Risky Mortgages

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EPFL EPFL

January 29, 2010

PRELIMINARY AND INCOMPLETE

Abstract

This paper develops a model with housing and risky mortgages. Housing is subject to an idiosyncratic shock and some mortgages are defaulted in equilibrium. An unexpected increase in the risk of mortgages produces a credit crunch in our model, where mortgage default rate increases, lending is curtailed and output remains below steady state for many periods. We compare economies that differ only in the riskiness of mortgages. We find that economies with lower volatilities have a lower rate of default of mortgages, bigger mortgages are therefore higher leverage ratios. The effects of an unexpected shock to the riskiness of mortgages are amplified in high-leverage economies. Address of corresponding author:

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1 Introduction

At the heart of the global financial crisis that began in August 2007 lies mortgages that were defaulted and put financial institutions into distress. The bursting of the housing bubble in the United States put many borrowers in a difficult financial position with mortgages they could not pay in the long run and larger than the value of the houses against which they were underwritten. As a result the rate of default on mortgages increased to 9.2% by August 2008 and to 14.4% by September 2009. Banks were forced to write down several hundred billion dollars in bad mortgages. These losses, the high degree of opacity surrounding mortgage-backed securities and a complicated web of interconnected obligations among financial institutions triggered a large liquidity crisis an a credit crunch that caused failure of a number of financial institutions and brought many others close to it. The turmoil that started in the mortgage market amplified and sparked a global recession.

The goal of this paper is to develop a model with housing and mortgages that are subject to idiosyncratic risk and therefore can be defaulted on. Within such model we study the propagation mechanism of shocks. Driven by recent events, we also analyze the propagation mechanism of an unexpected increase in the risk of mortgages. We model this increase in risk as an increase in the volatility of the idiosyncratic shocks of mortgages. Our model produces a credit crunch in response to an unexpected increase in risk. Mortgage default rates increase significantly, mortgages and output remains below steady state for many periods.

We compare two economies that differ only in the risk of mortgages, namely in the volatility of the idiosyncratic risk underlying mortgages. The economy with the lower volatility has a lower rate of default of mortgages at the steady state and, as a result, mortgages are larger and the economy is more leveraged. When the volatility of idiosyncratic risk unexpectedly increases, borrowers are hurt more and the effects of the credit crunch are amplified. Hence, more leveraged economies suffer deeper slumps as a result of a worsening of the distribution of mortgages.

Our housing model draws a number of features from Iacoviello (2005) and Iacoviello and Neri (2009). These papers feature two households that differ in terms of their discount factor. Savers have a higher discount factor than Borrowers. To ensure the existence of an equilibrium, these models feature an exogenous borrowing constraint according to which Borrowers can borrow a fraction of the expected discounted future value of their houses. In equilibrium Savers lend to Borrowers and mortgages are always repaid. We follow Iacoviello (2005) and Iacoviello and Neri (2009) in modeling two households with different discount factors. However, we do not impose a borrowing constraint. Our credit market friction generates an endogenous borrowing constraint.

In our model housing is subject to an idiosyncratic shock. Each household consists of many members. Each member starts with the same loan and housing value but then he experiences an idiosyncratic shock to the value of the house. Following Bernanke, Gertler and Gilchrist (1999) we assume that the realization of the idiosyncratic shock can only be observed privately by the household member itself. The lender, however, can pay a monitoring cost to to observe the Borrower's realized return. The introduction of idiosyncratic risk to housing has two effects. First, mortgage contracts must satisfy Savers' participation constraint that guarantees a predetermined rate of return on aggregate loans. This participation constraint boils down to a loan-to-value ratio for mortgages that depends on aggregate conditions. Hence, our model generates an endogenous borrowing constraint for Borrowers.

Second, the introduction of idiosyncratic risk to housing generates equilibrium default on mortgages. As in Bernanke, Gertler and Gilchrist (1999), the mortgage contract is truth-revealing. Borrowers experiencing low realization of the idiosyncratic shock default on their mortgages; Savers pay the monitoring cost and seize the houses whose loans have been defaulted. Borrowers who repay their mortgages pay a state-contingent rate that is above the predetermined one. Hence, our model is characterized by default on mortgages and a finance premium.

A growing literature has been incorporating housing in economic models. Iacoviello (2005) builds on Kiyotaki and Moore (1997) to model housing as a durable good that can be used as collateral in borrowing. Iacoviello and Neri (2010) expand the work of Iacoviello (2005) and write a DSGE model with housing that is estimated using U.S. data for the period 1965:1 to 2006:3. Calza, Monacelli and Stracca (2009) analyze how the transmission mechanism of monetary shocks in a housing model á la Iacoviello is affected by alternative values of the down-payment rate and the interest rate mortgage structure. Monacelli (2009) documents positive co-movement in durable and non-durable consumption in response to a monetary policy shock and shows that a DSGE model with an exogenous borrowing constraint is consistent with the empirical evidence. The novelty of our paper is to introduce idiosyncratic risk and default in a model with housing.

The literature on the financial accelerator is vast. Starting with Bernanke, Gertler and Gilchrist (1999) and then Carlstrom and Fuerst (1997), many papers have introduced this credit friction in DSGE models to analyze its effect on the transmission of shocks. We do not present an exhaustive review of this literature here but rather focus on few recent applications. Lawrence Christiano and Rostagno (2007) augment a standard monetary DSGE model to include financial markets and fit the model to EA and US data. Cohen-Cole and Martinez-Garcia (2008) consider a model with a financial accelerator as in Bernanke, Gerlter and Gilchrist and introduce systemic risk, namely an aggregate variable that affects the variance of idiosyncratic risk, and banking regulation. Our paper is the first, to our knowledge, to introduce the financial accelerator in a model with housing.

The rest of the paper is organized as follows. Section **?** presents the model. In section **?** we perform our benchmark calibration and in Section **?** we analyze the transmission mechanism in response to monetary and technology shocks. Section **?** analyzes the case of a credit crunch, namely the response to a change in the volatility of the idiosyncratic shocks, in high- and low-leverage economies. Section **?** concludes.

2 The Model

Our starting point is a model with patient and impatient households that consume non-durable goods and housing service and work. Many features of our model draw from the housing model of Iacoviello (2005), Iacoviello and Neri (2010) and Monacelli (2009). Our focus, however, is on the mortgage contract and on how its features matter for the transmission of shocks. Hence, we do not rely on an exogenous borrowing constraint but rather derive it endogenously from the lenders' participation constraint after explicitly introducing idiosyncratic risk and therefore default.

2.1 Households

The economy is populated by a continuum of households distributed over the $[0, 1]$ interval. A fraction ψ of households has discount factor β while the remaining fraction $1 - \psi$ has discount factor $\gamma > \beta$. We are going to refer to the households with the lower discount factor as Borrowers, as these households value current consumption relatively than the other agents and therefore want to borrow. We are going to refer to households with the higher discount factor as Savers.

