Estimating the output gap in the Polish economy: VECM approach[*]

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Abstract

This article presents three estimations of the output gap, one using the production function method, and the other two by establishing the long-term product using cointegration relationships (based on the production function of and the hypothesis of permanent income). It also presents an analysis of time-relationships between the estimated output gaps and selected measures of inflation using the covariance of a VAR-type stochastic process. The methods employed yield different estimates of the output gap. However, if the time-structure of the effects of these gap on inflation is taken into account, they lead to identical conclusions and indicate the absence of inflationary pressure from the demand side until at least the end of 2003.

Key words: Output gap, VECM, production function, Permanent-Transitory Decomposition

JEL classification: E32, C32

1. Introduction

This paper presents estimates of the output gap in the Polish economy in 1996-2002. The output gap, usually defined as the difference between the logarithm of the real GDP and that of the potential GDP, indicates the imbalance existing in a real economy. Thus the concept of output gap is based on the definition of unobserved potential output.

One can find various definitions of potential output in the literature. The applied definitions depend on the objectives of the investigator. From the monetary authority's point of view still the most influential is the definition proposed by Okun (1962), who described potential GDP as the maximum quantity of output the economy can produce under conditions of full employment (which is understood as the maximum level of employment not generating inflationary pressures). Later refinements of the definition stressed alternative aspects of the above-mentioned qualification, ranging from the intensity of use of labour and capital (Artus (1977)) and natural rate of unemployment (Gordon (1984)). Output gap is stressed in the neo-keynesian models of the DSGE type (*Dynamic Stochastic General Equilibrium*) with sticky prices (cf. Clarida, Gali, Gertler (2000)). Here the concept of potential output is different – the equilibrium level reached without nominal rigidities, that is, with fully flexible prices and wages.

Usefulness of the output gap concept for the monetary policy authorities is based on the interpretation of the potential output as economy's production capacity, that can be assumed to be constant in the short-run. Positive output gap means that the demand exceeds the supply, which implies inflationary pressure. In the medium-run the constancy of the production capacity is violated, particularly increased investment demand can itself generate an increse in capacity.

When interpreting the output gap, one should bear in mind that it reflects only the demand pressure (domestic and foreign). Positive output gap can for example appear with declining inflation in case of decreasing import prices or other cost-factors.

There is no commonly accepted methodology of calculating potential output. Methods based directly or indirectly on Okun's definition usually use the production function. On the other hand, methodologies that decompose the product into permanent and transitory components make use of concepts defining potential output as the long-run growth path of GDP.

Due to importance of output gap in decision making process of monetary authorities and the lack of commonly approved method of calculating the potential GDP, there is a huge degree of uncertainty in this process. Uncertainty is also added because of the GDP measuring errors, especially in the real-time (Orphanides, van Norden (2002)).

The authors decided to present the results of output gap calculations made using two methods:

- a method based on a two-factor dynamic production function (estimated in the co-integrated VECM system), in which the potential GDP is calculated as the product resulting from the long-term level of inputs of means of production (Chapter 2);
- the GDP Permanent-Transitory Decomposition, using long-term restrictions in the vector error correction model (VECM) imposed in an endogenous way by co-integrating relationships; Chapter 3 contains the results for two views: one based on the long-term production function and the other on the permanent income hypothesis.

As the output gap is connected with inflationary processes in the economy, Section 4 analyses time dependencies between the estimated time-series of output gaps and various inflation measures (CPI, PPI, GDP deflator and one of the core inflations). This analysis was made using determined cross-correlations of the inflation and the gap, which are implied by the covariance generating function of the VAR stochastic process.

2. Production function approach

Estimating the production function

In order to estimate the potential GDP of the Polish economy, the dynamic Cobb-Douglas function[1] was selected as the production function. In this function, production can be expressed as a combination of labour and capital inputs taking into account technical

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[1] The Cobb-Douglas function was selected because of its simplicity and because of the difficulty in the economic interpretation of alternative assumptions. Compare: Mc Morrow, Roeger (2001). The basic feature of the Cobb-Douglas function is the assumption of the unit elasticity of the substitution. This assumption seems to be proven by figures describing the Polish economy: in 1995-2001, the proportion of labour costs to gross value added was between 50% and 53%, and it was almost identical in 1995 and 2001.

development as understood by Hicks. If constant economies of scale are assumed, it can be represented as follows (cf. Żółtowska (1997)):

$$Y_t = A(\alpha_t L_t)^{\alpha} (\beta_t K_t)^{1-\alpha},$$

where Y_t is the production (GDP in this case), L_t and K_t are the inputs of labour and capital, respectively, while α_t and β_t are technical development functions in the meaning of Harrod and Solow, respectively. As it is difficult to separate labour productivity growth from capital productivity growth, the production function is often presented as follows:

$$Y_t = TFP_t L_t^{\alpha} K_t^{1-\alpha},$$

where TFP_t means the total factor productivity and reflects technical development increasing the productivity of both labour - due to the improved qualifications of employees, and capital - due to the introduction of advanced technologies. By having this structure, the TFP_t variable makes it possible to introduce variations in the A factor, and thus take into account factors which cannot be explained by technical development. One of such factors is the effectiveness of social resistance to the introduction of new technologies (cf. Prescot (1997)).

The direct estimation of the production function using the ordinary least square method (OLS) does not seem to be a proper method for at least two reasons. Firstly, the inputs of labour and capital cannot be treated as independent, so the assumption that explanatory variables are exogenous does not hold (cf. Griliches, Mairess (1995)). Secondly, according to the economic theory, at least GDP and capital are generated by non-stationary stochastic processes, so the use of OLS may lead to spurious regressions.

One of the methods allowing to avoid the above methodological errors is the cointegration analysis. In this paper, the system was estimated in the form of a vector error correction model (VECM), according to the Johansen (1991) procedure. The cointegration relationship between production and inputs of labour and capital estimated in the above way can be considered as a well-estimated production function provided that the model has been correctly specified.

