Monetary Policy in Small Open Economies
The case of disinflation*

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September 2004

Abstract

This paper discusses the sacrifice ratio, i.e. the output cost of given change in the trend inflation rate in a small open economy. Rather than assuming a particular pricing mechanism, we discuss the problem in a reduced form, which we believe incorporates most of the pricing mechanisms put forward in the litterature.

The paper shows that the best strategy with respect to minimising the sacrifice ratio depends strongly on the extent of inflation inertia. The sacrifice ratio is only positive if the the inflation process is mainly backward rather than forward looking. Under these conditions any policies that try to minimise the sacrifice ratio are likely to be time inconsistent.

Real appreciations in the early part of the disinflation process will raise the sacrifice ratio and income policies that limit the real appreciation might by beneficial.

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1 Introduction

Over the last decade, significant progress has been made in understanding monetary policy in closed and also open economies. In fact some authors claim that there is be something close to a consensus on how to model monetary policy. This litterature builds on the original time dependent price adjustment formulations of Taylor(1980) and Calvo (1983). Most of the research has focussed on the reaction of monetary policy to shocks to the economy and optimal rules for the policy instrument.

However whether the inflation specifications derived from such models can account for the empirical properties of inflation and output is not as clear as the word consensus suggests. In particular the standard formulation has enormous difficulty in accounting for the observed persistence in inflation rates. While this deficiency might have second order importance for the formulation of optimal monetary rules, this can not be said about a discussion of output cost of disinflations, a less actively researched area.

In fact the standard Calvo or Taylor price adjustments are not of much help to discuss this question.. The standard New Keynesian Phillips curve does not provide sufficient inertia to produce the observed output costs. In fact as we will see the standard New Keynesian framework allows for costless disinflations.

Given that we are relatively unsure about the way the prices are adjusted, it might be safer to discuss the problem of disinflations allowing for a variety of price setting mechanisms to find more robust results. In fact this approach has become quite popular in recent years. Thus Levin et al. discuss optimal monetary policy rules allowing for a variety of inflation process from the standard Calvo pricing process to processes that allow inflation to be partially backward looking such as that developed by Fuhrer and Moore.

This paper takes a similar approach to the problem of disinflation. We investigate what we can say about the optimal speed of disinflations and to which extent exchange rate dynamics actually matter for the cost of disinflations.

Few systematic empirical studies of disinflation are available. The best known one is probably the paper by Ball (1994). He investigated the determinants of output costs of disinflations systematically for OECD countries. Defining disinflations as instances in which trend inflation falls by at least two percent Ball identified 65 episodes in 19 countries between 1960 and 1992. The ouput costs of disinflation are measured by the sacrifice ration, defined as the undiscounted sum of deviations from trend output. He finds that the disinflation is decreasing in the speed of disinflation, i.e. gradualism makes disinflation more expensive. Second and not surprisingly the ratio is lower in countries with more flexible labour markets. Furthermore he finds no impact of openness on the cost of disinflation. This last finding is somehow surprising. Romer (1991) pointed out that there should be a relation between the output inflation tradeoff and the openess to trade. In a more open economy, the exchange rate appreciation arising from a monetary contraction has a larger direct effect on the price level. Consequently, inflation should fall more for a given policy shift and the sacrifice

ratio should be smaller.

We find that it is possible to say quite a bit about the cost of disinflation, and about ways of minimising that cost, simply by considering the inflation kernel. Other conclusions, for instance those related to the role of the exchange rate in the disinflation process can be reached on the basis of the inflation kernel plus a rather uncontroversial assumption about the long-run relationship between inflation and the value of the real exchange rate.

Because of this focus on the inflation kernel, there is no scope for considering such key issues as the institutional arrangement for the conduct of monetary and fiscal policy - central bank operational and/or target independence, budgetary rules etc. Nevertheless, our analysis does point to possible problems, when the inflation process is at least in part forward-looking, with the credibility of policy announcements and with the ability to commitment to effective policy rules.

The paper studies how different kinds of nominal rigidities (price level inertia and inflation inertia) should affect the design of efficient disinflation strategies. The various models of the inflation process considered in what follows are assumed to be structurally invariant under the kind of disinflation policies (or policy rules) considered in the paper. This assumption of a constant structure means that we must limit, in our interpretation of the models' properties, the range of inflationary experiences that we can hope to shed light on. The models studied here are only useful for the study of low and moderate inflations or deflations. High inflation, let alone hyperinflation, would destroy the nominal rigidities in the wage and price contracting processes that are central in what follows.

The paper considers a sequence of models, differentiated only by the nature of the core inflation process, that is, the by the specification of the augmentation term in a conventional-looking price Phillips curve. First, we revisit the New Classical Phillips curve or supply function. Second we consider core inflation processes which imply that current inflation is a function both of past inflation and of anticipated future inflation. We shall call these 'mixed models'. This includes the New Keynesian Phillips curve as the special case for which current inflation depends on anticipated future inflation but not on lagged inflation. We also argue that this framework captures the essentials of more recent attempts to model price setting dynamics that can match the inflation inertia found in the data. There are interesting differences within the class of mixed models between those for which the weight on future anticipated inflation exceeds that on past inflation ('mainly forward-looking mixed models) and those for which the opposite holds.

For the mainly backward-looking mixed model - the model we consider to be the most relevant and interesting for policy purposes - there is an interesting conflict between efficient and credible disinflation strategies.