Borrowers have a lifetime utility function given by

$$\max \sum_{t=0}^{\infty} \beta^t E_0 \left\{ U\left(X_t, N_t\right) \right\} \quad 0 < \beta < 1 \tag{1}$$

where N_t is hours worked and X_t is an index of non-durable and durable consumption services defined as

$$X_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} H_{t+1}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where C_t denotes consumption of non-durable goods, H_t denotes consumption of housing services, α is the share of housing in the consumption index and $\eta \geq 0$ is the elasticity of substitution between housing and non-durable services. We assume that housing services in period t are equal to the housing stock at the beginning of period t. Assuming that services are a fraction of the stock is not going to change our results qualitatively. Borrowers are subject to the sequence of budget constraints:

$$P_{C,t} C_t + P_{H,t} H_{t+1} + (1 + R_{L,t-1}) L_t = L_{t+1} + (1-\delta) \left[1 - \mu G(\bar{\omega}_{t+1}) \right] P_{H,t} H_t + W_t N_t + T_t, \tag{3}$$

where $P_{C,t}$ is the price of non-durable goods, $P_{H,t}$ is the price of housing, L_{t+1} are the loans taken from Savers at t to be repaid in period $t+1$ and $R_{L,t}$ is the interest rate to paid on them. We assume that $R_{L,t}$ is fixed at time t and therefore is non-state-dependent. The housing stock depreciates at the rate δ. In equilibrium some loans are going to be defaulted on. The term $[1 - \mu G(\bar{\omega}_t]$ represents the housing stock borrowers are left with at the end of period t following default on some of the loans. We explicitly derive this term later. W_t is the nominal wage and T_t are transfers from the government. Each household decides non-durable good consumption, investment and therefore consumption of housing, working hours and loans.

Each household consists of many members. Following Bernanke et al. (1999) the i-th member of the household manages the housing stock H^i_{t+1} and is responsible for the mortgage connected to that housing housing stock. After buying housing H^i_{t+1} the i-th household member experiences an idiosyncratic shock ω^i_{t+1} such that his ex-post housing stock is $\omega^i H^i_{t+1}$. The random variable ω^i is i.i.d. across members of the same household with mean equal to one, i.e. $E_t(\omega_{t+1}) = 1$, and its cumulative distribution function obeys standard regularities conditions.[1] This implies that while there is idiosyncratic risk at the household-member level, there is no risk at the household level and $E_t\left\{\omega_{t+1}H_{t+1}\right\} = H_{t+1}$.

After idiosyncratic shocks are realized, household members decide whether to repay their loans or default. Given the pre-determined and non-state-contingent interest rate $R_{L,t}$, the household members with high realizations of the idiosyncratic shock ω^j_{t+1} repay their loans while those with low realizations default. To be more precise, for $\omega^i_{t+1} \in [0, \bar{\omega}_{t+1}]$ loans are defaulted while for $\omega^i_{t+1} \in [\bar{\omega}_{t+1}, \infty]$ loans are paid off. Lenders pay a monitoring cost to assess the value of the houses whose mortgages have been defaulted on and to seize the collateral. As in Bernanke et al. (1999), we assume that the monitoring cost is equal to the proportion μ of the housing value and that the defaulting household member is left with nothing.

The participation constraint of lenders is therefore given by:

$$(1+R_{L,t})L_{t+1} = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta)P_{H,t+1}H_{t+1}f(\omega)d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} (1+R_{Z,t+1})L_{t+1}f(\omega)d\omega, \quad (4)$$

where $R_{Z,t+1}$ is the state-contingent effective interest rate paid by non-defaulting Borrowers on their loans and the threshold value of the idiosyncratic shock $\bar{\omega}_{t+1}$ is defined as the one at which the Borrower is able to repay at the contractual rate $R_{Z,t+1}$:

$$\bar{\omega}_{t+1}(1-\delta)P_{H,t+1}H_{t+1} = (1+R_{Z,t+1})L_{t+1}. \quad (5)$$

Let

$$G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}f(\omega)d\omega \quad (6)$$

[1] The c.d.f is continuous, at least once-differentiable and it satisfies

$$\frac{\partial \omega h(\omega)}{\partial \omega} > 0,$$

where $h(\omega)$ is the hazard rate.

be the expected value of the idiosyncratic shock conditional on the shock being less than or equal to the threshold value $\bar{\omega}_{t+1}$ and let

$$\Gamma(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega + G(\bar{\omega}_{t+1}) \tag{7}$$

be the expected gross share of housing value that goes to lenders. Then the participation constraint can be written more compactly as

$$(1 + R_{L,t})L_{t+1} = (\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))(1 - \delta)P_{H,t+1}H_{t+1}. \tag{8}$$

The loan-to-value ratio is given by

$$\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}), \tag{9}$$

and it measures the size of the loan (principal plus interests) as a fraction of the housing value net of depreciation. The loan-to-value ratio also measures the net share of the housing value that goes to the lender for repayment.

The loan contract above is one where the lender does not bear any risk. The contract specifies ex-ante the rate of return $R_{L,t}$ on loans given out at time t, L_{t+1}, and the lender is guaranteed this return. This happens because, after idiosyncratic shocks have realized, the threshold value $\bar{\omega}_{t+1}$ and the state-contingent return $R_{Z,t+1}$ are determined so as to satisfy the participation constraint (8). On the other hand, Borrowers absorb the entire risk.

Borrowers maximize (1) subject to the budget constraint (3) and participation constraint (8) with respect to the variables $C_t, H_{t+1}, N_t, L_{t+1}, \bar{\omega}_{t+1}$. The respective first-order conditions are

$$U_{C,t} - \lambda_{BC,t} P_{C,t} = 0, \tag{10}$$

$$U_{H,t+1} - \lambda_{BC,t} P_{H,t} + \beta(1 - \delta) E_t \left\{ \left[1 - \mu G(\bar{\omega}_{t+1})\right] P_{H,t+1} \lambda_{BC,t+1} + \right. \tag{11}$$

$$\left. + \lambda_{PC,t+1} P_{H,t+1} \left[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\right] \right\} = 0,$$

$$U_{N,t} + \lambda_{BC,t} W_t = 0 \tag{12}$$

$$\lambda_{BC,t} - (1 + R_{L,t}) \left[E_t \lambda_{PC,t+1} + \beta \lambda_{BC,t+1}\right] = 0, \tag{13}$$

$$-\lambda_{BC,t+1}(1-\delta)\mu P_{H,t+1}H_{t+1}G'(\bar{\omega}_{t+1})+\lambda_{PC,t}(1-\delta)P_{H,t+1}H_{t+1}\left[\Gamma'(\bar{\omega}_{t+1})-\mu G'(\bar{\omega}_{t+1})\right]=0, \quad (14)$$

where $\lambda_{BC,t}$ is the Lagrangian multiplier on the Borrowers' budget constraint and $\lambda_{PC,t}$ is the Lagrangian multiplier on the participation constraint (8). Notice that the first-order condition with respect to $\bar{\omega}_{t+1}$ is state-by-state and not in expected terms.