The quarterly data was used for empirical analysis. To eliminate the effect of seasonal movements on the results, all variables were adjusted seasonally using the multiplicative moving average method. The labour input (L_t) was assumed to be equal to the number of employed persons according to the Labour Force Survey (LFS). The variable describing the capital in the Polish economy (K_t) was assumed to be equal to the gross value of fixed assets

in the national economy in different quarters.[2] The data on real GDP (Y_t) is taken from publications of the Central Statistical Office.

The results of unit root testing based on the Dickey-Fuller test with the generalised least square method (GLS) indicates that the logarithms of the seasonally-adjusted Y_t, L_t and K_t variables are integrated of order one (Table 1). Thus there are grounds for looking for cointegrating relationships between these variables.

Table 1. Results of the unit root test

Variable	DF-GLS statistics	Conclusion at the 0.1 significance level
$\Delta\log(Y_t)$	-1.99**	I(1)
$\Delta\log(L_t)$	-2.00**	I(1)
$\Delta\log(K_t)$	-1.91*	I(1)

* significant at the 0.1 significance level
** significant at the 0.05 significance level

Source: Own calculations

In the first stage of estimation it was assumed that the TFP_t variable could be approximated by the exponential trend, i.e. a linear trend after calculating the logarithm. Cointegration test results indicate that there is one cointegrating relationship between the Y_t, L_t and K_t variables at the 1% significance level. This is so both in the case of the trace test and the test of maximum eigen-value (Table 2). Consequently, the VECM system was estimated with the assumption that there is one cointegrating relationship.

Table 2. Johansen cointegration test

Hypothesis: number of cointegrating relationships	Eigen-value	Trace statistic	Maximum eigen-value statistic
None	0.74	53.07***	37.38***
No more than one	0.28	15.68	9.06

*** hypothesis rejected at a 0.01 level of significance

Source: Own calculations

The approximation yielded the following estimate of the long-term relationship between GDP, labour input and capital input:

$$\hat{Y}_t = L_t^{0,57} K_t^{0,43} e^{0,29+0,01t}$$

The estimated long-term relationship parameters have the expected signs and are statistically significant. The same can be said of the α adjustment matrix parameters (for one

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[2] Calculations done by Marcin Kolasa and Roman Sawiński, DAMS, NBP.

cointegrating relationship it has the form of a vector), corresponding the equations describing labour and capital dynamics. The evaluations of adjustment parameters indicate that the equilibrium state is regained quicker through labour input (half of the adjustments lasted no more than 3 quarters) than through capital input (half of the adjustments after 10 quarters). This seems intuitively right and is consistent with observations of how the economy works.

The adjustment parameter in the equation describing GDP dynamics has the wrong sign, but statistically it is no different than zero. This characteristic has important implications for the entire system and is related to the idea of weak exogeneity (cf. S. Johansen (1991)). The weak exogenous character of GDP in relation to the cointegrating relationship can be interpreted by saying that adjustments stem only from production factors, and not from production itself. This result indicates the demand-related nature of the GDP, which means that the production factors are employed according to the aggregate demand.

For the estimation, two restrictions were imposed on the VECM system. The first has a normalising nature, the second is connected with the assumption of constant economies of scale. The quotient test of the likelihood ratio imposed on the restriction system indicates that there are no reasons for rejecting the hypothesis of their legitimacy (at the probability of 0.65). The system describes the dynamics of endogenous variables well, and this is proven by the high coefficients of determination and the tested lack of autocorrelation of the random component. The roots of the characteristic polynomial corresponding to the system show it to be stable.

The last stage in estimating the production function is to replace the exponential deterministic trend approximating the total factor productivity with the TFP_t^* estimation. For this purpose, after estimating the production function using the exponential trend, residual values of the production function without the trend were calculated and smoothed using an HP filter. At the end, the process of estimating the production function was repeated in an analogous VECM system, in which the dynamic structure and restrictions of the original system were maintained, but the exponential trend was replaced with the estimated TFP_t^*. The calculated elasticities of the product in relation to the input of production factors differ from those previously calculated only at the third decimal place. Consequently, the final estimation of the production function can be described by the following formula:

$$\hat{Y}_t = TFP_t^* L_t^{0,57} K_t^{0,43},$$

where TFP_t^* is the series of residuals after smoothing with the HP40 filter.

The estimated function was then used to calculate the potential production.

Potential labour input

According to the methods used by OECD (cf. C. Giorno et al. (1995)), the potential labour input used to calculate the potential production is obtained from the following formula:

$$L^* = LF(1 - NAWRU),$$

where *LF* is the labour force, while *NAWRU* is the non-accelerating wage rate of unemployment. Hence L^* corresponds to the number of employed persons at the natural unemployment rate[3].

Labour was assumed as the number of professionally active persons according to LFS, but it is more difficult to estimate the *NAWRU*. The starting point was the method proposed by Elmeskov (1993), in which the change in the rate of the wage inflation is proportional to the difference between the actual unemployment rate and *NAWRU*, which can be expressed as follows

$$\Delta^2 \log W = -a(U - NAWRU), \, a > 0$$

where *W* – wage level, *U* – unemployment rate. The underlying methodology is thus consistent with the Phillips curve supplemented with adaptative expectations, according to which the expected wage inflation in the current period is equal to the rise of wages in the previous period (cf. Staiger, Stock, Watson (1996)). Assuming that the *NAWRU* is constant between any two consecutive quarters allows one to calculate the *a* parameter for subsequent periods and as a result, to calculate the *NAWRU* series.

The method proposed by Elmeskov is used for annual data, so it seems right to modify it for calculations based on quarterly data. In this study, the modification consists in replacing the current value of the *U* variable with a distribution of its lags, i.e.

$$\Delta^2 \log W = -a(\varphi(L)U - NAWRU), \, a > 0$$

where $\varphi(L)$ is a lag polynomial of order four, while *L* is the lag operator. A hypergeometrical distribution of $\varphi(L)$ polynomial coefficients was assumed so as to take into account the delay with which the labour market situation impacts on the wage rise. The resulting formula is:

$$NAWRU = \varphi(L)U - \frac{\Delta\varphi(L)U - \Delta^3 \log W}{\Delta^2 \log W}.$$

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[3] The equilibrium unemployment, NAWRU and NAIRU terms, though used synonymously in many studies, are not really equivalent. Cf. Kwiatkowski (2002), p. 154.

