{g,i} g_t.$$

The following proposition shows under which conditions the economy converges to the REE

Figure 7: E-stability and the announcement of a T-period expected path



Proposition 8. *In an economy that (i) evolves according to equations (3.8), where*

(ii) private agents give weight $0 \leq (1-\lambda_1) \leq 1$ *to the central bank's announcement, the longer is the path revealed the more stringent are conditions for E-stability. In particular, the necessary and sufficient condition for the MSV solution (2.6) to be E-stable under contemporaneous-data reaction function (2.4) when the CB announces the* T*-period path is that*

$$\gamma_\pi > \frac{T}{(1+\lambda_1(T-1))} - \frac{(1-\beta)}{\alpha}\gamma_x \tag{3.12}$$

Proof. See appendix 9. □

In figure 7 we report the regions where we have an E-stable REE for different Ts, under perfect credibility.

6 Conclusions

In this paper we have analyzed analytically the effects of publishing the central bank's interest rate projection under asymmetric information.

We have shown that in a world where private agents are learning from past data, a central bank that publishes the interest rate projection consistent with the REE and it is not taking into account the fact that agents are learning, is acting as if the expected interest rates are determined by a "fundamental" policy rule and, therefore, is actually making conditions for determinacy and E-stability more stringent and the learning process slower than under no announcement. To offset this effect, the response of the policy to inflation has to be larger than in the case of no announcement.

We have also shown that announcement of expected inflation and output gap has an opposite effect on the stability and speed of the learning process. Particularly relevant for the actual debate on monetary policy transparency is the case where central banks that have already been publishing the expected output gap and inflation decide to publish also the interest rate path. In this case if the agents pay – or are induced to pay – too much attention to the announcement of the interest rate path with respect to the one they dedicate to expected inflation and output gap, the region of instability under learning may result larger, and the speed of learning lower, than under a communication strategy of no announcements.

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Appendix: Proofs of propositions

Appendix 1) The REE for the interest rate

Under the set of policies

$$i_t = \gamma + \gamma_x x_t + \gamma_\pi \pi_t$$

the economy evolves according to the following stochastic dynamic system:

$$HY_t = K + L \times E_t^* Y_{t+1} + M w_t,$$

with

$$\begin{aligned} y_t &= \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}, \quad w_t = \begin{bmatrix} u_t \\ g_t \end{bmatrix}, \quad \Psi = \begin{bmatrix} \rho_u & 0 \\ 0 & \rho_g \end{bmatrix}, \quad \varepsilon_t \sim N(0, \Omega_\varepsilon) \\ H &= \begin{bmatrix} 1 & -\alpha \\ \varphi\gamma_\pi & 1+\varphi\gamma_x \end{bmatrix}, K = \begin{bmatrix} 0 \\ -\varphi\gamma \end{bmatrix}, L = \begin{bmatrix} \beta & 0 \\ \varphi & 1 \end{bmatrix}, M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

To calculate the minimum state variable (MSV) solution, we rewrite the system as

$$Y_t = Q + F \times E_t^* Y_{t+1} + S w_t,$$

where

$$\begin{aligned} F &= H^{-1}L = \begin{bmatrix} \frac{\beta+\alpha\varphi+\beta\varphi\gamma_x}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{\alpha}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\varphi\frac{\beta\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \end{bmatrix} \\ Q &= H^{-1}K = \begin{bmatrix} -\alpha\gamma\frac{\varphi}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\gamma\frac{\varphi}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \end{bmatrix} \\ S &= H^{-1}M = \begin{bmatrix} \frac{\varphi\gamma_x+1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{\alpha}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\varphi\frac{\gamma_\pi}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \end{bmatrix} \end{aligned}$$

The MSV solution satisfies

$$A = Q + FA$$
$$B = FB\Psi + S$$

$$A = \begin{bmatrix} a_\pi \\ a_x \end{bmatrix}, \quad B = \begin{bmatrix} b_{\pi,u} & b_{\pi,g} \\ b_{x,u} & b_{x,g} \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_\pi \\ a_x \end{bmatrix} + \begin{bmatrix} b_{\pi,u} & b_{\pi,g} \\ b_{x,u} & b_{x,g} \end{bmatrix} \begin{bmatrix} u_t \\ g_t \end{bmatrix}$$

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} a_\pi \\ a_x \end{bmatrix} + \begin{bmatrix} b_{\pi,u} & b_{\pi,g} \\ b_{x,u} & b_{x,g} \end{bmatrix} \begin{bmatrix} \rho_u & 0 \\ 0 & \rho_g \end{bmatrix} \begin{bmatrix} u_t \\ g_t \end{bmatrix}$$

The REE is

$$A = (I-F)^{-1} Q = \begin{bmatrix} \frac{\alpha\gamma}{\alpha-\gamma_x-\alpha\gamma_\pi+\beta\gamma_x} \\ \frac{(1-\beta)\gamma}{\alpha-\gamma_x-\alpha\gamma_\pi+\beta\gamma_x} \end{bmatrix}$$

$$vec(B) = (I - \Psi' \otimes F)^{-1} vec(S)$$

$$B = \begin{bmatrix} \frac{-(1-\rho_u+\varphi\gamma_x)}{(\beta\rho_u-1)(1-\rho_u+\varphi\gamma_x)-\alpha\varphi(\gamma_\pi-\rho_u)} & -\frac{\alpha}{(\beta\rho_g-1)(1-\rho_g+\varphi\gamma_x)-\alpha\varphi(\gamma_\pi-\rho_g)} \\ \frac{\varphi(\gamma_\pi-\rho_u)}{(\beta\rho_u-1)(1-\rho_u+\varphi\gamma_x)-\alpha\varphi(\gamma_\pi-\rho_u)} & \frac{\beta\rho_g-1}{(\beta\rho_g-1)(1-\rho_g+\varphi\gamma_x)-\alpha\varphi(\gamma_\pi-\rho_g)} \end{bmatrix}$$

From the MSV solution for time t inflation and output gap we obtain the time t interest rate

$$i_t = a_i + b_{u,i}u_t + b_{g,i}g_t$$

where

$$a_i = \gamma + \gamma_x a_x + \gamma_\pi a_\pi$$
$$b_{u,i} = \gamma_x b_{x,u} + \gamma_\pi b_{\pi,u}$$
$$b_{g,i} = \gamma_x b_{x,g} + \gamma_\pi b_{\pi,g}$$

and the conditional interest rate at time $t+1$

$$E_t i_{t+1} = a_i + \rho_u b_{u,i} u_t + \rho_g b_{g,i} g_t$$

where a_i, $b_{u,i}$ and $b_{g,i}$ are easily obtained.