Savers

We denote Savers' variables with a ˜, except for loans. Savers maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \gamma^t E_0 \left\{ U(\widetilde{X}_t, \widetilde{N}_t) \right\} \quad 0 < \beta < \gamma < 1 \quad (15)$$

where $\widetilde{X}_t$ is defined similarly to (2) and subject to the sequence of budget constraints:

$$P_{C,t}\widetilde{C}_t + P_{H,t}\widetilde{H}_{t+1} + L_{t+1} = (1-\delta)P_{H,t}\widetilde{H}_t + (1+R_{L,t-1})L_t + W_t\widetilde{N}_t + \widetilde{T}_t + \widetilde{\Delta}_t, \quad (16)$$

where $\widetilde{\Delta}_t$ are profits in the non-durable and banking sector, which are taken as given; $\widetilde{T}_t$ are transfers from the government. Notice that we have used the same notation for Savers' (positive) and Borrowers' (negative) loans L_{t+1} because, in equilibrium, Savers lend to Borrowers because of the higher discount rate, without however restricting the equilibrium outcome in any way.

Savers maximize (15) subject to the budget constraint (16) with respect to $\widetilde{C}_t, \widetilde{H}_{t+1}, \widetilde{N}_t, L_{t+1}$. The first-order conditions, respectively, are

$$U_{\widetilde{C},t} - \widetilde{\lambda}_{BC,t}P_{C,t} = 0, \quad (17)$$

$$U_{\widetilde{H},t+1} + E_t\left[(1-\delta)\widetilde{\lambda}_{BC,t+1}P_{H,t+1}\right] - \widetilde{\lambda}_{BC,t}P_{H,t} = 0, \quad (18)$$

$$U_{\widetilde{N},t} + \widetilde{\lambda}_{BC,t}W_t = 0, \quad (19)$$

$$-\widetilde{\lambda}_{BC,t} + \gamma(1+R_{L,t})E_t\widetilde{\lambda}_{BC,t+1} = 0, \quad (20)$$

where $\widetilde{\lambda}_{BC,t}$ is the Lagrangian multiplier on Saver's budget constraint.

2.2 Firms and Technology

Both the non-durable C and the housing H sector have intermediate and final good producers.

Final Good Producers

Final good producers are perfectly competitive and produce $Y_{j,t}$, $j = C, H$. The technology in the $j-$th final good sector is given by

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}(i)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}}, \tag{21}$$

where $\varepsilon_j > 1$ is the elasticity of substitution among intermediate goods in sector j. Standard profit maximization implies that the demand for intermediate good i is given by

$$Y_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t}, \ \forall i \tag{22}$$

where the price index is

$$P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di \right)^{\frac{1}{1-\varepsilon_j}}.$$

Intermediate Good Sectors

There are two intermediate good sectors $j \in [C, H]$ and in each intermediate sector there is a continuum of firms each producing a differentiated good $i \in [0, 1]$. These firms are monopolistically competitive. We assume that intermediate good firms face a quadratic cost proportional to output given by

$$\frac{\theta_j}{2} \left(\frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t},$$

where θ_j measures the degree of price rigidity in sector j. Intermediate good firm i uses labor to produce according to the linear production function

$$Y_{j,t}(i) = A_{j,t} N_{j,t}(i), \tag{23}$$

where $A_{j,t}$ is the stochastic level of technology in sector j.

Firm i chooses labor and its nominal price so as to maximize expected nominal profits. The maximization problem for firm i is given by

$$\max_{P_{j,t}(i),N_{j,t}(i)} E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_t \left[P_{j,t}(i) Y_{j,t}(i) - W_t N_{j,t}(i) - \frac{\theta_j}{2} \left(\frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 P_{j,t} Y_{j,t} \right] \right. \tag{24}$$

$$\left. + mc_t(i) P_{j,t} \left[A_{j,t} N_{j,t}(i) - Y_{j,t}(i) \right] \right\},$$

where the demand is given in (22) and

$$\Lambda_t \equiv \frac{\gamma^t \widetilde{\lambda}_{BC,t}}{\widetilde{\lambda}_{BC,0}}$$

is the stochastic discount factor for Savers.

The first-order condition relative to labor is

$$-W_t + mc_t(i) P_{j,t} A_{j,t} = 0, \tag{25}$$

which states that the nominal marginal cost equals the ratio of the nominal wage to the marginal product of labor. Since the marginal productivity of labor and wages are the same across all firms, $mc_t(i) = mc_t$. The first-order condition relative to the price is given by

$$Y_{j,t} \left[1 - \varepsilon_j + \varepsilon_j mc_t - \theta_j \pi_{j,t} (\pi_{j,t} - 1) \right] + \theta_j \gamma E_t \left[\frac{\widetilde{\lambda}_{BC,t+1}}{\widetilde{\lambda}_{BC,t}} Y_{j,t+1} \pi_{j,t+1}^2 (\pi_{j,t+1} - 1) \right] = 0, \tag{26}$$

where $\pi_{j,t}$ denotes gross inflation in sector j prices.

2.3 Monetary Policy

We assume that monetary policy follows a simple Taylor-type rule for the nominal interest rate:

$$\frac{(1 + R_{L,t})}{(1 + R_L)} = A_{M,t} \pi_{C,t}^{\phi} \left(\frac{(1 + R_{L,t-1})}{(1 + R_L)} \right)^{\phi_r}, \quad \phi > 1, \;\; \phi_r < 1 \tag{27}$$

where R_L is the steady-state nominal interest rate and $A_{M,t}$ is a monetary policy shock. We have assumed that monetary policy targets only inflation in the non-durable goods sector.[2]

[2] Assuming that monetary policy targets inflation in both sectors does not affect our results.

2.4 Market Clearing

Equilibrium in the non-durable goods market requires that production of the final non-durable good net of adjustment costs equals aggregate demand:

$$Y_{C,t} = \psi C_t + (1-\psi)\widetilde{C}_t + \frac{\theta_C}{2}\left(\pi_{C,t}-1\right)^2 Y_{C,t}. \tag{28}$$

Similarly, equilibrium in the housing market requires

$$Y_{H,t} = \psi\left[H_{t+1} - (1-\delta)(1-\mu G(\bar{\omega}_t))H_t\right] + (1-\psi)\left[\widetilde{H}_{t+1} - (1-\delta)\widetilde{H}_t\right] + \frac{\theta_H}{2}\left(\pi_{H,t}-1\right)^2 Y_{H,t}. \tag{29}$$

Equilibrium in the labor market requires

$$\int_0^1 N_{H,t}(i)di + \int_0^1 N_{C,t}(i)di = \alpha N_t + (1-\alpha)\widetilde{N}_t. \tag{30}$$

3 Functional Forms, Calibration and Steady State

We the following standard utility function:

$$U(X_t, N_t) \equiv \ln X_t - \nu\frac{N_t^{1+\varphi}}{1+\varphi}, \quad \varphi > 0 \tag{31}$$

where φ is the inverse of the elasticity of labor supply to wages.