The drawback of the Elmeskov method is that the short-term *NAWRU* generated by this method changes to a great extent as real unemployment changes, which can be explained by the impact not only of the unemployment level, but also its changes, on wage inflation. This problem is usually reduced by smoothing the series using the Hodrick-Prescott filter (cf. Giorno et al. (1995)).

The use of the HP filter gives rise to the generally known doubts about choosing the right smoothing parameter and the end of sample bias. As there were no other premises, the smoothing parameter of 1600 was adopted, which is the same value as that chosen by Hodrick and Prescott (1980) and the standard parameter used for quarterly data[4]. The problem of the beginning of the series can be easily eliminated by adding data from 1992 – 1994 to the *NAWRU* calculation. The end of the series bias is far more problematic. Thus the most recent elements of sample have been adjusted so that the average *NAWRU* level in 2002 is approximately 16%[5], which the author believes corresponds to the current level of equilibrium unemployment in Poland.

Chart 1. NAWRU level in Poland, 1995-2002
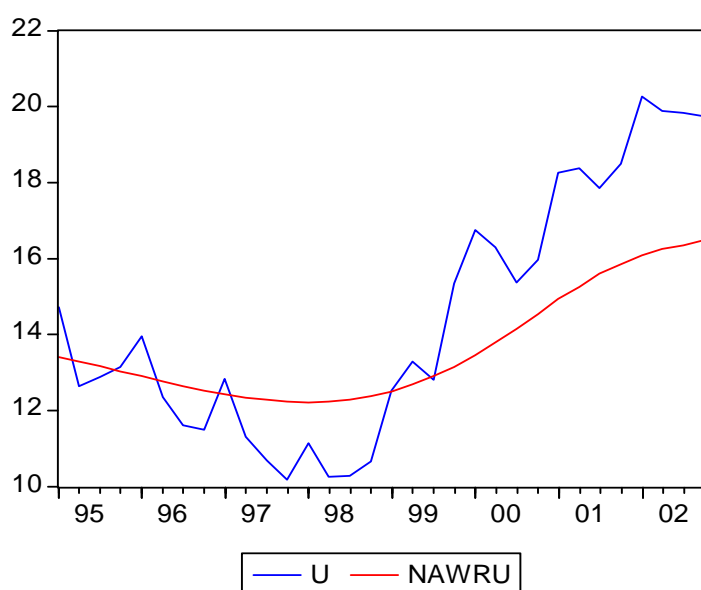
Source: Own calculations.

Since the *NAWRU* value for the end of the sample is assumed arbitrarily, the estimate of the potential GDP and the output gap should also include a sensitivity analysis taking

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[4] Discussion of the effect of using smoothing parameters in the HP filter in: Canova (1993).

account of a possible wrong estimate of the equilibrium unemployment. This analysis is presented later on in the text. The estimate obtained on the basis of the actual unemployment level according to BAEL is presented in Chart 1.

Estimating the potential production and the output gap

According to the methodology developed in OECD (cf. Giorno et al. (1995)), the following formula is used to calculate the potential GDP:

$$Y_t^* = f(K_t, L_t^*, TFP_t^*),$$

where Y_t^* is the potential GDP and f is the estimated production function.

If the estimated Cobb-Douglas production function is used, the potential GDP for the Polish economy can be calculated using the formula below

$$Y_t^* = TFP_t^* L_t^{*0,57} K_t^{0,43}.$$

After the potential GDP has been estimated, it is possible to calculate the output gap as the difference between the actual and potential GDP level. The results for Poland are presented in chart 2.

Chart 2. GDP, potential GDP and the output gap as a percentage of the potential GDP in Poland (seasonally adjusted data)
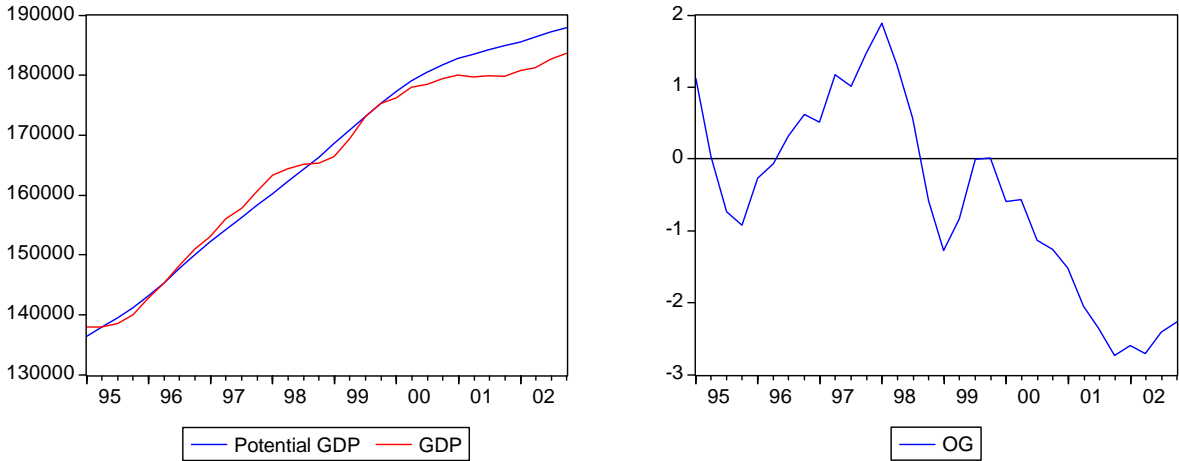
Source: Own calculations.

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[5] In comparison, the NAIRU rate for Poland estimated using the SVAR method was 15% in mid-2002 and grew by about 1 percent annually. Cf. BRE Bank S.A. (2002).

The results of output gap estimation using a method based on the production function show that this gap was positive until the third quarter of 1998, and then it fell below 1%. In the second half of 1999, GDP was almost equal to its potential level. From the beginning of 2000, the output gap was negative and kept worsening until the end of 2001. Since 2002, GDP has been returning to its potential level.