In order to analyze determinacy of the equilibrium we rewrite the system as

$$Y_t = Q + F \times E_t^* Y_{t+1} + S w_t,$$

and we focus on the eigenvalues of the matrix

$$F = H^{-1}L = \frac{1}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}\begin{bmatrix} \alpha\varphi + (\varphi\gamma_x + 1)\beta & \alpha \\ \varphi - \beta\varphi\gamma_\pi & 1 \end{bmatrix}.$$

For determinacy we need both the eigenvalues of the F matrix to be inside the unit circle (Bullard and Mitra, 2001). Since the characteristic polynomial of this matrix is

$$X^2 + a_1 X + a_2$$

$$a_1 = -\frac{(\beta + \alpha\varphi + \beta\varphi\gamma_x + 1)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}$$

$$a_2 = \frac{\beta}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}$$

a necessary and sufficient condition is $|a_2| < 1$ and $|a_1| < 1 + a_2$. This reduces to have

$$\gamma_\pi > 1 - \frac{(1-\beta)}{\alpha}\gamma_x$$

For E-stability we need the eigenvalues of the $F - I$ matrix and those of the $\left(\frac{dFB\Psi}{dB} - I\right)$ matrix to have the real part smaller than zero.

Since the characteristic polynomial of $F - I$ is

$$X^2 + a_1 X + a_2$$

$$a_1 = \frac{1 - \beta - \alpha\varphi + 2\alpha\varphi\gamma_\pi - \beta\varphi\gamma_x + 2\varphi\gamma_x}{(1 + \varphi\gamma_x + \alpha\varphi\gamma_\pi)}$$

$$a_2 = \frac{-\alpha\varphi + \varphi\gamma_x + \alpha\varphi\gamma_\pi - \beta\varphi\gamma_x}{(1 + \varphi\gamma_x + \alpha\varphi\gamma_\pi)}$$

and

$$a_1 = a_2 + \frac{1 - \beta + \alpha\varphi\gamma_\pi + \varphi\gamma_x}{(1 + \varphi\gamma_x + \alpha\varphi\gamma_\pi)}$$

a necessary and sufficient condition is $a_2 > 0$ and $a_1 > 0$. This reduces to have

$$\gamma_\pi > 1 - \frac{(1-\beta)}{\alpha}\gamma_x$$

We also need the eigenvalues of the $\left(\frac{dFB\Psi}{dB} - I\right)$ matrix to be smaller than zero.

$$vec\left(FB\Psi\right) = \left(\Psi' \otimes F\right) vec\left(B\right)$$

$$\left(\Psi' \otimes F\right) vec\left(B\right) = \left[\begin{array}{c} \frac{\alpha\varphi\rho_u + \beta\rho_u(\varphi\gamma_x+1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{\pi,u} + \alpha\frac{\rho_u}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{x,u} \\ \frac{\rho_u}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{\pi,u} + \frac{\varphi\rho_u - \beta\varphi\gamma_\pi\rho_u}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{x,u} \\ \frac{\alpha\varphi\rho_g + \beta\rho_g(\varphi\gamma_x+1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{\pi,g} + \alpha\frac{\rho_g}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{x,g} \\ \frac{\rho_g}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{\pi,g} + \frac{\varphi\rho_g - \beta\varphi\gamma_\pi\rho_g}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} b_{x,g} \end{array}\right]$$

$$\frac{\rho_u}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}\left[\begin{array}{cc} \alpha\varphi + \beta\left(\varphi\gamma_x + 1\right) & \alpha \\ \varphi - \beta\varphi\gamma_\pi & 1 \end{array}\right]\left[\begin{array}{c} b_{\pi,u} \\ b_{x,u} \end{array}\right]$$

$$\frac{\rho_g}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}\left[\begin{array}{cc} \alpha\varphi + \beta\left(\varphi\gamma_x + 1\right) & \alpha \\ \varphi - \beta\varphi\gamma_\pi & 1 \end{array}\right]\left[\begin{array}{c} b_{\pi,g} \\ b_{x,g} \end{array}\right]$$

As ρ_u and ρ_g are smaller than 1 a sufficient condition for eigenvalues of the $\left(\frac{dFB\Psi}{dB} - I\right)$ matrix to have the real part smaller than zero is that the eigenvalues of $(F - I)$ matrix to be smaller than zero.

Appendix 2) Proof of proposition 1 (determinacy of the REE under communication)

The economy evolves according to the following expression:

$$\widehat{H}\widehat{y}_t = \widehat{K} + \widehat{L} \times E_t^* \widehat{y}_{t+1} + \widehat{M} w_t, \tag{3.6}$$

$$\widehat{y}_t = \begin{bmatrix} \pi_t \\ x_t \\ E_t\pi_{t+1} \\ E_t x_{t+1} \end{bmatrix}, \ \widehat{H} = \begin{bmatrix} 1 & -\alpha & 0 & 0 \\ \varphi\gamma_\pi & (1+\varphi\gamma_x) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\widehat{K} = \begin{bmatrix} 0 \\ -\varphi\lambda_1\gamma - \varphi(1-\lambda)\,a_i \\ 0 \\ 0 \end{bmatrix}, \ \widehat{L} = \begin{bmatrix} 0 & \beta\alpha & \beta^2 & 0 \\ \varphi(1-\lambda_1\gamma_\pi) & -\varphi\lambda_1\gamma_x & \varphi & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\widehat{M} = \begin{bmatrix} 1+\beta\rho_u & 0 \\ -\varphi(1-\lambda_1)\,\rho_u b_{u,i} & 1+(1-\varphi(1-\lambda_1)\,b_{g,i})\,\rho_g \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

To find the conditions under which the REE is unique, we rewrite the system as

$$\widehat{y}_t = \widehat{Q} + \widehat{F} \times E_t^*\widehat{y}_{t+1} + \widehat{S}w_t,$$

where$\widehat{F} = \widehat{H}^{-1}\widehat{L}$, $\widehat{Q} = \widehat{H}^{-1}\widehat{K}$ and $\widehat{S} = \widehat{H}^{-1}\widehat{M}$. For determinacy we need the eigenvalues of the $\widehat{F}$ matrix to be inside the unit circle.

Since the characteristic polynomial of this matrix is

$$P(X) = \frac{(X+\beta)\,(a_0X^3 + a_1X^2 + a_2X + a_3)}{(\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1)}$$

with

$$\begin{aligned} a_0 &= (\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1) \\ a_1 &= -(\alpha\varphi + \beta\varphi\gamma_x + \beta - \lambda_1\varphi(\gamma_x + \alpha\gamma_\pi)) \\ a_2 &= -(1 + \alpha\varphi + \beta\lambda_1\varphi\gamma_x) \\ a_3 &= \beta \end{aligned}$$

necessary and sufficient conditions for determinacy are given by the Schur's theorem.

Let

$$\Delta_1 = \begin{bmatrix} a_0 & a_3 \\ a_3 & a_0 \end{bmatrix}, \Delta_2 = \begin{bmatrix} a_0 & 0 & a_3 & a_2 \\ a_1 & a_0 & 0 & a_3 \\ a_3 & 0 & a_0 & a_1 \\ a_2 & a_3 & 0 & a_0 \end{bmatrix}, \Delta_3 = \begin{bmatrix} a_0 & 0 & 0 & a_3 & a_2 & a_1 \\ a_1 & a_0 & 0 & 0 & a_3 & a_2 \\ a_2 & a_1 & a_0 & 0 & 0 & a_3 \\ a_3 & 0 & 0 & a_0 & a_1 & a_2 \\ a_2 & a_3 & 0 & 0 & a_0 & a_1 \\ a_1 & a_2 & a_3 & 0 & 0 & a_0 \end{bmatrix}$$

the Schur's theorem, says that a necessary and sufficient condition for all eigenvalues of F to be inside the unit circle is that $\det\Delta_1 > 0$, $\det\Delta_2 > 0$, $\det\Delta_3 > 0$.