2 Wage and Price Dynamics in an Open Economy

We base our analysis for the moment on an old-fashioned augmented Phillips curve model of an open economy. It turns out that both the New-Classical and the New-Keynesian inflation kernels are special cases of the final, quasi-reduced form of our model. We start with a conventional wage Phillips curve. Money wage inflation depends inversely on the unemployment rate, u, our index of deflationary pressure in the labour market. Money wage inflation also changes one-for-one with 'core inflation'. A vast range of alternative views on the nature of the wage-price process can be characterised transparently through different interpretations and specifications of 'core inflation'. 'Core inflation' refers to persistence or momentum in the inflationary process and is a direct consequence of how prices are set in specific models. This can be due to a variety of behavioural (e.g. expectational) and institutional (e.g. contracting) features of the wage-price setting process. The core inflation term in the open economy wage Phillips curve should be interpreted as the core inflation rate of a cost -of-living index, that is something like the consumer price index (CPI) or the retail price index (RPI). The rate of inflation of the consumer price index is denoted $\tilde{\pi}(t) \equiv \Delta \tilde{p}(t)$, where $\tilde{p}$ is the (logarithm of the) cost-of living index. Core CPI inflation is denoted $\tilde{\pi}_c$. The rate of inflation of the 'gross' price of domestic output (not the GDP deflator, because domestic output may be produced using imported intermediate and raw materials inputs) is denoted $\pi \equiv \Delta \tilde{p}$, where p is the (logarithm of the) domestic producer price level. The rate of inflation of world prices (in foreign currency) is denoted $\pi^* \equiv \Delta p^*$ where p^* is the (logarithm of the) foreign price level; $\pi_w \equiv \Delta w$, where w is the (logarithm of the) nominal wage, denotes the growth rate of money wages; the depreciation rate of the nominal exchange rate is denoted $\varepsilon \equiv \Delta e$ where e is the (logarithm of the) nominal spot exchange rate (the number of units of home currency per unit of foreign currency). The coefficient γ (the drift term in the wage Phillips curve), can be interpreted as the target growth of real wages pursued by workers.

$$
\begin{aligned}
\pi_w &= \gamma - \alpha u + \tilde{\pi}_c \\
\alpha &> 0
\end{aligned}
\tag{1}
$$

$$
\tilde{\pi} = \omega \pi + (1 - \omega)(\pi^* + \varepsilon) \\
1 \geq \omega \geq 0
\tag{2}
$$

The share of imports in the consumption bundle is denoted ω.

An empirically plausible price equation makes the domestic producer price a constant proportional mark-up on 'normal unit variable cost' an assumption that carries through most models discussed in the litterature. Unit variable cost is the sum of unit labour cost and unit import cost per unit of domestic gross output. Variables with *overbars* denote the normal, that is, cyclically adjusted,

values of the corresponding variable. The share of labour cost in total variable cost is λ, the growth rate of average labour productivity is γ_L and the growth rate of average import productivity is γ_N. The markup pricing model implies the following equation for the rate of inflation of domestic producer prices:

$$\pi = \lambda(\bar{\pi}_w - \bar{\gamma}_L) + (1 - \lambda)(\bar{\pi}^* + \bar{\varepsilon} - \bar{\gamma}_N) \tag{3}$$

As long as we can capture the cyclical impact through the difference of the current unemployment rate from the natural rate, the open economy Phillips curve can be written as

$$\tilde{\pi} = \tilde{\pi}_c - \beta(u - u_N) + \delta\pi_\rho \tag{4}$$
$$\beta, \ \delta > 0$$

where $\rho \equiv e + p^* - \tilde{p}$ denotes the real exchange rate.

Inflationary pressures depend on core inflation, the output gap and is lower if the country's terms of trade are improving (that is, if its price competitiveness is worsening). When domestic inflation exceeds world inflation $\tilde{\pi} > \pi^* + \varepsilon$, the worker's real consumption wage (which is defined in terms of the cost-of-living index $\tilde{p}$, rises relative to the real product wage that determines the demand for labour by firms, which is defined in terms of the domestic producer price index p. Workers' real wage aspirations can be satisfied to a greater extent through relatively cheaper import prices rather than through higher money wages. This has a dampening effect on inflation, that is of course only present as long as the terms of trade are improving. If and when the terms of trade improvement unwinds and $\tilde{\pi} < \pi^* + \varepsilon$, inflationary pressures are enhanced.

2.1 The Sacrifice Ratio

As is common in this litterature we adopt the sacrifice ratio as our operational measure of the output or employment cost of reducing inflation. This is defined as the *cumulative* increase in the unemployment rate (or cumulative reduction in output) required to achieve a one percentage point *sustained* reduction in the rate of inflation. Changes in unemployment at different dates are summed without any discounting. The principal reason for the absence of discounting is that, for the class of models we shall consider is that it is simpler to calculate the undiscounted cumulative total. The approach says nothing about the nature or magnitude of the gains from achieving a sustained reduction in inflation. It only studies the cumulative employment or output foregone in order to achieve such as sustained reduction.