It has already been mentioned that the estimation of the potential GDP and the output gap it implies depend on the estimation of the NAWRU unemployment rate. The elasticity of GDP in relation to labour input indicates that the underestimation (overestimation) of NAWRU by one percentage point decreases (increases) the potential GDP by 0.6% – 0.7%. As a result, the output gap decreases (increases) by 0.6 – 0.7 of a percentage point.

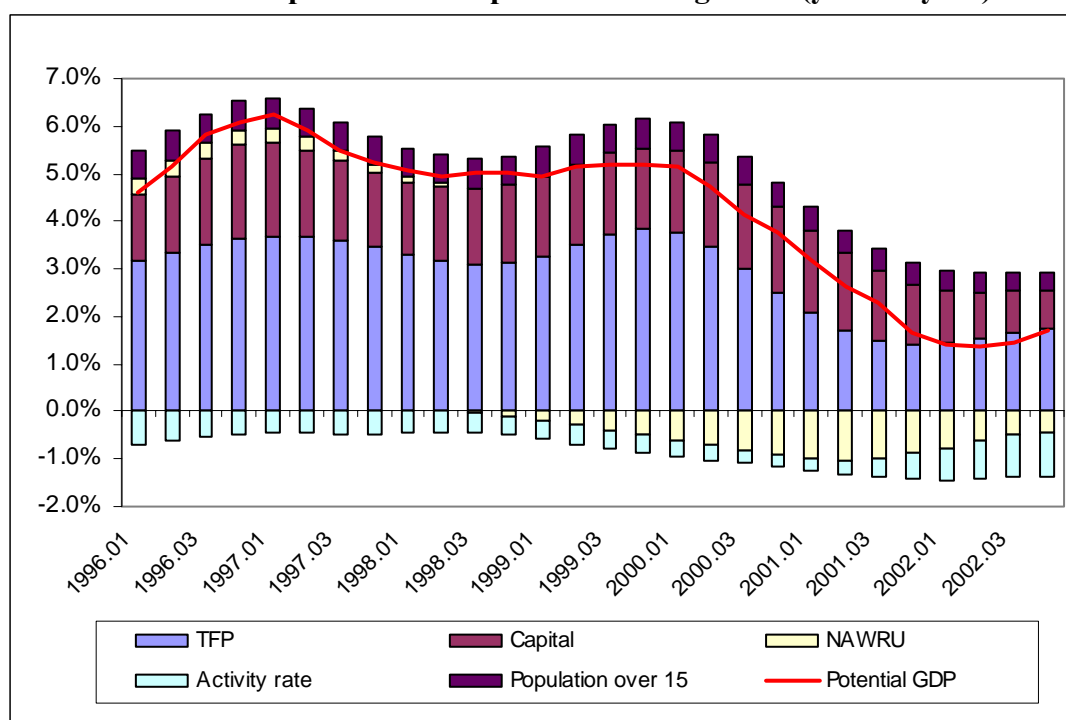**Decomposition of the potential GDP growth in Poland**

One of the advantages of the method based on the production function is that it can be used to calculate the contribution of particular factors to potential GDP growth. If the professional activity rate is defined as the proportion of employed persons to the number of inhabitants older than 15 and if the estimated production function is used, the rate of potential GDP growth can be broken down into five factors using the following formula:

$$\Delta\log(PKB^*) = \Delta\log(TFP^*) + 0{,}57\Delta\log(\overline{L}) + 0{,}57\Delta\log(s) + 0{,}57\Delta\log(1 - NAWRU) + 0{,}43\Delta\log(K)$$

where: $\overline{L}$ is the number of inhabitants older than 15 and s is the professional activity rate defined above.

Graph 3 shows that the long-term GDP growth in Poland in 1995-2002 was mainly driven by the increasing total factor productivity. In every quarter of this period, the share of TFP in the potential GDP growth exceeded 60%, or even 90% in 2002. Between 1995-2002, the potential GDP increased on the average by 2.9 percentage points a year due to the TFP growth. Over the entire period analysed, the contribution of the capital growth and the growth of population older than 15 were also positive, and amounted, respectively, to 1.6 and 0.6 of a percentage point. The breakdown reveals unfavourable trends on the Polish labour market. As a result of the decreasing professional activity rate, the long-term GDP growth rate was 0.5 percentage points lower every year. Since the end of 1998, the NAWRU unemployment level has been growing in Poland, slowing the potential GDP growth in this period by 0.6 percentage points a year on the average. The growing NAWRU can be interpreted as a symptom of unemployment hysteresis.

Chart 3. Decomposition of the potential GDP growth (year on year) in Poland

Source: Own studies.

3. Permanent-transitory decomposition of the GDP

An alternative approach to determining the potential output, used both in the economic theory and empirical research, is to treat it as the long-term GDP trend. The disputed issue is the method of determining the trend, as a theoretical concept, on the basis of time series of a relatively high frequency.

Econometric research based on the stochastic description of economic phenomena concentrates on the permanent-transitory (PT) decomposition of the product time series. In this approach, the potential product is treated as the permanent part of output established by eliminating the effect of transitory disturbances.

The starting point of the analysis is a VAR-type dynamic econometric model composed of variables integrated of order one. There should be long-term relationships between the variables making up this system, consistent with the selected economic theory[6]. This condition imposes recursive cross-restrictions on the parameters on the moving average

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[6] In the econometric sense, the existence of long-term relationships means that there is a stationary linear combination of I(1) variables. A relationship like this is called a cointegrating relationship.

representation of original model, leading to the representation of the vector error correction (VECM). The next step in the analysis is the PT decomposition of disturbances affecting the system. The decomposition itself and identification of both types of disturbances is made endogenously in the system, using the long-run restrictions imposed not implicitly on the VAR system by VECM. The last element of the identification process is the assumption that short- and long-run factors are independent (uncorrelated). The number and type of both sorts of disturbances is implied by the number of cointegrating relationships in the system (permanent disturbances are called common trends). The mathematical details of the described decomposition, developed by Yang (1998), are presented in Appendix.