It is straightforward to show that $\det\Delta_1 > 0$ for all values of γ_x and γ_π; sufficient condition for the $\det\Delta_2 > 0$ are

$$\gamma_\pi > 1 - \frac{(1-\beta\lambda_1)}{\alpha}\gamma_x$$

and

$$\gamma_\pi > \frac{(1-\beta)}{(1-\beta\lambda_1)} - \frac{(1+\beta^2-2\beta\lambda_1)}{\alpha\,(1-\beta\lambda_1)}\gamma_x.$$

Since necessary and sufficient condition for $\det\Delta_3 > 0$ is

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x,$$

and when this condition is satisfied, also the conditions for $\det\Delta_2 > 0$ are satisfied, necessary and sufficient conditions for determinacy is

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x.$$

Appendix 3) Proof of proposition 2 (E-stability of the REE under communication)

To study how the economy evolves under learning we rewrite the system as

$$\widetilde{H}y_t = \widetilde{K} + \widetilde{L} \times E_t^* y_{t+1} + \widetilde{P} \times E_t^* y_{t+2} + \widetilde{M}w_t,$$

where

$$\begin{aligned} y_t &= \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}, \quad \widetilde{K} = \begin{bmatrix} 0 \\ -\varphi\gamma(1+\lambda_1) - \varphi(1-\lambda_1)a_i \end{bmatrix}, \quad \widetilde{P} = \begin{bmatrix} \beta^2 & 0 \\ \varphi & 1 \end{bmatrix} \\ \widetilde{H} &= \begin{bmatrix} 1 & -\alpha \\ \varphi\gamma_\pi & (1+\varphi\gamma_x) \end{bmatrix}, \quad \widetilde{L} = \begin{bmatrix} 0 & \beta\alpha \\ \varphi(1-\lambda_1\gamma_\pi) & -\varphi\lambda_1\gamma_x \end{bmatrix}, \\ \widetilde{M} &= \begin{bmatrix} 1+\beta\rho_u & 0 \\ -\varphi(1-\lambda_1)\rho_u b_{u,i} & 1+(1-\varphi(1-\lambda_1)b_{g,i})\rho_g \end{bmatrix} \end{aligned}$$

or

$$Y_t = \widetilde{Q} + \widetilde{F} \times E_t^* Y_{t+1} + \widetilde{V} \times E_t^* Y_{t+2} + \widetilde{S} w_t,$$

where

$$\widetilde{F} = \widetilde{H}^{-1}\widetilde{L} \text{ and } \widetilde{V} = \widetilde{H}^{-1}\widetilde{P}.$$

For E-stability we need the eigenvalues of the $\widetilde{F} + \widetilde{V} - I$ matrix and of the $\left(\frac{d(FB\Psi+VB\Psi'\Psi)}{dB} - I\right)$ matrix to have the real part smaller than zero.

Since the characteristic polynomial of the $\widetilde{F} + \widetilde{V} - I$ matrix is

$$X^2 + a_1 X + a_2$$

with

$$a_1 = \frac{(2\varphi\gamma_x - 2\alpha\varphi - \beta^2 - \beta^2\varphi\gamma_x + 2\alpha\varphi\gamma_\pi + \varphi\lambda_1\gamma_x + \alpha\beta\varphi\gamma_\pi + \alpha\varphi\lambda_1\gamma_\pi + 1)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}$$

and

$$a_2 = \frac{\varphi(\beta+1)(\gamma_x - 2\alpha + \alpha\gamma_\pi - \beta\gamma_x + \lambda_1\gamma_x + \alpha\lambda_1\gamma_\pi - \beta\lambda_1\gamma_x)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1},$$

necessary and sufficient conditions are $a_2 > 0$ and $a_1 > 0$. As

$$\gamma_\pi > -\frac{(1-2\alpha\varphi-\beta^2)}{(2+\beta+\lambda_1)\alpha\varphi} - \frac{(2-\beta^2+\lambda_1)}{(2+\beta+\lambda_1)\alpha}\gamma_x$$

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x$$

Since

$$\frac{(2-\beta^2+\lambda_1)}{(2+\beta+\lambda_1)\alpha} - \frac{(1-\beta)}{\alpha} = \frac{(\lambda_1+1)\beta}{\alpha(\beta+\lambda_1+2)} > 0$$

and

$$\frac{2}{(1+\lambda_1)} + \frac{(1-2\alpha\varphi-\beta^2)}{(2+\beta+\lambda_1)\,\alpha\varphi} = \frac{(\beta+1)\,(\lambda_1\,(1-\beta)+(1-\beta)+2\alpha\varphi)}{\alpha\varphi\,(\lambda_1+1)\,(\beta+\lambda_1+2)} > 0$$

this reduces to have $a_2 > 0$, that is

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x$$

We also need the eigenvalues of the $\left(\frac{d\left(\widetilde{F}\widetilde{B}\Psi+\widetilde{V}\widetilde{B}\Psi'\Psi\right)}{d\widetilde{B}} - I\right)$ matrix to be smaller than zero.

$$vec\left(\widetilde{F}\widetilde{B}\Psi\right) + vec(\widetilde{V}\widetilde{B}\Psi'\Psi) = \left(\Psi' \otimes \widetilde{F} + \Psi\Psi' \otimes \widetilde{V}\right) vec\left(\widetilde{B}\right)$$

$$\left(\rho_u\widetilde{F} + \rho_u^2\widetilde{V}\right)\begin{bmatrix} b_{\pi,u} \\ b_{x,u} \end{bmatrix}$$

$$\left(\rho_g\widetilde{F} + \rho_g^2\widetilde{V}\right)\begin{bmatrix} b_{\pi,g} \\ b_{x,g} \end{bmatrix}$$

As ρ_u and ρ_g are both smaller than we just focus on the first expression. A sufficient condition for eigenvalues of the $\left(\frac{d\left(\widetilde{F}\widetilde{B}\Psi+\widetilde{V}\widetilde{B}\Psi'\Psi\right)}{d\widetilde{B}} - I\right)$ matrix to have the real part smaller than zero is that the eigenvalues of

$$\rho_u\widetilde{F}+\rho_u^2\widetilde{V}-I = \begin{bmatrix} -\alpha\varphi\rho_u\frac{\lambda_1\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} + \rho_u^2\frac{\left(\beta^2+\alpha\varphi+\beta^2\varphi\gamma_x\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} - 1 & \alpha\rho_u\frac{\beta+\beta\varphi\gamma_x-\varphi\lambda_1\gamma_x}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} + \rho_u^2\frac{\alpha}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\varphi\rho_u\frac{\lambda_1\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} - \rho_u^2\varphi\frac{\beta^2\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & -\varphi\rho_u\frac{\lambda_1\gamma_x+\alpha\beta\gamma_\pi}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} + \rho_u^2\frac{1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} - 1 \end{bmatrix}$$

have the real part smaller than zero. Since the characteristic polynomial of the $\rho_u\widetilde{F} + \rho_u^2\widetilde{V} - I$ matrix is

$$X^2 + a_1X + a_2$$

with

$$a_1 = \frac{(2\varphi\gamma_x - \beta^2\rho_u^2 - \rho_u^2 - \alpha\varphi\rho_u^2 - \alpha\varphi\rho_u - \beta^2\varphi\gamma_x\rho_u^2 + \varphi\lambda_1\gamma_x\rho_u + \alpha\varphi\gamma_\pi\,(2+\beta\rho_u+\lambda_1\rho_u)+2)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}$$

and

$$a_2 = \frac{(\beta\rho_u+1)\,((1-\beta\rho_u)\,(1-\rho_u^2) - (1+\rho_u)\,\alpha\varphi\rho_u + \varphi\gamma_x\,(1-\beta\rho_u)\,(1+\lambda_1\rho_u) + \alpha\varphi\gamma_\pi\,(1+\lambda_1\rho_u))}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1},$$

necessary and sufficient conditions are $a_2 > 0$ and $a_1 > 0$.