Given our notation the sacrifice ratio is given by

$\Sigma_{t=1}^{\infty}(u_t - u_n) = \frac{1}{\beta}\Sigma_{t=1}^{\infty}[\tilde{\pi}_{c_t} - \tilde{\pi}_t] + \frac{\delta}{\beta}\Sigma_{t=1}^{\infty}\pi_{\rho_t}$

For our augmented open economy Phillips curve model, the sacrifice ratio depends on four key features. First, the responsiveness of wage inflation to unemployment. Second, the determinants of the core inflation rate, and in particular the degree of inertia or persistence in core inflation and the extent to

which core inflation is forward-looking. Third, the behaviour of the real exchange rate from the beginning till the end of the disinflation process. Fourth, the determinants of the natural rate of unemployment, and especially the question as to whether the disinflation process itself can affect the natural rate, that is, whether the natural rate is *hysteretic*. However we abstract in this paper from the last point.

We will maintain in what follows that the process of achieving a sustained reduction in the rate of inflation has no permanent effect on the level of the real exchange rate, that is

$$\sum_{t=1}^{\infty} \pi_\rho(t) = 0 \tag{5}$$

Assumption 5 asserts that, in the long run, our economy has classical features. In the short run, with a floating nominal exchange rate, a process of disinflation is likely to be associated with an appreciation of the real exchange rate. Many models, including the 'overshooting' models have this property. What 5 asserts is that such short-run anti-inflationary benefits of real exchange rate appreciation have to be handed back in at some time later during the disinflation process. It does not affect the (undiscounted) cumulative unemployment cost of achieving a sustained reduction in the rate of inflation.

3 The sacrifice ratio as a function of different core inflation processes

In what follows we will discuss the implications of different price setting assumptions on the sacrifice ratio. Furthermore we want to see to which extent answers to the question of how quickly a country should disinflate and the if the sacrifice ratio dependso on the exchange rate depend on these assumptions. As pointed out before we consider this excercise as useful since there is little agreement in the litterature on how to model the price setting process. More as an illustration rather than that we believe that this model captures the real world we will start with the neo classical assumption of rational expectations and no price stickiness. We then move on to processes that allow for price stickiness and argue that pretty much all inflation processes that have been put forward in the litterature can be captured by a simple partiall forward and partially backward looking inflation process.

3.1 Costless disinflation 1: the New Classical world

The first example of a wage-price process for which costless disinflation is possible, is the open economy version of the 'surprise supply function' cum rational expectations, popularised by the New Classical macroeconomics literature of the 1970s and 1980s. In our open economy Phillips curve model, we obtain the New Classical variant by equating core inflation in period t with the rate of inflation expected, in period $t-1$, to prevail in period t, that is

$$\tilde{\pi}_c(t) = E_{t-1}\tilde{\pi}(t) \tag{6}$$

where E_{t-1} is the conditional expectation operator for expectations formed at time $t-1$. Rational, or model-consistent expectations imply that

$$\tilde{\pi}(t) = E_{t-1}\tilde{\pi}(t) + \varepsilon(t-1,t)$$

where $\varepsilon(t-1,t)$ is the rational forecast error in period t for forecasts made in period $t-1$. Since $\varepsilon(t-1,t)$ is a rational forecast error,

$$E_{t-1}\varepsilon(t-1,t) = 0$$

Inflation is not a predetermined state variable. There is no structural inflation persistence or inertia. Inflation can adjust instantaneously and costlessly to credible announcements about future policy. As long as we assume that the disinflation process has no permanent impact on the equilibrium real exchange rate $\frac{\dot{\lambda}}{\beta}\Sigma_{t=1}^{\infty}\pi_{\rho_t} = 0$ it follows that the sacrifice ratio is zero: costless disinflation only requires credibility:

$$\sigma = 0$$

Given that disinflation can come about immediately and costlessly the optimal policy is alway cold turkey as long as there are some long term gains to a lower inflation rate (something that is outside of our framework).

In a closed economy ($\delta = 0$), with a surprise supply function and rational expectations, policy cannot influence the first moment of the distribution of real output, employment or unemployment.[1] In an open economy, equilibrium unemployment can be affected by policy to the extent that policy (whether anticipated or unanticipated) can affect the rate of depreciation or appreciation of the real exchange rate. However any temporary loss in employment that might arise through a short term real appreciation will be compensated for when the exchange rate moves back to equilibrium.

$$u(t) = u_N - \beta^{-1}\varepsilon(t-1,t) + \beta^{-1}\delta\pi_\rho(t)$$

3.2 Mixed backward-looking and forward-looking core inflation

In our view. neither an old-style, backward-looking core inflation models nor the New Classical world are satisfactory vehicles for analysing the inflation processes and the design of efficient disinflation policies in advanced industrial countries or in the advanced transition countries that stand on the threshold of EU accession. Policy-relevant models should incorporate the following two key features of the inflation process in these countries. First, the inflation rate is, in part,

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[1] Even with a surprise supply function, and rational expectations, it may still be possible to influence higher moments of the conditional and unconditional distribution of real variables. See e.g. Buiter [4].

anchored in the past. For low and moderate of inflation rates, there is a structurally invariant degree of persistence or inertia in the inflation process. Second, the inflation process is in part forward-looking. Anticipations of future policy actions do influence current wage and price setting. Credibility matters for the sacrifice ratio but is not in general sufficient to achieve costless disinflation as argued by Ball (1994). In what follows we abstract from the problem of imperfect credibility.