There are three exogenous shocks in our model that evolve according to the following first-order autoregressive processes

$$\ln A_{C,t} = \rho_C \ln A_{C,t-1} + \epsilon_{C,t}, \quad \rho_C \in (-1,1), \tag{32}$$

$$\ln A_{H,t} = \rho_H \ln A_{H,t-1} + \epsilon_{H,t}, \quad \rho_H \in (-1,1), \tag{33}$$

$$\ln A_{M,t} = \rho_M \ln A_{M,t-1} + \epsilon_{M,t}, \quad \rho_M \in (-1,1), \tag{34}$$

where $\epsilon_C, \epsilon_H, \epsilon_M$ are i.i.d. innovations with mean zero, standard deviation $\sigma_C, \sigma_H, \sigma_M$, respectively.

The parameters values for our calibration are specified in Table 1. Borrower's and Saver's discount factors, the rate of depreciation for housing, the elasticities of substitution between non-durable goods and housing services and the share of housing in the consumption bundle are taken by Monacelli (2009). The Saver's discount factor pins down the annual real rate of return at the steady state to $R_L = 0.0101$. For the degree of price stickiness, we assume that housing prices are fully flexible, which is in line with the empirical estimation of Iacoviello and Neri (2010) and the empirical evidence on price stickiness for durable goods. For non-durable goods, we assume $\theta_C = 75$, which is standard in the literature. Moreover, this implies the same slope of the Phillips curve that would emerge in the typical Calvo-Yun model with a probability of not changing prices equal to 0.75, which is typically assumed in the literature. We set the parameters in the interest rate rule $\phi = 1.5$ and $\phi_r = 0.9$, as standard in the literature on Taylor-type rules.

As for the idiosyncratic risk in the housing sector, we follow Bernanke et al. (1999) and assume that ω is distributed log-normally:

$$\ln \omega \sim N(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2). \tag{35}$$

In our benchmark calibration we set $\sigma_\omega = 0.7$, which is consistent with a steady-state default rate of 4.6%. We set $\mu = 0.07$ and therefore assume that monitoring costs are 7 percentage points of the housing value.

At the steady state, the nominal interest is $R_L = 0.0101$, the state-contingent interest rate paid by non-defaulting households is $R_Z = 0.0234$. We define the external finance premium at t as $R_{Z,t} - R_{L,t}$, namely the difference between the ex-post state-contingent rate paid by non-defaulting households at time t and the pre-set rate lenders receive on their total loans L_{t+1}.[3] This premium captures the additional cost that Borrowers must pay for their mortgages relative to Savers, whose relevant rate is the opportunity cost of funds and therefore the risk-free lending rate R_L. At the steady state, the external finance premium is equal to 0.0133.

[3] Our definition of external finance premium differs somewhat from that in Bernanke et al. (1999), where the premium is the difference between the costs of funds raised externally and the opportunity costs of funds internal to the firm.

Parameter	Value	Description
γ	0.99	Discount factor of Savers
β	0.98	Discount factor of Borrowers
δ	0.0025	Rate of depreciation for housing
ε_C	7.5	Elasticity of substitution for C goods
ε_H	7.5	Elasticity of substitution for H goods
ψ	0.5	Relative size of borrower group
α	0.16	Share of housing in consumption bundle
ν	2.5	Disutility from work
η	1	Elasticity of substitution between C and H goods
φ	1	Inverse of elasticity of labor supply
θ_C	75	Price adjustment cost in C
θ_H	0	Price adjustment cost in H
ϕ	1.5	Taylor-rule coefficient on inflation
ϕ_r	0.9	Taylor-rule serial correlation of the interest rate
ρ_C	0.9	Serial correlation of productivity shocks in C
ρ_H	0.9	Serial correlation of productivity shocks in H
ρ_M	0.7	Serial correlation of monetary policy shocks
σ_ω	0.7	Standard deviation of idiosyncratic shocks
μ	0.07	Monitoring cost

Table 1: Parameter Values

4 Dynamic Response to Monetary and Technology Shocks

Figure 1 illustrates the impulse responses of the model under the benchmark calibration in response to a monetary shock, namely a 25 basis point increase in the nominal interest rate $R_{L,t}$. Savers, who are consumption smoothers, reduce consumption of non-durable goods and hours worked and raise their demand for housing. Borrowers, on the other hand, experience an increase in the cost of borrowing and reduce their loans, consumption of non-durable goods and housing but raise hours worked. Driven by the fall in non-durable goods demand, total output falls. Higher interest rates on mortgages raise $\bar{\omega}$, the threshold value below which households do not repay, and households' default rate. As more mortgages are defaulted on, monitoring costs increase. Since a lower fraction of households repays the loans and since monitoring costs increase, the external financial premium increases by ten percentage points. This means that the quarterly external finance premium increases by 250 basis points from 0.0133 to 0.0309. The loan-to-value ratio increases on impact as a result of higher interest rate and lower housing value.

Figure 2 illustrates the impulse responses of the model under the benchmark calibration in