PT decomposition methods are often based on strong assumptions. Multivariate Beveridge-Nelson decompositions (Evans, Reichlin (1994)) or unobservable factor methods (usually solved using the Kalman filter) assume that the trend of product is a random walk process and ignore the adjustments of potential after occurring of a permanent shock. The potential is then an imagined level of the product achieved after all the transitory adjustments died out. However, research, using Real Business Cycle models, indicates that the transitory dynamics of permanent productivity shocks also influence potential output (Lippi, Reichlin (1994)). The method of trend determination used in this study takes into account not only the long-term impact of permanent disturbances, but also the accompanying transitory adjustments. Compared to other methods based on cointegrating relationships (incl. Cochrane (1994), Duspaquier, Guay, St-Amant (1999)), which take transitory adjustments into account, this methodology developed by Yang properly identifies the number of permanent disturbances affecting the system (which is equal to the number of common stochastic trends governing the behaviour of the whole system). In addition, this procedure does not require multiple estimations (as do the above mentioned methods), thus it's relative efficiency.

The basis for the decomposition is the estimation of the VECM, in which the permanent relationship is advocated by a certain economic theory. A decision was made to estimate two systems:

- The PT-PF model, based on the long-term production function hypothesis;
- The PT-PIH model, based on the permanent income hypothesis.

The PT-PF model

The analysed dynamic system is composed of 3 variables: the real GDP, the number of employed persons (data from LFS study) and the capital[7]. All variables are measured quarterly and were adjusted seasonally using the TRAMO/SEATS[8] method. The sample range is 1995-2002.

The variables making up the system are integrated of order 1 (cf. table 1) and the economic theory defines the cointegration relationship in this system as the Cobb-Douglas production function (discussed in more detail in Chapter 2). This justifies estimating the system as a vector error correction mechanism (VECM).

A restriction of constant returns to scale was imposed on the cointegration relationship parameters (validated successfully by the Likelihood Ratio test at the probability level of around 0.82). The estimated GDP elasticity of labour input is 0.493 (with error 0.09), and product elasticity of capital input is 0.507 (with error 0.09). Johansen tests of rank of cointegration (trace and the maximum eigenvalue) confirm, just as the economic theory suggests, that there is one cointegrating vector for the described system (with a slope and trend in the cointegration relationship) – see table 3. The existence of one cointegration relationship was also confirmed by eigenvalues of the VAR(4) corresponding to the analysed VECM system: two of them reached values close to 1, while the remaining eigenvalues were definitely lower, implying the existence of two common stochastic trends, which, in a three-dimensional system, means one cointegration relationship. The precise form of the long-run relationship is as follows:

$$\hat{Y}_t = L_t^{0,493} K_t^{0,507} e^{-0,055+0,009t}$$

Values of adjustment coefficients to the previous period's disequilibria (the error correction term) for labour and capital inputs have the appropriate signs, implying convergence to the long-run path, and the following respective values: –0.20 (meaning that 50% of adjustments is done after 3 quarters) and –0.05 (one half of adjustments take place in 3 years). The speed of adjustments of the labour is greater than of capital, which is consistent with the theory. Estimation results imply that the parameter measuring the strength of output adjustments to the long-run level is statistically insignificant, which is referred to as the weak

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[7] Calculations done by Marcin Kolasa and Roman Sawinski, DAMS, NBP.
[8] The choice of different seasonal adjustment was an effect of desire to get more smooth data without random deviations, which is important when using methods making use of long-run properties of time series. On the other side, with the PF approach, the emphasis was on leaving annual dynamics of the data intact. The different character of employed methodologies is thus the reason for estimating two production functions using different data sets.

exogenity of product with respect to the cointegration vector. As the production function describes the supply side of the economy, the implications of weak exogenity of product seem to suggest the demand driven nature of product: in reaction to a disequilibrium, the adjustment is done only by production inputs, whereas changes in labour are definitely faster.

**Table 3. Johansen cointegration test for the PT-PF model**

Hypothesis: number of cointegrating relationships	Eigenvalue	Trace statistic	Maximum eigen-value statistic
None	0.73	51.58***	37.06***
No more than one	0.33	14.52	11.36

*** hypothesis rejected at a 0.01 significance level

Source: Own calculations.

Because of the quarterly frequency of data used for estimation, the short-term dynamic is reflected by rates of growth to the 3rd order (this corresponds to a four lag VAR in levels). A zero restriction on lags of the 2nd order was imposed on the short-term dynamics (this was successfully validated by the Wald significance test of regressor group). This restriction was imposed on the short-term dynamics to gain degrees of freedom while assuring the description of the seasonal pattern. All free (not implied by the restrictions of common trends) roots of the characteristic polynomial are located within a unit circle, which implies system stability. Lagrange Multiplier tests of the auto-correlation indicate that there are no reasons for rejecting the hypothesis of the zero autocorrelation of disturbances up to the 12th order.

Chart 4. Output gap in the PT-PF model as a percentage of the potential
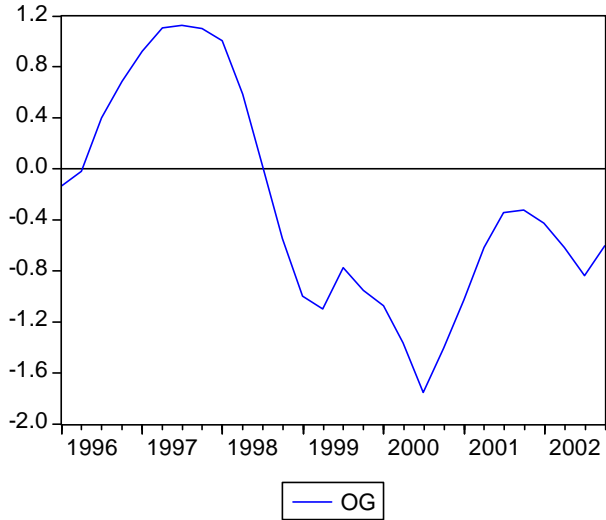
Source: own calculations

The course of the output gap in 1996-2002 is shown on chart 4. In accordance with the assumptions for the decomposition, the output gap is stationary in the econometric sense. Both the ADF non-stationary test and the complementary KPSS stationary test successfully validated the appropriate hypothesis at the 0.05 significance level.