Equation 7 below makes current core inflation a convex combination of last period's inflation and of current expectations of next period's inflation. This specification is in the spirit, if not the letter, of Taylor-style staggered, overlapping contract models (see Taylor [16], [17]) The model of equation 7 is closest to the staggered overlapping *real* wage contracting variant of the Taylor model proposed by Buiter and Jewitt [6] and popularised by Fuhrer and Moore [11].

$$\tilde{\pi}_c(t) = (1 - \theta)E_t\tilde{\pi}(t+1) + \theta\tilde{\pi}(t-1)$$
$$0 \leq \theta \leq 1 \tag{7}$$

Substituting this into **??** yields

$$\tilde{\pi}(t) = (1 - \theta)E_t\tilde{\pi}(t+1) + \theta\tilde{\pi}(t-1) - \beta[u(t) - u_N] + \delta\pi_\rho(t) \tag{8}$$

We will refer to the model of equation 8 as the *mixed* model. The version with $0 \leq \theta < \frac{1}{2}$ will be called the *mainly forward-looking* mixed model; the version with $\frac{1}{2} < \theta \leq 1$ will be referred to as the *mainly backward-looking* mixed model.

Almost all models in the litterature can be incorporated into the above framework. A few examples should suffice

1. The new keynesian Phillipscurve

   Gali and Monacelli have shown that the Phillips Curve in a small open economy setting is hardly different from that for closed economy. The textbook Keynesian framework with Calvo type price setting leads to the standard new Keynesian Phillips curve

   $$\pi_t = E_t\pi_{t+1} - \beta(u_t - u_n)$$

   with the only difference that π_t here refers to domestic inflation. The Phillips curve expressed using the cpi will have the same form as 8 with $\theta = 0$ and the share of imported costs in domestic production λ being zero.

2. The Phillips Curve according to Mankiw and Reiss or Calvo (mainly backward looking)

   Mankiw and Reiss (2002) have suggested that the pricing process is best characterised by sticky information rather than by sticky prices. While under the common Calvo pricing process firms are randomly selected to adjust their current price, they assume that firms only infrequently update the information on which their pricing depends. Whenever they optimise

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their prices they set pricing schedules for the future that depend on their expectation of future economic conditions. Implicitly firms choose firm specific inflation rates that they will follow until they update their information base. Under this assumption the Phillips curve takes the form

$$\tilde{\pi}_t = -\beta[u_t - u_N] + \lambda \Sigma_{j=0}^{\infty}(1-\lambda)^j E_{t-1-j}(\tilde{\pi}_t)$$

The coreinflation process depends on expectations in the past of current inflation. Thus assuming that disinflation involves moving from one level of money growth to another, inflation will depend mainly on inflation in the past as long as the switch is not unticipated. Even if the switch in policy is announced in advance, past monetary conditions will still have a positive weight both directly through the share of firms that reoptimised prior to the announcement data and indirectly since these early price setters prices will have to be taken into account by current price setters. In any case the inflation process will display considerable inertia as it will be at least partially backward looking.

Calvo et al. (2003) uses a similar framework in a micro founded open economy model to study disinflations. They postulate that just like under the usual Calvo pricing model firms are randomly selected to change their prices. However when they do they also determine their future individual price adjustments, that will prevail until they are chosen to adjust their prices again. Formally this assumption is identical to the assumption of sticky information. The authors apply this framework only to the production of nontradables. Tradable production instead is assumed to be perfectly competitive and prices always match marginal costs.

The inflation rate for nontradables π becomes a function of the firm specific inflation rates Ψ that have been set in the past and the prices p set by current price setters

$$(\pi_t - \bar{\pi}) = \delta(p_t - \bar{p}) + (\Psi_t - \bar{\pi})$$

where the overbar denotes the steady state level of a variable and δ denotes the share of firms that update their prices each period. Persistence in the inflation rate arises from two sources, first directly through the inflation rate followed by firms that are not reoptimising in a given period. Second, firms that are able to reoptimise have to take into account their competitive position, which in turn will depend on the price setting of other firms. Thus even the price p will depend on inflation rates set in the past.

Either of the two formulations, the one by Calvo and the one by Mankiw and Reiss will lead to inflation dynamics where the core inflation process arguably will depend mainly on information on economic conditions that were known in the past. Assuming unanticipated disinflation policies, this will effectively make the inflation process mainly backward looking as it will depend on past monetary policies rather than current ones.

Furthermore some authors have recently develop models of learning that can

account for inflation inertia. Woodford (2001) and Erceg and Levin (2002) are two examples.

We now turn to the question of what we can say about the sacrifice ratio given an inflation process that is both forward and backward looking and discuss the optimal speed of disinflation and the role of exchange rate policies. As it turns out the answer depends strongly on the assumption if the inflation process in mainly backward (like in Mankiw and Reiss) or mainly forward looking (like in the standard Calvo pricing model.

3.2.1 The inflation process when the core inflation process is both forward and backward looking

The inflation process is given by

$$\tilde{\pi}(t) = (1-\theta)E_t\tilde{\pi}(t+1) + \theta\tilde{\pi}(t-1) - \beta[u(t)-u_N] + \delta\pi_\rho(t) \qquad (9)$$

We will refer to the model of equation 8 as the *mixed* model. The version with $0 \leq \theta < \frac{1}{2}$ will be called the *mainly forward-looking* mixed model; the version with $\frac{1}{2} < \theta \leq 1$ will be referred to as the *mainly backward-looking* mixed model. We have a linear rational expectations model with constant coefficients whose homogeneous part is a second order difference equation. Its order can be reduced using the method of undetermined coefficients (see annex). Choosing the stable roots we arrive at the following solution.