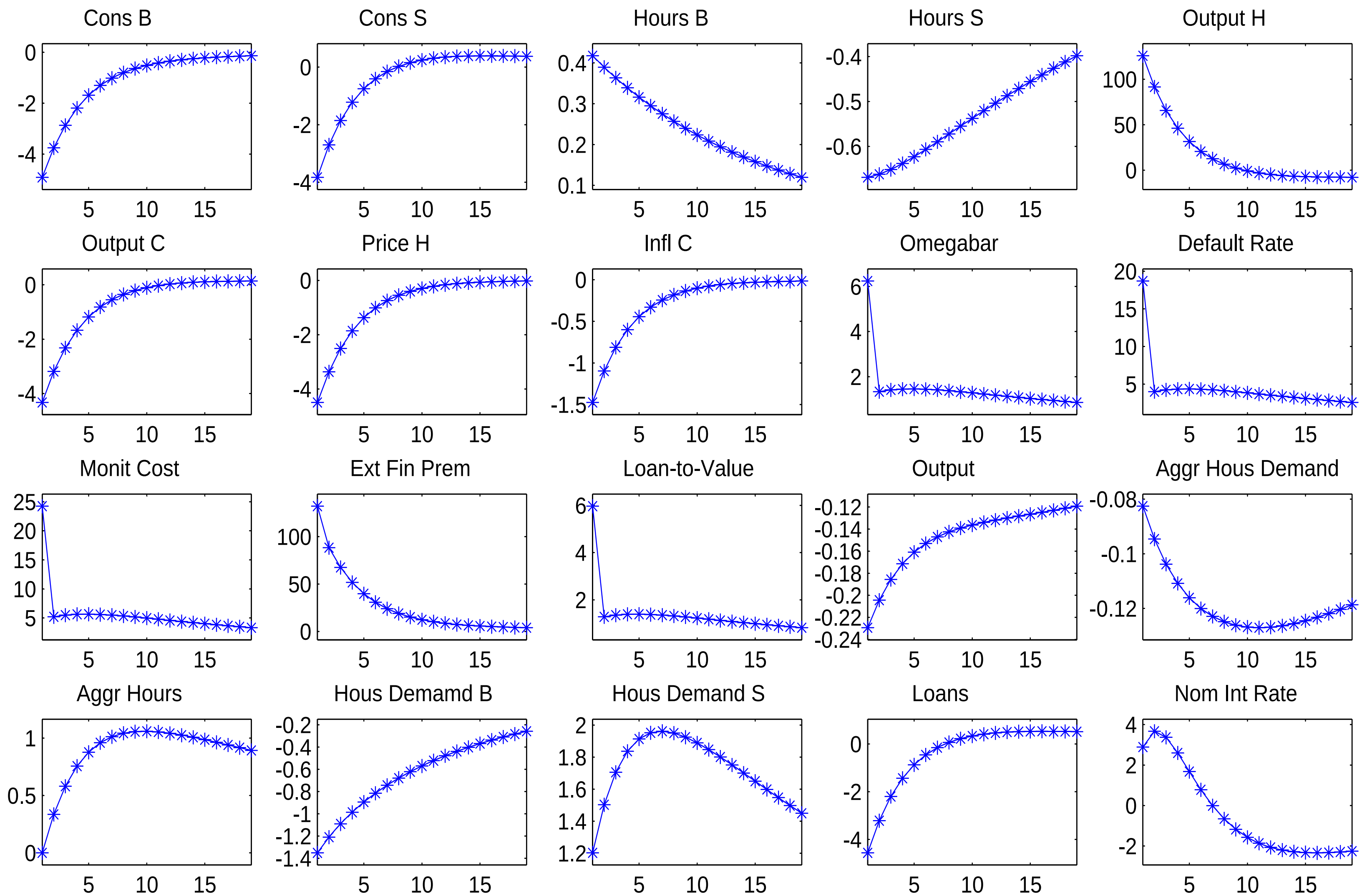

Figure 1: Impulse Responses to a 25 basis points Monetary Shock
The vertical axis is the % deviation from steady state; the horizontal axis are quarters after the shock. LTV: Loan-to-Value Ratio.

response to a negative technological shock in the non-durable sector, namely a one percentage point decrease in $A_{C,t}$. Output falls and prices increase in the C sector, which in turn raises the nominal interest rate via the Taylor rule. As a result, Savers reduce consumption and slightly raise hours worked. Borrowers face higher costs of borrowing and therefore reduce loans, consumption and housing. The increase in nominal interest rate resulting from a negative technological shock affects the mortgage market much along the same lines of a monetary shock, as the default rate, monitoring costs and the loan-to-value ratio all go up. Interestingly, and differently from a monetary shock, the external finance premium falls. This is because housing prices fall more than the threshold $\bar{\omega}$, thereby reducing $R_{Z,t}$, the ex-post interest rate paid by non-defaulting households.

Figure 3 illustrates the impulse responses of the model under the benchmark calibration in response to an increase in the level of technology in the housing sector, namely a one percentage point increase in $A_{H,t}$. Production increases and prices fall and in the housing sector. Since housing is cheaper, Savers increase their demand. Borrowers, on the other hand, take fewer loans as the value of their houses is now lower. Hence, they consume less but nevertheless raise their housing demand. Total output increases because the increase in production in the housing sector more than compensates the reduction in production in the non-durable goods sectors following an increase in real wages and a fall in demand. Inflation in the non-durable goods sector raises nominal interest rates that, in turn, raise the rate of default, monitoring costs and the loan-to-value ratio. In line with the negative technological shock in the C sector, the external finance premium falls.

5 Credit Crunch

This section analyzes the dynamic response of the model to an unexpected increase in the standard deviation of the distribution of $\ln \omega$, the idiosyncratic shock in the housing market. Intuitively, we want to capture the situation in which loans are made on the basis of an expected distribution for idiosyncratic shocks *but* the actual distribution turns out to be characterized by a higher standard deviation. To do this, we assume that the standard deviation of $\ln \omega$ is