As

$$\gamma_\pi > \frac{-2 + (1+\beta^2)\rho_u^2 + (1+\rho_u)\alpha\varphi\rho_u}{(1+\beta\rho_u + 1 + \lambda_1\rho_u)\alpha\varphi} - \gamma_x \frac{(2 - \beta^2\rho_u^2 + \lambda_1\rho_u)}{(1+\beta\rho_u + 1 + \lambda_1\rho_u)\alpha}$$

$$\gamma_\pi > \frac{-(1-\beta\rho_u)(1-\rho_u^2) + (1+\rho_u)\alpha\varphi\rho_u}{(1+\lambda_1\rho_u)\alpha\varphi} - \frac{(1-\beta\rho_u)}{\alpha}\gamma_x$$

Since

$$\frac{(2 - \beta^2\rho_u^2 + \lambda_1\rho_u)}{(1+\beta\rho_u + 1 + \lambda_1\rho_u)\alpha} - \frac{(1-\beta\rho_u)}{\alpha} = \frac{(\lambda_1\rho_u + 1)\beta\rho_u}{(\beta\rho_u + \lambda_1\rho_u + 2)\alpha} > 0$$

and

$$\frac{-(1-\beta\rho_u)(1-\rho_u^2) + (1+\rho_u)\alpha\varphi\rho_u}{(1+\lambda_1\rho_u)\alpha\varphi} - \frac{-2 + (1+\beta^2)\rho_u^2 + (1+\rho_u)\alpha\varphi\rho_u}{(1+\beta\rho_u + 1 + \lambda_1\rho_u)\alpha\varphi} > 0$$

this reduces to have $a_2 > 0$, that is

$$\gamma_\pi > \frac{-(1-\beta\rho_u)(1-\rho_u^2) + (1+\rho_u)\alpha\varphi\rho_u}{(1+\lambda_1\rho_u)\alpha\varphi} - \frac{(1-\beta\rho_u)}{\alpha}\gamma_x$$

Finally by comparing this last condition with the

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x$$

we observe that

$$\frac{(1-\beta\rho_u)}{\alpha} - \frac{(1-\beta)}{\alpha} = \frac{\beta(1-\rho_u)}{\alpha} > 0$$

and

$$\frac{2}{(1+\lambda_1)} - \frac{-(1-\beta\rho_u)(1-\rho_u^2) + (1+\rho_u)\alpha\varphi\rho_u}{(1+\lambda_1\rho_u)\alpha\varphi} > 0$$

Therefore a sufficient condition for eigenvalues of the $\left(\frac{d\left(\widetilde{F}\widetilde{B}\Psi + \widetilde{V}\widetilde{B}\Psi'\Psi\right)}{d\widetilde{B}} - I\right)$ matrix to have the real part smaller than zero is that the eigenvalues of $\left(\widetilde{F} + \widetilde{V} - I\right)$ matrix to be smaller than zero, that is

$$\gamma_\pi > \frac{2}{(1+\lambda_1)} - \frac{(1-\beta)}{\alpha}\gamma_x$$

Appendix 4) proof of proposition 3 (speed of convergence and com-

munication of the path)

From Marcet and Sargent (1992) it follows that a necessary condition for root-t convergence is that the eigenvalues of $\widetilde{F}+\widetilde{V}$ and those of $\frac{d\left(\widetilde{F}\widetilde{B}\Psi+\widetilde{V}\widetilde{B}\Psi'\Psi\right)}{d\widetilde{B}}$ have the real part smaller than $\frac{1}{2}$. On the other hand, agents beliefs will not converge to the MSV solution at root-t speed if any eigenvalue of $\widetilde{F}+\widetilde{V}$ or $\frac{d\left(\widetilde{F}\widetilde{B}\Psi+\widetilde{V}\widetilde{B}\Psi'\Psi\right)}{d\widetilde{B}}$ has real part more than 1/2. Similarly to the proof in appendix 3, it turns out that it is sufficient to have the real part of the eigenvalues of $\widetilde{F}+\widetilde{V}$ smaller than $\frac{1}{2}$. Let

$$\widetilde{F}+\widetilde{V}-\frac{1}{2}I=\begin{bmatrix} \frac{\left(\beta^2+2\alpha\varphi+\beta^2\varphi\gamma_x-\alpha\varphi\lambda_1\gamma_\pi\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}-\frac{1}{2} & \frac{\alpha(\beta+\beta\varphi\gamma_x-\varphi\lambda_1\gamma_x+1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\varphi\frac{\gamma_\pi\beta^2+\lambda_1\gamma_\pi-2}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & -\frac{\varphi\lambda_1\gamma_x+\alpha\beta\varphi\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}-\frac{1}{2} \end{bmatrix}$$

$$X^2+a_1X+a_2$$

$$a_1=\frac{-\left(2\alpha\varphi+\beta^2\right)+\varphi\gamma_x\left(1-\beta^2+\lambda_1\right)+\alpha\varphi\gamma_\pi\left(1+\beta+\lambda_1\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}$$

$$a_2=\frac{-\left(1-2\beta^2\right)-4\alpha\varphi\left(1+2\beta\right)+\varphi\gamma_x\left(1+2\lambda_1\right)\left(1-2\beta^2\right)+\alpha\varphi\gamma_\pi\left(1+2\lambda_1\right)\left(1+2\beta\right)}{4\left(\varphi\gamma_x+\alpha\varphi\gamma_\pi+1\right)}$$

By applying the Routh theorem, we have the necessary and sufficient conditions for root-t convergence: both eigenvalues of $\widetilde{F}+\widetilde{V}$ have real parts less than 1/2 (i.e. both eigenvalues of $\widetilde{F}+\widetilde{V}-\frac{1}{2}I$ have negative real parts) if and only if $a_1>0$ and $a_2>0$. Under the assumption that γ_x and γ_π are non negative, the necessary and sufficient conditions are:

$$\gamma_\pi>\max\begin{bmatrix} \frac{\left(\beta^2+2\alpha\varphi\right)}{\alpha\varphi(1+\lambda_1+\beta)}-\frac{\left(1+\lambda_1-\beta^2\right)}{\alpha(1+\lambda_1+\beta)}\gamma_x \\ \frac{4(2\beta+1)\alpha\varphi-\left(2\beta^2-1\right)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi}+\frac{\left(2\beta^2-1\right)}{(1+2\beta)\alpha}\gamma_x \end{bmatrix}$$