For $\theta \leq \frac{1}{2}$, that is, for the mainly forward-looking model, the solution looks as follows:

$$\tilde{\pi}(t) = \left[\frac{\theta}{1-\theta}\right]\tilde{\pi}(t-1) - (1-\theta)^{-1}\beta\sum_{i=0}^{\infty}E_t[u(t+i)-u_N] \qquad (10)$$

$$+(1-\theta)^{-1}\delta\sum_{i=0}^{\infty}E_t\pi_\rho(t+i) + S_t$$

$$S(t) = E_tS(t+1) \qquad (11)$$

For $\theta \geq \frac{1}{2}$, that is, for mainly backward-looking model, the solution looks as follows:

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \theta^{-1}\beta\sum_{i=0}^{\infty}\left[\frac{1-\theta}{\theta}\right]^i E_t[u(t+i)-u_N] \qquad (12)$$

$$+\theta^{-1}\delta\sum_{i=0}^{\infty}\left[\frac{1-\theta}{\theta}\right]^i E_t\pi_\rho(t+i) + S(t)$$

$$S(t) \equiv 0 \text{ for } \theta > \frac{1}{2} \qquad (13)$$

$$= E_tS(t+1) \text{ for } \theta = \frac{1}{2}$$

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Note that, when $\theta = \frac{1}{2}$, that is, when future expected inflation has the same weight as past inflation, the solution given by 39 and 40 is the same as the solution given by 41 and 42. When $\theta > \frac{1}{2}$, that is, when the model is mainly backward-looking, the sunspot component of the solution has to equal zero if the sunspot is not to blow up in expectation (see 38).

3.2.2 Disinflation when the inflation process is 'mainly forward-looking'

Consider first the case where $\theta \leq \frac{1}{2}$ and the inflation process is mainly forward-looking. Assume that condition 5 holds and the disinflation process does not permanently affect the real exchange rate.

The solution for the inflation rate becomes

$$\tilde{\pi}(t) = \left[\frac{\theta}{1 - \theta}\right] \tilde{\pi}(t - 1) - (1 - \theta)^{-1} \beta \sum_{i=0}^{\infty} E_t[u(t + i) - u_N] + S_t \tag{14}$$

$$S(t) = E_t S(t + 1) \tag{15}$$

Solving 14 forward we find

$$\lim_{t \to \infty} \tilde{\pi}(t) = \lim_{t \to \infty} \left[\frac{\theta}{1 - \theta}\right]^t \tilde{\pi}(0) \tag{16}$$

$$+ (1 - \theta)^{-1} \beta \lim_{t \to \infty} \sum_{k=0}^{t} \left[\frac{\theta}{1 - \theta}\right]^k \lim_{\ell \to \infty} \sum_{i=0}^{\ell} E_t[u(t - k + i) - u_N]$$

$$+ \lim_{t \to \infty} \sum_{k=0}^{t} \left(\frac{\theta}{1 - \theta}\right)^k S_{t-k}$$

The first term on the RHS of 16 vanishes because $0 \leq \theta \leq \frac{1}{2}$. Equation 16 has to hold for all processes driving the gap between the actual and natural unemployment rates. We assume that, in the long-run, the actual and natural unemployment rates are the same and that there is a constant long-run rate of inflation. The second term on the RHS of 16 therefore also vanishes

Using 15, this implies

$$S_t = \left(\frac{1 - 2\theta}{1 - \theta}\right) \lim_{i \to \infty} E_t \tilde{\pi}(t + i) \tag{17}$$

Before considering the mainly forward-looking case in detail, we consider the special case of the purely forward-looking process, $\theta = 0$. This is of interest because it is prominent in the New Keynesian literature.

Costless disinflation 2: the New Keynesian world

When $\theta = 0$, that is, when only expected future inflation affects current inflation and past inflation has no weight at all, the model reduces to the canonical time-contingent New Keynesian model developed by Calvo [9] (see also Ball [1], Ball, Mankiw and Romer [2] and Mankiw [15]). The model simplifies to

$$\tilde{\pi}(t) = E_t \tilde{\pi}(t+1) - \beta[u(t) - u_N] + \delta \pi_\rho(t) \qquad (18)$$

Its solution is

$$\tilde{\pi}(t) = -\beta \sum_{i=0}^{\infty} E_t[u(t+i) - u_N] + \delta \sum_{i=0}^{\infty} E_t \pi_\rho(t+i) + S_t \qquad (19)$$

$$S(t) = \lim_{i \to \infty} E_t \tilde{\pi}(t+i) \qquad (20)$$

While it is not apparent from 18, the Calvo [9] model that generates the (closed economy version of) 18, does have nominal price *level* rigidity or inertia. In any given period, the initial price level is predetermined. It does not, however, have *inflation* rigidity or inertia in the rate of inflation. Given the appropriate monetary policy support therefore, costless disinflation can be achieved. Given the proper monetary policy, there is no trade-off between inflation and unemployment in the New Keynesian universe characterised by equation 18, or, equivalently, by equations 19 and 20.[2] Note that, if 5 holds and the disinflation policy does not have a permanent effect on the real exchange rate, the solution to the New Keynesian inflation equation can be written as:

$$\sum_{i=0}^{\infty} E_t[u(t+i) - u_N] = \beta^{-1} \left[\lim_{i \to \infty} E_t \tilde{\pi}(t+i) - \tilde{\pi}(t) \right] \qquad (21)$$

Credible policies and policy announcements can lower both the long-run equilibrium rate of inflation, $\lim_{i \to \infty} E_t \tilde{\pi}(t+i)$ and the current rate of inflation $\tilde{\pi}(t)$ by equal amounts without the need to go through any transitional (let alone permanent) unemployment. The kind of monetary policy that would support costless disinflation in the Calvo model of equation 18 for which the price *level* is predetermined, would be an unanticipated immediate and permanent reduction in the *rate of growth* of the nominal money stock, accompanied by an immediate (unanticipated) one-off increase in the *level* of the nominal money stock of just the right magnitude to accommodate the increased demand for real money balances at the lower rate of inflation (and lower nominal interest rates) associated with the successful implementation of the policy.