In 1996-1998 the gap is positive and reaches a peak equal to 1.15% of the potential GDP, later the gap became negative, with a local minimum (-1.8%) in the third quarter of 2000. In 2001, the tendency of the economy to regain equilibrium was interrupted again, with another minimum at −0.8%. However, recent observations again show signs of the economy returning to equilibrium.

The PT-PIH model

The construction of the model and the nature of long-term restrictions are consistent with the permanent income hypothesis (PIH). The system consists of three variables, measured quarterly (the sample range is 1995 - 2002) and seasonally adjusted using the TRAMO/SEATS method: the real GDP, consumption and the short-term real interest rate. These variables are integrated of order one (cf. Table 4). For GDP and consumption, the appearance of the unit root in time series is economically justified (cf. King, Plosser, Stock, Watson (1991)), but there is no clear agreement among economists as to the real interest rate.

Table 4. Results of the unit root test

Variable	DF-GLS statistic	Conclusion at 0.1 significance level
$\Delta\log(Y_t)$	-1,99**	I(1)
$\Delta(r_t)$	-2,58**	I(1)
$\Delta\log(C_t)$	-3,59** (trend)	I(1)

** significant at a 0.05 significance level

Source: Own calculations

In this system, the long-term relationship is defined as a stationary consumption/GDP ratio, independent on the interest rate. Imposing a restriction on the system, that eliminates the interest rate from the long-run relationship (successfully validated by the Wald test at the 0.4 probability level), led to the estimated elasticity of consumption in relation to the product

equal to 1.0076. This confirms the hypothesis of the stationary proportion of consumption to the GDP[9].

The results of the Johansen cointegration rank test (see Table 5) indicate that there is one cointegration relationship at a 0.08 significance level. The existence of one long-term relationship is proven by an analysis of the eigenvalues of a corresponding VAR. Two eigenvalues are close to one (equal in module to 0.96), while the other are lower, which implies that there are two common stochastic trends in the system, and hence one co-integrating relationship.

Table 5. The Johansen cointegration test for the PT-PIH model

Hypothesis: number of cointegrating relationships	Eigenvalue	Trace statistic	Maximum eigenvalue statistic
None	0.43	22.38*	15.802*
No more than one	0.207	6.58	6.51

* hypothesis rejected at a 0.1 significance level

Source: Own calculations

The analysis of coefficients of the model at the error correction mechanism leads to the following conclusions:

- consumption adjusts very quickly to the equilibrium level (coefficient equal to –0.35) – more than half of the adjustments is finished after 2 quarters and 75% of adjustments take place after 3 quarters;

- as in the previous system, the product is weakly exogenous (statistically insignificant in terms of adjustment coefficient);

- there are also strong movements of the interest rate, which does not directly participate in the error correction mechanism, but its effect is visible in the short-term dynamics of the system.

The short-term dynamics of the system was approximated by rates of growth of variables up to 3rd order, which is consistent with the quarterly frequency of the data. On the basis of tests of the Lagrange Multiplier of the joint residual auto-correlation, one can say that the adopted scheme of transitional dynamics correctly approximated the dynamics of this phenomenon, eliminating auto-correlation from disturbances (up to the 12th order). All free eigenvalues of
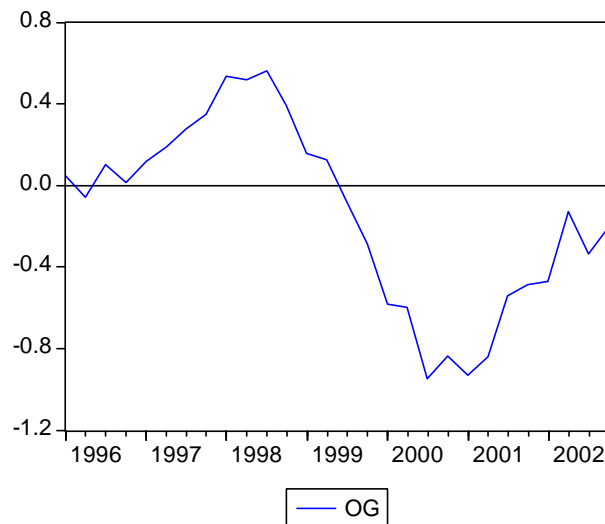
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[9] Stationarity of consumption-GDP ratio doesn't imply validating the Permanent-Income Hypothesis, especially in its strong version of Hall (1978)

the system are smaller than one, so the described system is stable (the variables in the system return to sustainable growth paths).

Chart 5. PT-PIH model output gap in percentages of the potential

Source: own calculations

The output gap generated by the PT-PIH model (see chart 5) is stationary (both the ADF and the KPSS tests at the 0.05 significance level successfully validate the hypothesis of the stationary nature of the gap). Its course in time is similar to the PF model (cf. chart 4), but the scale of imbalance is smaller (the maximum deviation of about 1% of the potential GDP). It is worth noting that the moment when the gap changes the sign is in the second quarter of 1999, it reaches minimum in 2000 and the break in the growth trend in 2002. Recent observations confirm that economy is heading towards equilibrium path.

4. Links between the calculated output gaps and inflation

The output gap determines the disequilibrium in the economy, and its size affects the course of inflationary processes. A positive output gap means that the aggregate demand exceeds the short-term production capacity, which results in an increased utilisation of production factors in subsequent periods and through the cost-price spiral causes an

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inflationary pressure in the economy. The mechanism by which the gap affects the inflation has been demonstrated in theoretical models with imperfect price-elasticity (cf. Clarida, Gali, Gertler (2000)).

The impact of the output gap on inflation can be measured using several tools. One of them is the Phillips curve, i.e. the direct estimation of the above relationship. However, it imposes *a priori* restrictions on number of lags in equation.. One way to avoid this problem is to calculate the cross-correlations between the current inflation and the delayed gap, ordered according to the increasing time-interval between the two series.