Appendix 5) proof of proposition 4 (root-t convergence and communication of the path)

Given the set

$$S_1=\left\{\gamma_\pi,\gamma_x:\gamma_\pi>\max\begin{bmatrix} \frac{\left(\beta^2+2\alpha\varphi\right)}{\alpha\varphi(1+\lambda_1+\beta)}-\frac{\left(1+\lambda_1-\beta^2\right)}{\alpha(1+\lambda_1+\beta)}\gamma_x \\ \frac{4(2\beta+1)\alpha\varphi-\left(2\beta^2-1\right)}{(2\beta+1)(1+2\lambda_1)\alpha\varphi}-\frac{\left(1-2\beta^2\right)}{(1+2\beta)\alpha}\gamma_x \end{bmatrix}\right\}$$

under which we have root-T convergence, we take derivatives of the two terms inside the max operator with respect to λ_1 and we show that the larger the λ_1, that is the smaller the weight to the announcement (the extreme cases of $\lambda_1 = 1$ coincide with the case of no announcement), the smaller is the S_1 set, since the larger has to be γ_π in order to stay in the S_1 set.

As

$$\frac{\partial}{\partial\lambda_1}\frac{(1+\lambda_1-\beta^2)}{\alpha\,(1+\lambda_1+\beta)} = \frac{\alpha\beta\,(\beta+1)}{[\alpha\,(1+\lambda_1+\beta)]^2} > 0$$

and

$$\frac{\partial}{\partial\lambda_1}\frac{(\beta^2+2\alpha\varphi)}{(1+\beta+\lambda_1)\,\alpha\varphi} < 0$$

always. The first term of the max function is larger for smaller λ_1.

And since

$$\frac{\partial}{\partial\lambda_1}\frac{4\,(2\beta+1)\,\alpha\varphi-(2\beta^2-1)}{(2\beta+1)\,(1+2\lambda_1)\,\alpha\varphi} < 0$$

for

$$\alpha\varphi > \frac{(2*0.99^2-1)}{4*(1+2*0.99)} = 0.0806,$$

condition that is always satisfied for all reasonable values of $\alpha\varphi$ (for example under CGG (1999) parametrization $\alpha\varphi = 0.3$; under Woodford (2003) parametrization $\alpha\varphi = 0.15$). That is the second term of the max function is larger for smaller λ_1.

Therefore the set of (γ_π, γ_x) combinations under which we have root-T convergence is smaller, the larger is λ_1.

Appendix 6) Speed of convergence isoquants

The idea is the following: we look at combinations of (γ_π, γ_x) that results in the same value for the real part of the largest eigenvalue of the $F+V$ matrix. Let $0 < k < 1$, then

$$z^2 + a_1^k z + a_2^k$$

with

$$a_1^k = \frac{(2k-\beta^2-2\alpha\varphi-\beta^2\varphi\gamma_x+2k\varphi\gamma_x+\varphi\lambda_1\gamma_x+2k\alpha\varphi\gamma_\pi+\alpha\beta\varphi\gamma_\pi+\alpha\varphi\lambda_1\gamma_\pi-1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}$$

and

$$a_2^k = \frac{(\beta^2-k)\,(1-k)-2\alpha\varphi\,(k+\beta)+(k-\beta^2)\,(\lambda_1+k)\,\varphi\gamma_x+\alpha\varphi\gamma_\pi\,(k+\beta)\,(k+\lambda_1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}$$

is the characteristic polynomial of $F+V-kI$.

Both eigenvalues of F have real parts less than k (i.e. both eigenvalues of $F-kI$ have negative real parts) if and only if $a_1^k > 0$ and $a_2^k > 0$. That is,

$$\gamma_\pi = \max\left[\begin{array}{c} \frac{\left(1-2k+\beta^2+2\alpha\varphi\right)}{(2k+\beta+\lambda_1)\alpha\varphi} - \frac{\left(-\beta^2+2k+\lambda_1\right)}{(2k+\beta+\lambda_1)\alpha}\gamma_x \\ \frac{2\alpha\varphi(k+\beta)-\left(\beta^2-k\right)(1-k)}{\alpha\varphi(k+\beta)(k+\lambda_1)} - \frac{\left(k-\beta^2\right)}{\alpha(k+\beta)}\gamma_x \end{array}\right].$$

Appendix 7) proof of proposition 5 (speed of convergence and communication of the path)

Let's consider an isoquant in the S_2 set:

$$\gamma_\pi^k = \max\left[\begin{array}{c} \frac{\left(1-2k+\beta^2+2\alpha\varphi\right)}{(2k+\beta+\lambda_1)\alpha\varphi} - \frac{\left(-\beta^2+2k+\lambda_1\right)}{(2k+\beta+\lambda_1)\alpha}\gamma_x \\ \frac{2}{(k+\lambda_1)} - \frac{\left(\beta^2-k\right)(1-k)}{\alpha\varphi(k+\beta)(k+\lambda_1)} - \frac{\left(k-\beta^2\right)}{\alpha(k+\beta)}\gamma_x \end{array}\right] \quad \text{for } \frac{1}{2} \leq k \leq 1$$

We take derivatives of the two terms inside the max operator with respect to λ_1 and we show that for a given k, the larger the λ_1, that is the smaller the weight to the announcement (the extreme cases of $\lambda_1 = 1$ coincide with the case of no announcement), the smaller is γ_π^k.

As

$$\frac{\partial}{\partial\lambda_1}\frac{(-\beta^2+2k+\lambda_1)}{(2k+\beta+\lambda_1)\,\alpha} = \frac{\alpha\beta\,(\beta+1)}{\left[(2k+\beta+\lambda_1)\,\alpha\right]^2} > 0$$

and

$$\frac{\partial}{\partial\lambda_1}\frac{(1-2k+\beta^2+2\alpha\varphi)}{(2k+\beta+\lambda_1)\,\alpha\varphi} = \frac{-\alpha\varphi\,(1-2k+\beta^2+2\alpha\varphi)}{\left[(2k+\beta+\lambda_1)\,\alpha\varphi\right]^2} < 0$$

for

$$k < \frac{1+\beta^2}{2} + \alpha\varphi$$

and since

$$\frac{1+\beta^2}{2} + \alpha\varphi > 1$$

for all reasonable values of α, φ, β, it is $\frac{\partial}{\partial\lambda_1}\frac{\left(1-2k+\beta^2+2\alpha\varphi\right)}{(2k+\beta+\lambda_1)\alpha\varphi} < 0$

The first term of the max function is larger for smaller λ_1.

And since

$$\frac{\partial}{\partial\lambda_1}\frac{2\alpha\varphi\,(k+\beta)-(\beta^2-k)\,(1-k)}{\alpha\varphi\,(k+\beta)\,(k+\lambda_1)} < 0$$

for

$$\left[2\alpha\varphi\left(k+\beta\right)-\left(\beta^{2}-k\right)\left(1-k\right)\right]>0$$

that for $\frac{1}{2}\leq k\leq 1$ is always satisfied if

$$\alpha\varphi>\frac{(2*0.99^{2}-1)}{4*(1+2*0.99)}=0.0806,$$

condition that we have already seen is always satisfied for all reasonable values of $\alpha\varphi$.