The conclusion that the sacrifice ratio is zero in the purely forward-looking model carries over to the entire class of 'mainly forward-looking' mixed models ($\frac{1}{2} \le \theta < 1$).

This can be seen from equations 14 and 17. These imply

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[2] In general, the implementation of the appropriate monetary policies may also require suitably supportive fiscal policies, owing to the inextricable interconnection between intertemporal monetary and fiscal policy through the government solvency constraint.

$$\tilde{\pi}(t) \;\; = \;\; \left[\frac{\theta}{1-\theta}\right]\tilde{\pi}(t-1) - (1-\theta)^{-1}\beta\sum_{i=0}^{\infty}E_t[u(t+i)-u_N] \qquad (22)$$

$$+ \left(\frac{1-2\theta}{1-\theta}\right)\lim_{i\to\infty}E_t\tilde{\pi}(t+i) \qquad (23)$$

Assume that the government targets an inflation rate of $\tilde{\pi}^*$. We determine whether, for the inflation process given in 22, the transition from $\tilde{\pi}(t-1)$ to $\tilde{\pi}^*$ can be achieved with the unemployment rate always at the natural rate. From 22, this would require that the following process converges to $\tilde{\pi}^*$:

$$\tilde{\pi}(t) = \left[\frac{\theta}{1-\theta}\right]\tilde{\pi}(t-1) + \left(\frac{1-2\theta}{1-\theta}\right)\tilde{\pi}^* \qquad (24)$$

Therefore

$$\lim_{t\to\infty}\tilde{\pi}(t) = \lim_{t\to\infty}\left[\frac{\theta}{1-\theta}\right]^t\tilde{\pi}(0) + \left(\frac{1-2\theta}{1-\theta}\right)\tilde{\pi}^*\lim_{t\to\infty}\sum_{i=0}^{t}\left[\frac{\theta}{1-\theta}\right]^i \qquad (25)$$

Since $0 \le \theta < \frac{1}{2}$,
$$\lim_{t\to\infty}\tilde{\pi}(t) = \tilde{\pi}^* \qquad (26)$$

Thus, asymptotically, the inflation rate can be brought to any level, without incurring any unemployment rate different from the natural rate. Unlike the fully forward-looking New Keynesian case, however, for which convergence to the new inflation rate can be instantaneous, convergence with $u = u_N$ will only be gradual (indeed asymptotic) when $\theta > 0$.[3]

Just like in the neo classical case, costless disinflation implies that a cold turkey is likely to be the best option in the mainly forward looking model.

Conclusion 1 *In both the New Keynesian (purely forward-looking) model and in the mainly forward-looking mixed model, the sacrifice ratio is zero. Achieving any desired change in the rate of inflation can be instantaneous in the New Keynesian model, but only asymptotic in the mainly forward-looking mixed model.*

3.2.3 Disinflation when the inflation process is 'mainly backward looking'

The previous subsection showed that, given intelligently designed monetary policy, the sacrifice ratio is zero in the mainly forward-looking mixed model $(0 \le \theta \le \frac{1}{2})$. We now focus on the most interesting case - that of an inflation process - the mainly backward-looking mixed process $(\frac{1}{2} < \theta \le 1)$. It is obvious

---

[3] The simple macromodel of Footnote 1 can be used to demonstrate the validity of the zero sacrifice ratio and asymptotic convergence to the new inflation rate. One policy experiment that supports this is the unanticipated announcement of an immediate and permanent reduction in the proportional growth rate of the nominal money stock.

from 18 and 41 and 42, that the sacrifice ratio is finite: it takes (at most) a temporary increase in unemployment to achieve a permanent reduction in inflation. It is also clear that the sacrifice ratio depends only on β and θ, and, when $\theta = \frac{1}{2}$, on $\lim_{i \to \infty} E_t S(t+i)$.

When $\theta > \frac{1}{2}$ (which implies $A_0 = 1$), it is always possible, since there only is one lag in the inflation process, and the coefficient on lagged inflation in 41 is unity, to achieve a permanently lower rate of inflation by increasing (expected) unemployment for just one period. Consider the case where unemployment is raised only in period t, that is, $u(t+i) = u_N$, $i \geq 1$. Assume also, until further notice, that the current and expected future real exchange rate depreciation rates are held constant. From 41 it then follows that

$$\sigma = \theta \beta^{-1} \tag{27}$$

As expected, the sacrifice ratio is smaller when the responsiveness of inflation to unemployment is higher and when the relative weight of past inflation is lower. Note that the desired full reduction in the rate of inflation is achieved immediately in this case, that is, in period t. Note that, the path of inflation over time is given by

$$\tilde{\pi}_{t+i} = \tilde{\pi}_{t-1} - 1 \quad , i \geq 0 \tag{28}$$

Forward-looking expectations and the case for 'delay in implementation' Again considering the case where $\theta > \frac{1}{2}$ and where the path of the real exchange rate is held constant. We can determine the sacrifice ratio when unemployment is increased by the same amount in two successive periods, t and $t+1$, say. For all subsequent periods, unemployment is kept at the natural rate. This means