In the case of the short time series, an important limitation on using cross-correlations to assess the strength and time lag of the gap effect on inflation is the significant decrease in the precision of estimators as the delay grows. This problem can be solved by constructing a two-dimensional stochastic process generating output gap and inflation data, and then determining its covariance-generating function (cf. Hamilton (1994), pp. 261-268). Normalisation of the elements of this function generates theoretical correlation relationships in time between the analysed time series, implied by the features of the stochastic process describing the joint development of these series.

Charts 6, 7 and 8 present the theoretical coefficients of correlation between the estimated output gaps and different price change measures, obtained by assuming a VAR-type data generation process. The horizontal axis shows the quarterly lag of the gap in comparison to the specific inflation rate. The following four measures of annual inflation were used in the analysis:

- the consumer price index (CPI);
- the producer price index (PPI);
- the GDP deflator;
- one of the core inflation measures (IBNZP) – core inflation excluding the most variable and fuel prices.

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Chart 6. Correlations – the PF model
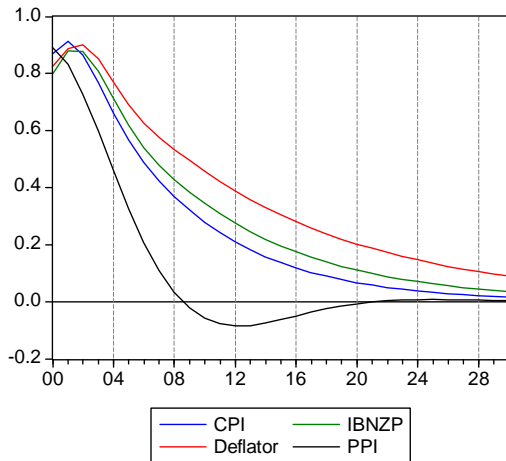
Source: own calculations

Theoretical correlations corresponding to the model for estimating the potential production using the production function (PF) method indicate that there is a strong link between the output gap and the inflation measures. In the case of CPI and IBNZP, the maximum effect is seen with a one quarter time lag, for the GPD deflator – 2 quarters lag, while the PPI reacts the most strongly in the current quarter. The strong correlation between the gap and inflation (above 0.5) lasts for a year in the case of PPI and for 18 months in the case of the other measurements. This indicates that the effect of demand-side shocks causing inflationary pressure in the economy is relatively long-lasting.
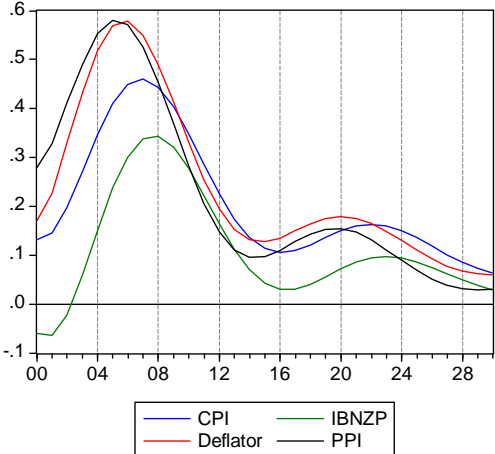
Chart 7. Correlations – the PT-PF model
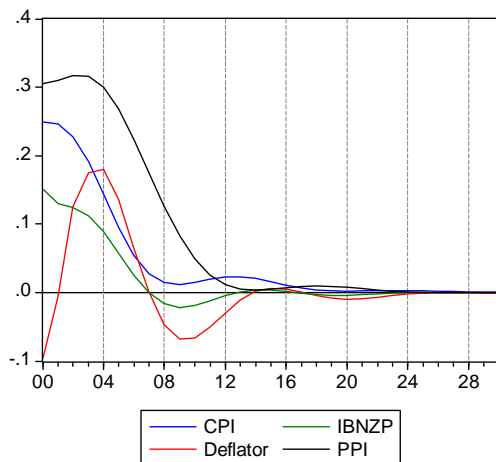
Source: own calculations

The interactions between the output gap generated by the PT-PF model and inflation are weaker, and the time structure of these links is different. The maximum effect of the gap on CPI and IBNZP is seen in the current quarter, while for the PPI and the GDP deflator it occurs in the third and fourth quarters respectively. It is worth noting the relatively strong link between the gap and CPI, PPI and IBNZP inflations in the current period, while the effect of the gap on the GDP deflator only becomes visible after half a year has passed. The links between the gap and the analysed inflation measures expire in 6 – 9 quarters, depending on the index. Due to low levels of correlation coefficients (especially in the case of GDP deflator and IBNZP) one should be careful in interpreting the PT-PF model results.

Chart 8. Correlations – the PT-PIH model

Source: own calculations

The theoretical correlations corresponding to the gap calculated by the PT-PIH model show that it is more strongly connected with inflation than in the PT-PF model. The gap has the strongest impact on the GDP deflator and the PPI index, and a weaker one on inflation measured by the CPI and IBNZP. In comparison with the two previous models, the gap from the PT-PIH model has the longest delay in exerting its effect, reaching maximum values after 5 quarters for the PPI and 8 quarters for the IBNZP. In the case of the GDP deflator and the CPI inflation, the maximum effect is noted after 6 and 7 quarters, respectively. The impact of the gap on inflation effectively disappears in four years.

5. Summary

The alternative methods of estimating the output gap presented above differ with regard to the concept and method of calculation. The potential GDP estimated using the production function reflects the supply-side of the economy, i.e. the GDP level stemming from long-term inputs of production factors. On the other hand, methods based on the permanent-transitory decomposition of GDP take advantage of long-term relationships between macroeconomic aggregates and yield a potential GDP of permanent nature, which is a product of accumulated shocks.

Consequently, it is not surprising that the alternative methods used lead to different estimates of the output gap. However, it turns out that if the different time-structures of the effect of the calculated gaps on the inflation are taken into account, the conclusions on the inflationary pressure in the economy drawn from those gaps are similar. In particular, the development of all three gaps over the last quarters shows the lack of any demand-side inflationary pressure, which may be the case till the end of 2003.

Thus, when forecasting inflationary phenomena in the economy, it seems right to use the signals produced by both methods, using the output gap calculated on the basis of the production function to assess the inflationary pressure in the coming quarters, while the gap generated by the permanent-transitory decomposition can be used to forecast inflation over a longer period of time - exceeding one year.