That is, also the second term of the max function is larger for smaller λ_1.

Therefore for a given γ_x, the larger is λ_1, that is the smaller the weight that private agents give to the policy path projections, the smaller has to be γ_π in order to reach the same speed of convergence denoted by the k-isoquant. Or in other terms, for a given combination of γ_x,γ_π, the larger is λ_1 the smaller is k, and the fastest the learning process.

Appendix 8) proof of proposition 6 (announcing expected inflation and output gap)

Under learning the economy evolves according to the system of equation

$$\widetilde{\widetilde{H}}y_t=\widetilde{\widetilde{K}}+\widetilde{\widetilde{L}}\times E_t^{*}y_{t+1}+\widetilde{\widetilde{P}}\times E_t^{*}y_{t+2}+\widetilde{\widetilde{M}}w_t,$$

and the $\widetilde{\widetilde{H}}$, $\widetilde{\widetilde{L}}$ and $\widetilde{\widetilde{P}}$ matrices that are relevant in order to study stability under learning are

$$\begin{aligned}\widetilde{\widetilde{H}}&=\begin{bmatrix}1&-\alpha\\\varphi\gamma_\pi&(1+\varphi\gamma_x)\end{bmatrix},\quad\widetilde{\widetilde{P}}=\begin{bmatrix}\lambda_2\beta^2&0\\\varphi\lambda_2&\lambda_2\end{bmatrix}\\\widetilde{\widetilde{L}}&=\begin{bmatrix}0&\beta\alpha\lambda_2\\\varphi\lambda_2\left(1-\lambda_1\gamma_\pi\right)&-\varphi\lambda_2\lambda_1\gamma_x\end{bmatrix}\end{aligned}$$

or

$$Y_t=\widetilde{\widetilde{Q}}+\widetilde{\widetilde{F}}\times E_t^{*}Y_{t+1}+\widetilde{\widetilde{V}}\times E_t^{*}Y_{t+2}+\widetilde{\widetilde{S}}w_t,$$

where

$$\widetilde{\widetilde{F}}=\widetilde{\widetilde{H}}^{-1}\widetilde{\widetilde{L}}=\begin{bmatrix}-\alpha\varphi\lambda_2\frac{\lambda_1\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}&\alpha\lambda_2\frac{\beta+\beta\varphi\gamma_x-\varphi\lambda_1\gamma_x}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}\\-\varphi\lambda_2\frac{\lambda_1\gamma_\pi-1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}&-\varphi\lambda_2\frac{\lambda_1\gamma_x+\alpha\beta\gamma_\pi}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}\end{bmatrix}$$

$$\tilde{\tilde{V}} = \tilde{\tilde{H}}^{-1}\tilde{\tilde{P}} = \begin{bmatrix} \frac{\lambda_2\left(\beta^2+\alpha\varphi+\beta^2\varphi\gamma_x\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \alpha\frac{\lambda_2}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\frac{\varphi\lambda_2\left(\beta^2\gamma_\pi-1\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{\lambda_2}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \end{bmatrix}$$

for E-stability we need the eigenvalues of the $\tilde{\tilde{F}} + \tilde{\tilde{V}} - I$ matrix to have the real part smaller than zero.

$$\tilde{\tilde{F}} + \tilde{\tilde{V}} - I = \begin{bmatrix} \frac{\lambda_2\left(\beta^2+2\alpha\varphi+\beta^2\varphi\gamma_x-\alpha\varphi\lambda_1\gamma_\pi\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} - 1 & \frac{\alpha\lambda_2(\beta+\beta\varphi\gamma_x-\varphi\lambda_1\gamma_x+1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\frac{\varphi\lambda_2\left(\gamma_\pi\beta^2+\lambda_1\gamma_\pi-2\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & -\frac{\lambda_2(\varphi\lambda_1\gamma_x+\alpha\beta\varphi\gamma_\pi-1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} - 1 \end{bmatrix}$$

Since the characteristic polynomial of this matrix is

$$X^2 + a_1X + a_2$$

with

$$a_1 = \frac{\left(2 - \lambda_2 - \beta^2\lambda_2 - 2\alpha\varphi\lambda_2 + \varphi\gamma_x\left(2 - \beta^2\lambda_2 + \lambda_1\lambda_2\right) + \alpha\varphi\gamma_\pi\left(2 + \beta\lambda_2 + \lambda_1\lambda_2\right)\right)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}$$

$$a_2 = \frac{\left(\left(1 - \lambda_2\right)\left(1 - \beta^2\lambda_2\right) - 2\alpha\varphi\lambda_2\left(1 + \beta\lambda_2\right) + \varphi\gamma_x\left(1 - \beta^2\lambda_2\right)\left(1 + \lambda_1\lambda_2\right) + \alpha\varphi\gamma_\pi\left(1 + \beta\lambda_2\right)\left(1 + \lambda_1\lambda_2\right)\right)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}$$

necessary and sufficient conditions are $a_2 > 0$ and $a_1 > 0$. As

$$a_1 = a_2 + c$$

and

$$c = \frac{\left(1 - \beta^2\lambda_2^2 + 2\alpha\varphi\beta\lambda_2^2\right) + \left(\lambda_1\beta^2\lambda_2^2 + 1\right)\varphi\gamma_x + \alpha\varphi\gamma_\pi\left(1 - \beta\lambda_1\lambda_2^2\right)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1} > 0$$

this reduces to have $a_2 > 0$, that is

$$\gamma_\pi > \frac{2\lambda_2\left(\beta\lambda_2 + 1\right)\alpha\varphi - \left(\beta^2\lambda_2 - 1\right)\left(\lambda_2 - 1\right)}{\left(\beta\lambda_2 + 1\right)\left(1 + \lambda_1\lambda_2\right)\alpha\varphi} - \frac{\left(1 - \beta^2\lambda_2\right)}{\left(\beta\lambda_2 + 1\right)\alpha}\gamma_x$$

In order to determine what is the effect of announcing only the expected inflation and expected output gap, we impose $\lambda_1 = 1$ and obtain the following condition:

$$\gamma_\pi > h - z\gamma_x$$

with

$$
\begin{aligned}
h &= \frac{2\lambda_2(\beta\lambda_2+1)\alpha\varphi-(\beta^2\lambda_2-1)(\lambda_2-1)}{(\beta\lambda_2+1)(1+\lambda_2)\alpha\varphi} \\
z &= \frac{(1-\beta^2\lambda_2)}{(\beta\lambda_2+1)\alpha}
\end{aligned}
$$

Since

$$\frac{\partial h}{\partial\lambda_2}=\frac{(\beta(1+\beta)(1-\lambda_2^2)+2(1+\beta\lambda_2)(1-\beta^2\lambda_2)+\alpha\varphi(1+\beta^2\lambda_2^2+2\beta\lambda_2))}{[\alpha\varphi(\lambda_2+1)(\beta\lambda_2+1)]^2}>0$$

and

$$\frac{\partial z}{\partial\lambda_2}=\frac{(1-\beta^2\lambda_2)}{(\beta\lambda_2+1)\alpha}=\frac{(\beta^3\lambda_2^2-\beta^3\lambda_2-\beta^2-\beta\lambda_2)}{\alpha(\beta\lambda_2+1)^2}<0$$

We have that the larger the weight to the projection (the lower λ_2), the lower the intercept h and the steeper the slope (in absolute value) z. Therefore, for all values of the parameters, we have that by announcing the expected inflation and output gap, the combinations of (γ_x,γ_π) that imply E-instability is a subset of those obtained when the central bank does not announce the inflation and output gap.