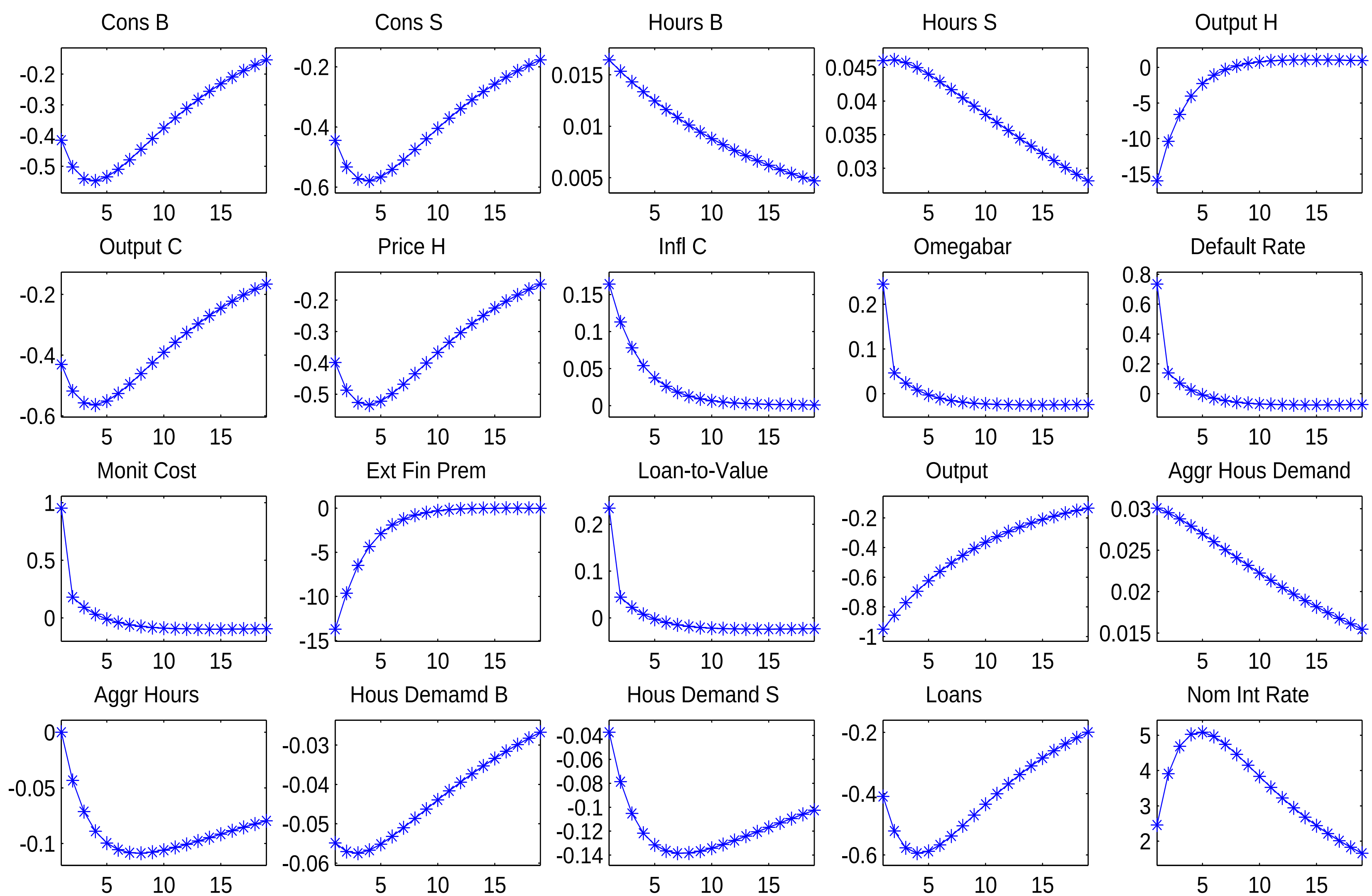

Figure 2: Impulse Responses to a -1% Technology Shock in the Consumption-Good Sector

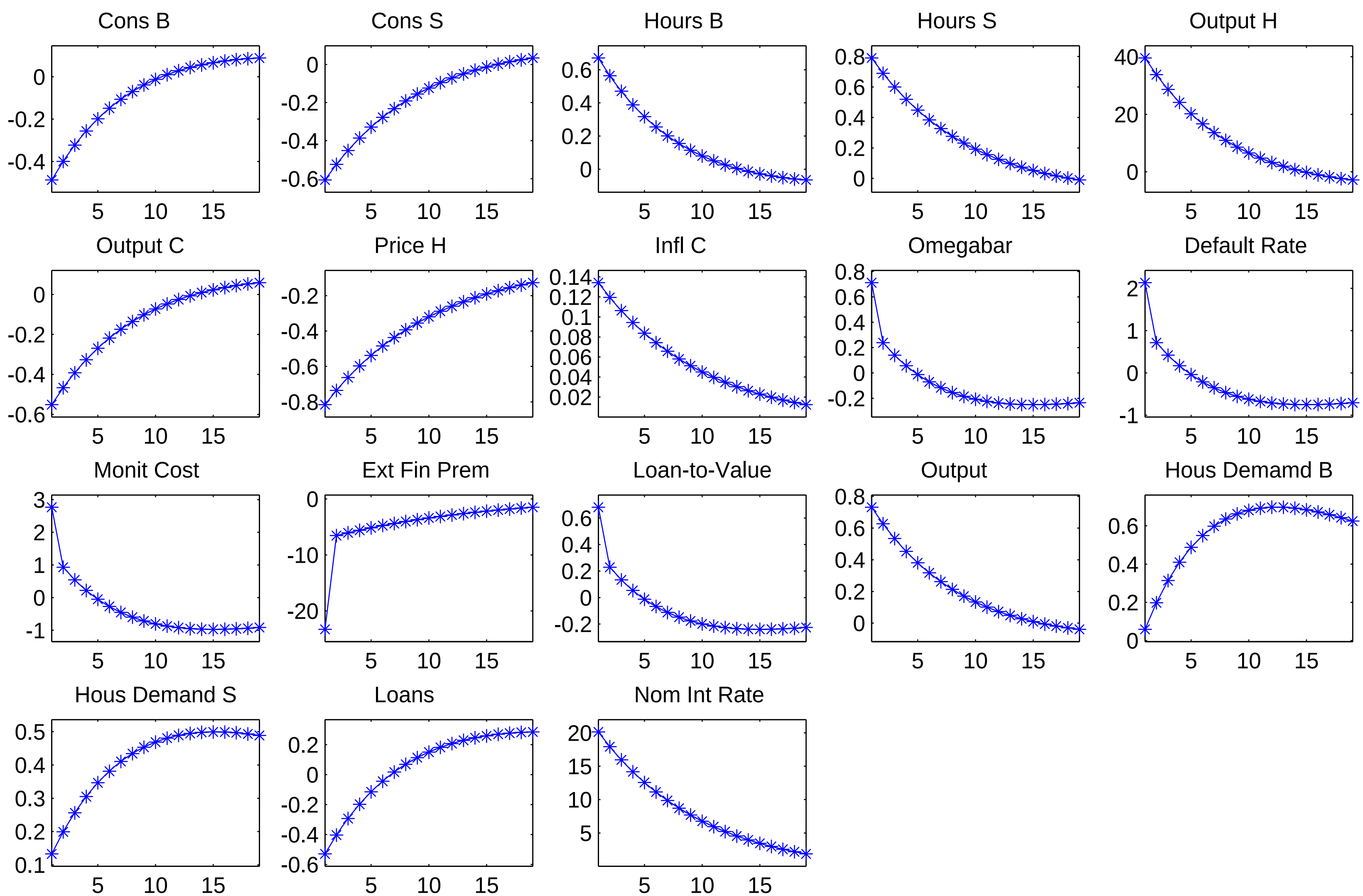

Figure 3: Impulse Responses to a 1% Technology Shock in the Housing Sector

itself an exogenous shock subject to a first-order autoregressive process:

$$\ln \sigma_{\omega,t} = \rho_\sigma \ln \sigma_{\omega,t-1} + \epsilon_{\sigma_{\omega,t}}, \tag{36}$$

where ϵ_{σ_ω} is an i.i.d. shock with mean zero, finite standard deviation σ_{σ_ω} and bounded support. We set $\rho_\sigma = 0.9$. Notice that an increase in σ_ω, the standard deviation of the distribution of $\ln \omega$, has two effects on the distribution itself. First, it increases the variance. Second, it lowers the mean, which implies a leftward shift of the distribution. Since the log-normal distribution does not take negative values, a fall in the mean implies a thicker lower tail of the distribution. Thus for a given value of $\bar{\omega}$ an increase in σ_ω raises the default rate on mortgages, which in turn reduces Borrowers' wealth.

An increase in σ_ω affects the economy through two main channels. On one hand, the loan-to-value ratio, which is also the net share of the housing value going to Savers for repayment, falls. This makes the participation constraint more binding. On the other hand, since a lower fraction of Borrowers repay their debts, monitoring costs increase sharply and the housing stock of Borrowers falls.