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \frac{\beta}{\theta} \left[u(t) - u_N \right] - \frac{\beta}{\theta} \left(\frac{1-\theta}{\theta} \right) \left[u(t+1) - u_N \right]$$

$$\begin{aligned} \tilde{\pi}(t+1) &= \tilde{\pi}(t) - \frac{\beta}{\theta} \left[u(t+1) - u_N \right] \\ &= \tilde{\pi}(t-1) - \frac{\beta}{\theta} \left[u(t) - u_N \right] - \frac{\beta}{\theta^2} \left[u(t+1) - u_N \right] \end{aligned}$$

Since unemployment is, by assumption, the same in periods t and $t+1$,

$$u(t) - u_N = u(t+1) - u_N = \bar{u} - u_N$$

Therefore,

$$\tilde{\pi}(t+1) = \tilde{\pi}(t-1) - \frac{\beta}{\theta^2} (1+\theta) (\bar{u} - u_N)$$

The unemployment gap we need to maintain for *2* periods in order to reduce inflation by 1% in two periods (and keep it at that lower level forever after) is therefore given by

$$\bar{u} - u_N = \frac{\theta^2}{\beta(1 + \theta)}$$

The sacrifice ratio is

$$\sigma = \left(\frac{2\theta}{1 + \theta}\right)\frac{\theta}{\beta} < \frac{\theta}{\beta} \text{ for } \theta < 1$$

This is intuitively plausible. Unemployment in period $t + 1$ works twice. First, directly, in period $t + 1$. Second, during period t, when the expectation of period $t + 1$ unemployment depresses period t inflation. Since policy has both *direct effects* and *announcement effects* when expectations are forward-looking, there is a case for a disinflationary policy taking the form of the *immediate, credible announcement* of *future* contractionary policy measures that will result in *future* unemployment. Thus the *credible* policy announcement should be forthcoming immediately, but the recession it announces should be postponed into the future, in order for the total, cumulative unemployment cost of achieving a sustained reduction in the rate of inflation to be as low as possible. Note that this constitutes an argument for *gradualism* over *cold turkey,* provided announcements concerning future policy are credible, that is, provided the policy maker is able to commit to future policy actions or policy rules. Gradualism means that the restrictive policy measures and their effect on unemployment and inflation are delayed and spread out over a number of periods. Cold turkey means that implementation is immediate and concentrated in a short period of time.

The inflation profile is of course not the same when part of the contractionary package required to achieve a given reduction in inflation (permanently but by some unspecified date) is postponed. When unemployment is increased by equal amounts in periods t and $t + 1$, the profile of inflation is as follows

$$
\begin{align}
\tilde{\pi}(t) &= \tilde{\pi}(t - 1) - \frac{1}{1 + \theta} \tag{29} \\
\tilde{\pi}(t + i) &= \tilde{\pi}(t - 1) - 1 \ , \ \ i \geq 1
\end{align}
$$

Comparing 28 and 29, we note that, while the inflation reduction in period $t + 1$ and beyond is the same for both unemployment profiles, the reduction in inflation in period t is larger when all the unemployment in concentrated in period t rather than being spread equally over periods t and $t + 1$.

Starting in period t, the authorities could also achieve a permanent reduction in inflation starting in period $t + 1$ by just increasing unemployment in period $t + 1$, and making a credible announcement to that effect in period t. In this case:

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \frac{\beta}{\theta}\left(\frac{1-\theta}{\theta}\right)[u(t+1) - u_N]$$

$$\begin{aligned}
\tilde{\pi}(t+1) &= \tilde{\pi}(t) - \frac{\beta}{\theta}[u(t+1) - u_N] \\
&= \tilde{\pi}(t-1) - \frac{\beta}{\theta^2}[u(t+1) - u_N]
\end{aligned}$$

In this case the sacrifice ratio is

$$\sigma = \frac{\theta^2}{\beta} < \left(\frac{2\theta}{1+\theta}\right)\frac{\theta}{\beta} < \frac{\theta}{\beta} \text{ for } \theta < 1$$

As expected, concentrating the necessary unemployment increase completely in period $t+1$ results in an even lower sacrifice ratio than spreading it evenly over periods t and $t+1$. The sequence of inflation rates in this case is

$$\begin{aligned}
\tilde{\pi}(t) &= \tilde{\pi}(t-1) - (1-\theta) \qquad\qquad (30) \\
\tilde{\pi}(t+i) &= \tilde{\pi}(t-1) - 1 , \;\; i \geq 1
\end{aligned}$$

Again, while the same inflation rate is ultimately achieved and sustained, period t inflation is highest when all the unemployment is concentrated in the later period.

What the logic of the model therefore calls for, if the purpose were to minimise the sacrifice ratio, is the immediate, credible announcement of (and commitment to), a delayed implementation of policy measures some time in the future (the further in the future, the better for the sacrifice ratio).

Note that there is likely to be a credibility problem or commitment problem here. Once the (credible) announcement (in period t) of higher unemployment in period $t+1$ (or even further into the future) has had its desired announcement effect on inflation in period t, the authorities may no longer wish to incur the higher unemployment when period $t+1$ arrives, just to ensure that their period t commitments are honoured. How would the public respond to the following policy statement?