In order to study the speed of convergence, we consider the k-isoquant. obtained from the $F+V-kI$ matrix

$$F+V-kI=\begin{bmatrix} \frac{\lambda_2\left(\beta^2+2\alpha\varphi+\beta^2\varphi\gamma_x-\alpha\varphi\lambda_1\gamma_\pi\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}-k & \frac{\alpha\lambda_2(\beta+\beta\varphi\gamma_x-\varphi\lambda_1\gamma_x+1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\frac{\varphi\lambda_2\left(\gamma_\pi\beta^2+\lambda_1\gamma_\pi-2\right)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & -\frac{\lambda_2(\varphi\lambda_1\gamma_x+\alpha\beta\varphi\gamma_\pi-1)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}-k \end{bmatrix}$$

We look at combinations of (γ_π,γ_x) that results in the same value for the real part of the largest eigenvalue of the $F+V$ matrix. Let $0<k<1$, then

$$z^2+a_1^kz+a_2^k$$

with

$$a_1^k=\frac{(2k-\lambda_2-\beta^2\lambda_2+2k\varphi\gamma_x-2\alpha\varphi\lambda_2-\beta^2\varphi\lambda_2\gamma_x+2k\alpha\varphi\gamma_\pi+\varphi\lambda_1\lambda_2\gamma_x+\alpha\beta\varphi\lambda_2\gamma_\pi+\alpha\varphi\lambda_1\lambda_2\gamma_\pi)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}$$

and

$$a_2^k=\frac{((\beta^2\lambda_2-k)(\lambda_2-k)-2\lambda_2(\beta\lambda_2+k)\alpha\varphi+(k-\beta^2\lambda_2)(k+\lambda_1\lambda_2)\varphi\gamma_x+(\beta\lambda_2+k)(k+\lambda_1\lambda_2)\alpha\varphi\gamma_\pi)}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1}$$

is the characteristic polynomial of $(F+V-kI)$. Both eigenvalues of $F+V$ have real parts less than k (i.e. both eigenvalues of $F+V-kI$ have negative real parts) if and only if $a_1^k>0$ and $a_2^k>0$. That is, both conditions

$$\gamma_\pi > \frac{\left(1+\beta^2+2\alpha\varphi\right)\lambda_2-2k}{\alpha\varphi\left(2k+\left(\beta+\lambda_1\right)\lambda_2\right)}-\frac{2k-\left(\beta^2-\lambda_1\right)\lambda_2}{\alpha\left(2k+\left(\beta+\lambda_1\right)\lambda_2\right)}\gamma_x$$

$$\gamma_\pi > \frac{2\lambda_2\left(\beta\lambda_2+k\right)\alpha\varphi-\left(\beta^2\lambda_2-k\right)\left(\lambda_2-k\right)}{\left(\beta\lambda_2+k\right)\left(k+\lambda_1\lambda_2\right)\alpha\varphi}-\frac{\left(k-\beta^2\lambda_2\right)}{\left(\beta\lambda_2+k\right)\alpha}\gamma_x$$

must be satisfied.

A k-isoquant in the S_2 set satisfies:

$$\gamma_\pi^k = \max\begin{bmatrix} \frac{\left(1+\beta^2+2\alpha\varphi\right)\lambda_2-2k}{\alpha\varphi(2k+(\beta+1)\lambda_2)}-\frac{2k-\left(\beta^2-1\right)\lambda_2}{\alpha(2k+(\beta+1)\lambda_2)}\gamma_x, \\ \frac{2\lambda_2(\beta\lambda_2+k)\alpha\varphi-\left(\beta^2\lambda_2-k\right)(\lambda_2-k)}{(\beta\lambda_2+k)(k+\lambda_2)\alpha\varphi}-\frac{\left(k-\beta^2\lambda_2\right)}{(\beta\lambda_2+k)\alpha}\gamma_x \end{bmatrix} \text{ for } \frac{1}{2}\leq k\leq 1$$

We take derivatives of the two terms inside the max operator with respect to λ_2 and we show that for a given k, the larger the λ_2, that is the smaller the weight to the announcement (the extreme case of $\lambda_2=1$ coincides with the case of no announcement), the larger is γ_π^k.

As

$$\frac{\partial}{\partial\lambda_2}\frac{2k-\left(\beta^2-1\right)\lambda_2}{\alpha\left(2k+\left(\beta+1\right)\lambda_2\right)}=\frac{-2k\alpha\beta\left(\beta+1\right)}{\left(\alpha\left(2k+\beta\lambda_2+\lambda_2\right)\right)^2}<0$$

and

$$\frac{\partial}{\partial\lambda_2}\frac{\left(\left(1+\beta^2+2\alpha\varphi\right)\lambda_2-2k\right)}{\alpha\varphi\left(2k+\left(\beta+1\right)\lambda_2\right)}=\frac{2k\left(\beta^2+\beta+2\alpha\varphi+2\right)}{\alpha\varphi\left(2k+\lambda_2+\beta\lambda_2\right)^2}>0$$

The first term of the max function is larger for larger λ_2.

And since

$$\frac{\partial}{\partial\lambda_2}\frac{\left(k-\beta^2\lambda_2\right)}{\left(\beta\lambda_2+k\right)\alpha}=\frac{-k\alpha\beta\left(\beta+1\right)}{\left(\left(\beta\lambda_2+k\right)\alpha\right)^2}<0$$

TO BE COMPLETED

Appendix 9) proof of proposition 7 (publishing interest rate, inflation and output gap projections)

Condition for E-stability becomes

$$\gamma_\pi > h - z\gamma_x$$

with

$$h = \frac{2\lambda_2(\beta\lambda_2+1)\alpha\varphi - (\beta^2\lambda_2-1)(\lambda_2-1)}{(\beta\lambda_2+1)(1+\lambda_1\lambda_2)\alpha\varphi}$$

$$z = \frac{(1-\beta^2\lambda_2)}{(\beta\lambda_2+1)\alpha}\gamma_x$$

Here we are interested on the combination of between λ_1 and λ_2 that make announcement worse than no announcement.

Under no announcement we have

$$\gamma_\pi > 1 - \frac{(1-\beta)}{\alpha}\gamma_x.$$

The slope

$$\frac{(1-\beta^2\lambda_2)}{(\beta\lambda_2+1)\alpha} - \frac{(1-\beta)}{\alpha} = \frac{(1-\lambda_2)\beta}{(\beta\lambda_2+1)\alpha} \geq 0$$

for all values of $0 \leq \lambda_2 \leq 1$.