Figure 4 shows how these two mechanisms are incorporated into the impulse responses of the model to a ten percentage points increase in the standard deviation σ_ω (namely an increase on impact of the standard deviation from 0.7 to 0.77). As a result of the shift in the distribution, the rate of default and monitoring costs increase by around fifty percentage points whereas the loan-to-value ratio drops by around ten percentage points. Borrowers cut their expenditure of both housing and the non-durable good and increase their labor supply in response to the reduction in wealth. Borrowers' leverage is reduced by the tighter participation constraint and part of their housing stock is lost because of default. Lower inflation reduces the nominal interest rate and Savers raise current consumption of housing and non-durable goods, reduce saving and enjoy more leisure. Notice that for both types of consumers the response of housing demand is significantly larger than that of consumption and labor. This is because housing as durable good provides additional services. When σ_ω increases, the loan-to-value ratio falls and Borrowers value housing less as a collateral. Savers instead value housing more as a substitute for loans to transfer wealth into the future.

The different behavior of the two groups of households explains the opposite responses of output in the H and C sectors. Interestingly, on impact *production in the non-durable sector contracts*. The drop in Borrowers' demand for C goods more than compensate the slight increase in that of Savers. Conversely, the production of new houses raises. This is due mainly to the need to replenish the housing stock lost through the monitoring process than to a net increase in residential demand. Indeed, the change in housing demand of Savers an Borrowers almost compensate each other. Aggregate output and employment increase initially as the increase in residential investment dominates. After few periods, however, aggregate output and employment enter a long-lasting contraction below their steady-state levels.

5.1 Credit Crunch: High- and Low-Leverage Economies

In this section we compare two economies characterized by different standard deviations of the distribution of the idiosyncratic shocks to housing. In the first economy (the "High-Leverage Economy") the standard deviation is $\sigma_\epsilon = 0.7$, which is the value used in our benchmark economy. In the second economy (the "Low-Leverage Economy") $\sigma_\epsilon = 1.4$.[4] We show that higher leverage amplifies the macroeconomic effects associated with a sudden increase in the rate of default on mortgages due to higher-than-anticipated volatility of the idiosyncratic shocks to housing.

Table 2 reports the steady-state levels of a number of endogenous variables in the two economies. The last column of Table 2 reports the percentage point difference between High- and Low-Leverage economies. Intuitively, a lower standard deviation of the idiosyncratic shocks is associated with a lower rate of default on mortgages and therefore a lower external finance premium and an higher loans-to-value ratio. In this scenario Borrowers demand more residential housing, which can be used as a collateral, and borrow more substituting out consumption for the non-durable goods and leisure. We calculate the leverage ratio as the fraction of total expenses, namely expenses on consumption of C and H goods plus loan repayment, that is financed by loans. The leverage ratio therefore captures the dependence of Borrowers from external funding. Because loans are bigger when idiosyncratic volatility is lower, the steady-state leverage ratio is almost 100 percentage points higher in the High-Leverage economy. Total monitoring costs are $\mu G(\bar{\omega})$, namely the monitoring cost multiplied by the average ω among

[4] All the other parameters are still set according to the values in table 1.