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Appendix

Consider an *n*-dimensional stochastic process $\{x_t\}$ of integrated variables and its error correction (VECM) representation:

$$\Delta x_t = \delta + \theta x_{t-1} + \sum_{i=1}^{p} \Pi_i \Delta x_{t-i} + \varepsilon_t \tag{1}$$

where $\Delta = 1 - L$ is the difference operator and L is the lag operator, δ is deterministic part of model, θ and Π_i are matrices of parameters. ε_t is a vector of random disturbances from individual equations with variance-covariance matrix Ω. The system (1) could be estimated by ML procedure developed by Johansen (1988). In case of r cointegrating relations, tested by Johansen procedures, θ has a reduced rank and can be decomposed as $\theta = \alpha\beta'$ where α and β are full-rank $(n \times r)$ matrices. Columns of β are cointegrating relations between the variables of the system and elements of α are system reactions to previous period disequilibria. The system should converge monotonically to the long-run relationship $\beta'x_t$ with adjustment rate given in α. Defining $\Pi(L) = I_n - \sum_{i=1}^{p} \Pi_i L^i$ and $A(L) = \Pi(L)(1-L) + \theta L$ one can convert VECM into corresponding VAR representation: $A(L)x_t = \delta + \varepsilon_t$.

As elements of x_t are I(1), the Wold theorem assures that its first differences have an infinite Vector Moving Average representation (VMA), showing the way disturbances of previous periods affect the current value of variables:

$$\Delta x_t = C(L)(\delta + \varepsilon_t) = \mu + \varepsilon_t + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + \ldots \tag{2}$$

where $\mu = C(1)\delta$ is a deterministic part, $C(1) = \sum_{i=0}^{\infty} C_i$ is a sum of all short-run multipliers and the matrix polynomial is of the form: $C(L) = \sum_{i=0}^{\infty} C_i L^i$ with normalization $C(0) = I_n$.

Engle and Granger (1987) showed that on defining $C^*(L) = (C(L) - C(1))(1-L)^{-1}$ equation (2) can be represented as:

$$\Delta x_t = C(L)(\delta + \varepsilon_t) = \mu + C(1)\varepsilon_t + C^*(L)\Delta\varepsilon_t \tag{3}$$

In case of C(1) being of reduced rank k ($k < n$), there are $r = n - k$ cointegrating relations. C(1) can be decomposed as: $C(1) = hg'$ where h and g are $(n \times k)$ matrices. Equation (3) shows

the decomposition of matrix polynomial C(L) into a permanent part C(1) and a transitory lag distribution $C^*(L)\Delta\varepsilon_t$. It is clear that there are only k linear combinations of disturbances permanently affecting x_t - they are of the form $g'\varepsilon_t$.

Johansen (1992) showed that $C(1) = \beta_+ (\alpha_+^T (I - \sum_{i=1}^{p} \Pi_i)\beta_+)^{-1}\alpha_+^T$ where α_+ and β_+ are orthogonal complements[10] to corresponding matrices. Equation (3) can be then presented as:

$$\Delta x_t = \mu + \beta_+ (\alpha_+^T (I_n - \sum_{i=1}^{p} \Pi_i)\beta_+)^{-1}\alpha_+^T\varepsilon_t + C^*(L)\Delta\varepsilon_t \qquad (4)$$

with $\alpha_+^T\varepsilon_t$ being a group of permanent shocks and matrix $\beta_+ (\alpha_+^T (I - \sum_{i=1}^{p} \Pi_i)\beta_+)^{-1}$ showing their long-run impact on the variables creating the system.

In order to obtain components of x_t, that are created by permanent disturbances, taking into account not only their long-run, but also short-run impact, one should find a connection between long-run shocks $\alpha_+^T\varepsilon_t$ and MA representation of (2). The stochastic component $C(L)\varepsilon_t$ can be divided into 2 components, of which one is the permanent one $\alpha_+^T\varepsilon_t$:

$$C(L)\varepsilon_t = C(L)\bar{\alpha}_+ \cdot \alpha_+^T\varepsilon_t + C(L)\bar{\gamma} \cdot \gamma^T\varepsilon_t \qquad (5)$$

where γ is any $(n \times r)$ matrix chosen to assure that $\begin{bmatrix} \alpha_+ & \gamma \end{bmatrix}$ is invertible and satisfying the equation[11]: $\begin{bmatrix} \bar{\alpha}_+ & \bar{\gamma} \end{bmatrix} = \{\begin{bmatrix} \alpha_+ & \gamma \end{bmatrix}^T\}^{-1}$. Representation showed in (5) assumes that multipliers of permanent and transitory disturbances are linear combinations of multipliers of model (3). Dynamic influence of shocks on variables forming x_t was then divided into coming from long-run and short-run disturbances. To finish the decomposition one must choose matrix γ. Assuming that permanent disturbances $\alpha_+^T\varepsilon_t$ and transitory ones $\gamma^T\varepsilon_t$ are orthogonal

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[10] Orthogonal complement of $(n \times r)$ matrix α is a $(n \times n - r)$ matrix α_+ that satisfy the relation $\alpha^T\alpha_+ = 0$. It's a matrix which columns form basis of subspace, which is orthogonal to the subspace generated by columns of α matrix.

[11] It's solution to the equation of the form: $\bar{\alpha}_+ \cdot \alpha_+^T + \bar{\gamma} \cdot \gamma^T = I_n$

(independent), which is desired in impulse-response analysis, one can get the following relation defining γ:

$$\gamma = \alpha - \alpha_+ (\alpha_+^{\mathrm{T}} \Omega \alpha_+)^{-1} \alpha_+^{\mathrm{T}} \Omega \alpha \qquad (6)$$

Permanent components of vector x_t could be obtained as deterministic part of equation (3) with the whole stochastic component of permanent disturbances:

$$x_t^* = \mu + (C(L)\bar{\alpha}_+)\alpha_+^{\mathrm{T}} \varepsilon_t$$

Thus x_t^* is defined to be that part of x_t, that was generated by only permanent disturbances, so according to our definition, it could be treated as trends of elements of vector x_t.