The intercept

$$1 - \frac{2\lambda_2}{(1+\lambda_1\lambda_2)} + \frac{(\beta^2\lambda_2-1)(\lambda_2-1)}{(\beta\lambda_2+1)(1+\lambda_1\lambda_2)\alpha\varphi} < 0$$

for

$$0 \leq \lambda_1 < 2 - \frac{(1-\lambda_2)(1-\beta^2\lambda_2)}{\lambda_2\alpha\varphi(1+\beta\lambda_2)} - \frac{1}{\lambda_2}$$

where

$$2 - \frac{(1-\lambda_2)(1-\beta^2\lambda_2)}{\lambda_2\alpha\varphi(1+\beta\lambda_2)} - \frac{1}{\lambda_2} \leq 1$$

for all values of $\lambda_2 \leq 1$, since

$$\left(1-\beta^2\lambda_2\right)(1-\lambda_2) + \alpha\varphi(1-\lambda_2) + (1-\lambda_2)\alpha\beta\varphi\lambda_2 \geq 0$$

for all values of $\lambda_2 \leq 1$.

We have that the larger the weight to the projection (the lower λ_2), the lower the intercept h and the steeper the slope (in absolute value) z. Therefore, for all values of the parameters, we have that by announcing the expected inflation and output gap, the combinations of (γ_x, γ_π) that imply E-instability is a subset of those obtained when the central bank does not announce the inflation and output gap.

Appendix 10) Proof of proposition 8 (announcement of a T-period path)

First of all notice that

$$\pi_t = \beta^T E_t^* \pi_{t+T} + E_t^P \sum_{j=0}^{T-1} \beta^j \left(\alpha x_{t+j} + u_{t+j}\right)$$

$$x_t = E_t^* x_{t+T} - E_t^P \sum_{j=0}^{T-1} \left(\varphi i_{t+j} - \varphi \pi_{t+j+1} - g_{t+j}\right)$$

$$E_t^P i_{t+j} = \left(1 - \lambda_1\right) E_t^{CB} i_{t+j} + \lambda E_t^* i_{t+j},$$

can be written in matrix notation as

$$A_0 Y_t = K + A_1 E_t^* Y_{t+1} + ... + A_{T-1} E_t^* Y_{t+T-1} + A_T E_t^* Y_{t+T} + S w_t$$

with

$$\begin{aligned}
A_0 &= \begin{bmatrix} 1 & -\alpha \\ \varphi \gamma_\pi & \left(1 + \varphi \gamma_x\right) \end{bmatrix} \\
A_T &= \begin{bmatrix} \beta^T & 0 \\ \varphi & 1 \end{bmatrix} \\
A_1 &= \begin{bmatrix} 0 & \alpha\beta \\ \varphi \left(1 - \lambda_1 \gamma_\pi\right) & -\varphi \lambda_1 \gamma_x \end{bmatrix} \\
A_{T-1} &= \begin{bmatrix} 0 & \alpha\beta^{T-1} \\ \varphi \left(1 - \lambda_1 \gamma_\pi\right) & -\varphi \lambda_1 \gamma_x \end{bmatrix}
\end{aligned}$$

Since the perceived law of motion (PLM) is

$$E_t^* Y_{t+T} = A + B \Psi^T w_t$$

we can rewrite the previous expression as

$$A_0 Y_t = K + A_1 \left(A + B \Psi w_t\right) + ... + A_{T-1} \left(A + B \Psi^{T-1} w_t\right) + A_T \left(A + B \Psi^T w_t\right) + S w_t.$$

Therefore, the actual law of motion (ALM) is

$$Y_t = A_0^{-1}K + A_0^{-1}\left(A_1 + ... + A_T\right)A + \left(A_0^{-1}A_1 B\Psi + ... + A_0^{-1}A_T B\Psi^T + A_0^{-1}S\right)w_t,$$

And the mappings from the PLM to the ALM that drive the properties of the equilibrium under learning are

$$T(A) - A = A_0^{-1}\left(A_1 + ... + A_T\right)A - A$$

$$T(B) - B = A_0^{-1}\left(A_1 B\Psi + ... + A_T B\Psi^T\right) - B$$

To study the E-stability conditions we just need to focus on the $A_0^{-1}\left(A_1 + ... + A_T\right)$ matrix

$$\begin{bmatrix} \frac{\varphi\gamma_x+1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{\alpha}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \\ -\varphi\frac{\gamma_\pi}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} & \frac{1}{\varphi\gamma_x+\alpha\varphi\gamma_\pi+1} \end{bmatrix} \begin{bmatrix} \beta^T & \alpha\beta + \alpha\beta^2 + ... + \alpha\beta^{T-1} \\ \varphi\left(1 + (T-1)\left(1 - \lambda_1\gamma_\pi\right)\right) & 1 - (T-1)\varphi\lambda_1\gamma_x \end{bmatrix}.$$

In particular necessary and sufficient condition for E-stability is that the eigenvalues of the $\left[A_0^{-1}\left(A_1 + A_2 + ... + A_{T-1} + A_T\right) - I\right]$ matrix to have the real part smaller than zero. Let define

$$\begin{aligned} a &= \alpha\beta + \alpha\beta^2 + ... + \alpha\beta^{T-1} \\ b &= \varphi\left(1 + (T-1)\left(1 - \lambda_1\gamma_\pi\right)\right) \\ c &= 1 - (T-1)\varphi\lambda_1\gamma_x. \end{aligned}$$

Since the characteristic polynomial of the matrix $\left[A_0^{-1}\left(A_1 + A_2 + ... + A_{T-1} + A_T\right) - I\right]$ is

$$X^2 + a_1 X + a_2$$

with

$$\begin{aligned} a_1 &= \frac{\left(-c - b\alpha - \beta^T + 2\varphi\gamma_x - \beta^T\varphi\gamma_x + a\varphi\gamma_\pi + 2\alpha\varphi\gamma_\pi + 2\right)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1} \\ a_2 &= \frac{\left(-c - b\alpha - \beta^T + \varphi\gamma_x + c\beta^T - ab - \beta^T\varphi\gamma_x + a\varphi\gamma_\pi + \alpha\varphi\gamma_\pi + 1\right)}{\varphi\gamma_x + \alpha\varphi\gamma_\pi + 1}, \end{aligned}$$

necessary and sufficient conditions are $a_2 > 0$ and $a_1 > 0$.

The first condition implies that

$$\gamma_\pi > \frac{T\alpha\varphi - \left(1-\beta^T\right)}{\left(2+(T-1)\,\upsilon+\beta+\beta^2+...+\beta^{T-1}\right)\alpha\varphi} - \frac{\left(2+(T-1)\,\lambda_1-\beta^T\right)}{\left(2+(T-1)\,\lambda_1+\beta+\beta^2+...+\beta^{T-1}\right)\alpha}\gamma_x.$$

The second condition implies

$$\gamma_\pi > \frac{T}{(1-\lambda_1+\lambda_1 T)} - \frac{(1-\beta)}{\alpha}\gamma_x.$$

Since the latter condition implies the former, necessary and sufficient condition for E-stability is

$$\gamma_\pi > \frac{T}{(1-\lambda_1+\lambda_1 T)} - \frac{(1-\beta)}{\alpha}\gamma_x.$$