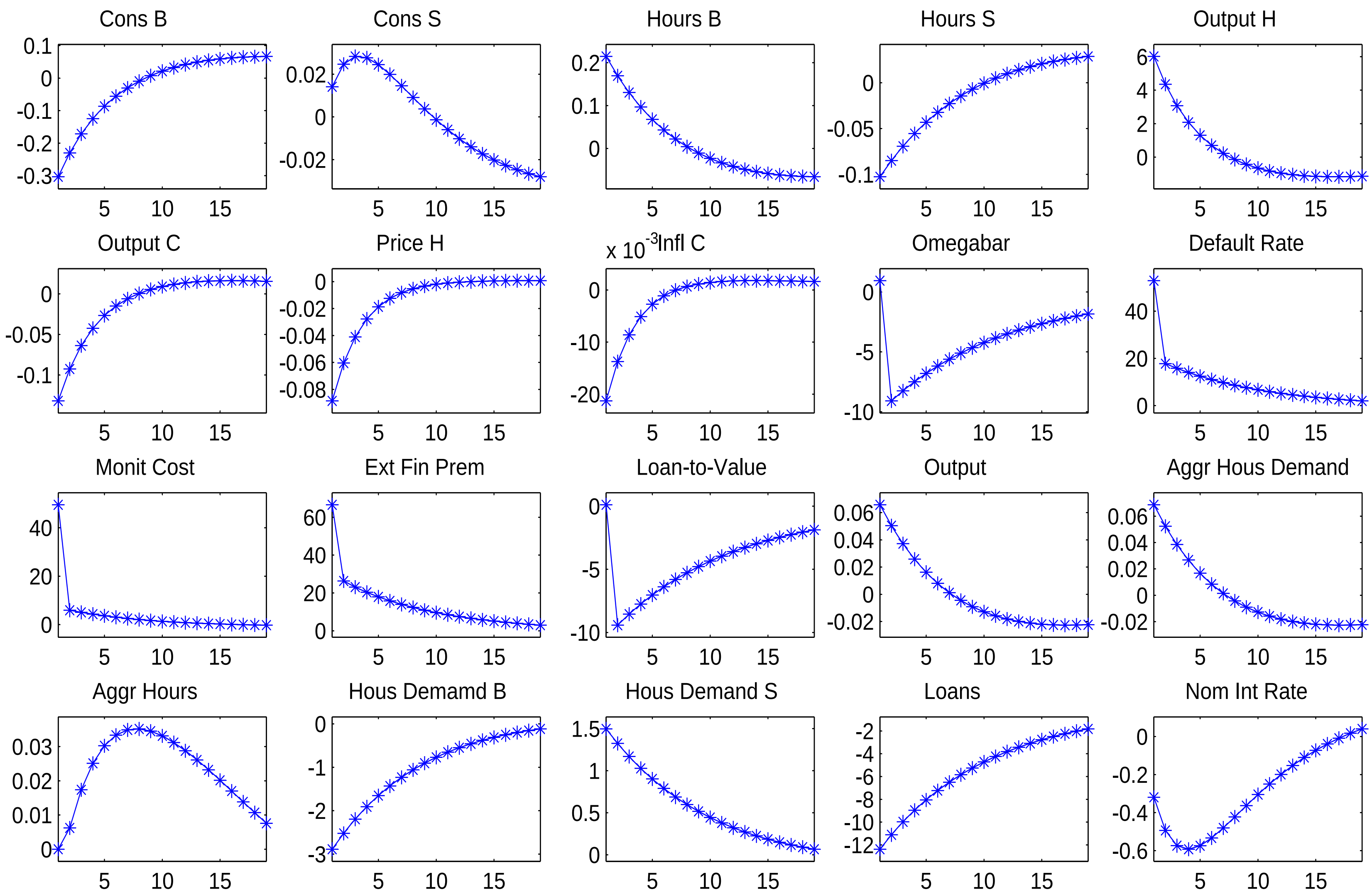

Figure 4: Impulse Responses to a 10% Increase in σ_ω

Variable	High Leverage	Low Leverage	% Difference
Output C	0.6881	0.6887	-0.09
Output H	0.0233	0.0219	6.39
Consumption, Borrowers	0.6127	0.6217	-1.45
Consumption, Savers	0.7634	0.7557	1.02
Housing Demand, Borrowers	5.6478	5.3570	5.43
Housing Demand, Savers	11.6556	11.5390	1.01
Hours Worked, Borrowers	0.7426	0.6801	9.19
Hours Worked, Savers	0.7383	0.6829	8.11
Loans	1.3241	0.3252	307
Loan-to-Value Ratio	0.2374	0.0615	286
Leverage Ratio	0.6729	0.3370	99.67
Default Rate on Mortgages	0.0459	0.1043	-55.99
Total Monitoring Cost	0.06%	0.03%	100
External Finance Premium	0.0133	0.0505	-73.66

Note: The Leverage Ratio is calculated as $L/(L + NW/P_C)$

Table 2: High- and Low-Leverage Economies: Steady States Comparison

defaulting households. We keep the monitoring cost $\mu = 0.07$ for both economies – see Table 1. Hence, the difference in total monitoring costs between High- and Low-Leverage economies is explained entirely by the difference in $G(\bar{\omega})$. Intuitively, even though the default rate is higher in Low-Leverage economies, the total amount of loans is much lower and therefore the total amount of defaulted loans is also lower.

Next we analyze how differences in steady-state leverage ratios translate into different responses over the business cycle. Figure 5 plots the impulse responses of the endogenous variables to a ten percentage points increase in σ_ω for the "High-Leverage" and the "Low-Leverage" economy. The effect of an increase in the risk of the mortgages are amplified in the high-leverage scenario. With higher leverage ratios, the credit crunch caused by an unexpected increase in σ_ω harms Borrowers more. Borrowers must reduce their loans substantially. Since loans finance almost 70 percent of Borrowers' spending (see Table 2) in this scenario, Borrowers must cut their consumption of both houses and non-durable goods and work more. Even though loans fall by less in High-Leverage economies in percentage deviation of the steady state, they fall more in absolute value because steady-state loans are much higher. As a consequence Borrowers' demands for both housing and non-durable goods fall by more than in economy with low leverage. The sharper contraction in demand explains the deeper slump in output in the non-durable sector.

Aggregate output increases in the first four periods, somewhat counter-factually. Once again, this is due to the increase in production in residential housing, which in turn is explained by the increase in monitoring costs and the need to replenish the housing stock. Adding adjustment costs to the housing production sector would make output smoother and likely eliminate the short-term increase in aggregate output.

6 Conclusions

To be written.

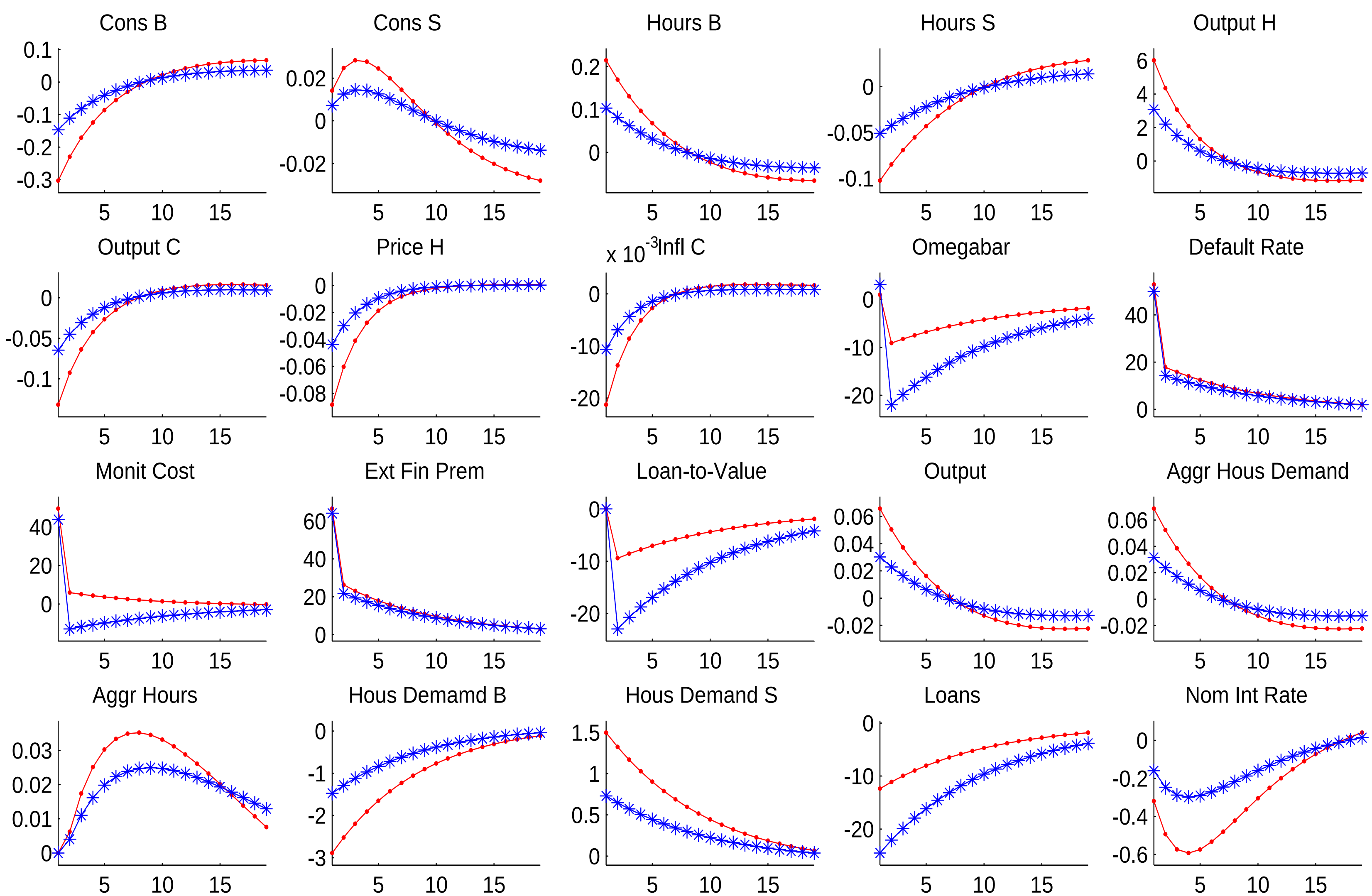

Figure 5: Impulse Responses to a 10% Increase in σ_ω
Red line: High-Leverage Economy; Blue line: Low-Leverage Economy

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