> "Dear electorate. Several years ago my government announced that we would create a recession, starting today, in order to achieve a lasting reduction in the rate of inflation. You believed us. Your wage and price setting practices moderated and the desired reduction in inflation has now been achieved without, thus far, any increase in unemployment. There is therefore no reason, from the point of view of our inflation performance, to go ahead now and implement the measures that will produce the recession we promised you all those years ago. However, I cannot tell a lie. My credibility matters to me. We'll have the recession anyway. I look forward with confidence to you support at the next general elections"

Clearly, the policy announcement of a future policy-induced recession, with the costs postponed until after the gains have been reaped, would not in fact be credible. It is true that politics is a repeated game. Therefore, considerations of repetition and reputation can, in principle, overcome time inconsistency problems of the kind that arise here, or indeed whenever policy has announcement effects. It seems questionable, however, that a government would create a recession when there is no longer any use for it from the point of view of reducing inflation, simply to preserve its credibility for possible future policy announcements.

There are two further qualifications to the proposition that the postponement of disinflationary pain may be good for you. First, the sacrifice ratio sums the *undiscounted* increases in unemployment. The authorities may not in fact be indifferent about the timing of unemployment.[4] Second, the inflation profiles are different for the three cases, before period $t+1$. Inflation in period t is lowest when the unemployment is all concentrated in period t and highest when it is all concentrated in period $t + 1$.

3.2.4 The role of the real exchange rate

Can the behaviour of the real exchange rate make a difference to the sacrifice ratio in the mainly backward-looking model? When $\theta > \frac{1}{2}$, and therefore $S(t) = 0$, the inflation process looks as follows:

$$
\tilde{\pi}(t) = \tilde{\pi}(t-1) - \theta^{-1}\beta \sum_{i=0}^{\infty} \left[\frac{1-\theta}{\theta}\right]^i E_t[u(t+i) - u_N] + \theta^{-1}\delta \sum_{i=0}^{\infty} \left[\frac{1-\theta}{\theta}\right]^i E_t\pi_\rho(t+i)
$$

(31)

Even granting the assumption that the disinflation process does not have a lasting effect on the real exchange rate, that is $\sum_{i=0}^{\infty} E_t\pi_\rho(t + i) = 0$, a given real exchange rate appreciation at time t followed by a later depreciation of the same magnitude will have a lasting effect on the inflation rate. Indeed, an initial real appreciation followed by a later real depreciation that restores the original real exchange rate will raise inflation and the sacrifice ratio. For instance, in the Dornbusch 'overshooting' model of Footnote 1, when 8(with $\theta > \frac{1}{2}$) is substituted for the conventional Phillips curve, the real exchange rate 'overshooting' that occurs in response to the unanticipated, immediate and permanent reduction in the growth rate of the nominal money stock, raises the sacrifice ratio relative to a policy that pursues a constant real exchange rate throughout the disinflation process.[5] The real overshooting does, of course, provide the anti-inflationary gains earlier. For instance, a 1% real appreciation in period t followed by the expectation of a 1% real depreciation in period $t + 1$ (and the actual occurrence of that depreciation in period $t + 1$) would *reduce* inflation in period t by $\delta(2\theta - 1)\theta^{-2}$%, but would also *raise* it during period

---

[4] If the subjective discount rate of the authorities is positive, this would further favour postponing the unemployment.

[5] This would require the use of time-varying fiscal policy.

$t + 1$ by $\delta(1 - \theta)\theta^{-2}\%$ relative to where it would have been with a constant real exchange rate. The reason is that, from 31, the future real exchange rate depreciation works twice, once when it actually occurs, during period $t + 1$, and once when it is anticipated, in period t. The earlier appreciation does not have these announcement effects.

Conclusion 2 *In the mainly backward-looking model, costless disinflation is impossible. The sacrifice ratio is always positive (but finite).*

Conclusion 3 *In the mainly backward-looking model, the unexpected, credible announcement of a given cumulative increase in current or future unemployment has a stronger sustained effect on the rate of inflation the further in the future it is applied. Equivalently, a given sustained reduction in inflation can be achieved with a lower sacrifice ratio if the unemployment is postponed further into the future. However, postponing the unemployment also delays the achievement of the full reduction in inflation.*

Conclusion 4 *Technically more efficient disinflation plans that involve the achievement of some or all of the full anti-inflationary gains through announcement effects of unemployment to be incurred after the anti-inflationary gains have been achieved, are unlikely to be credible.*

Conclusion 5 *In the mainly backward-looking model, temporary real exchange rate changes can affect the sacrifice ratio. An early real appreciation followed by a later real depreciation that takes the real exchange rate back to its initial level will raise the sacrifice ratio.*

3.3   Conclusion

Under most reasonable assumptions the benefits from eliminating moderate inflation cannot be enjoyed without incurring the pain of increased unemployment and lost production. All models considered in this review share this property, with the exception of the New Classical and New Keynesian models.[6]   The richest model, and the one we consider most appropriate as a framework for monetary policy evaluation and design in low and moderate inflation countries is the mainly backward-looking mixed model - henceforth the *preferred* model. Because it incorporates forward-looking behaviour as well as inflation inertia, it permits the explicit consideration of key issues faced by central bankers throughout the industrial world and in many candidate countries for accession.

The preferred model has the feature that painless disinflation is impossible. It also has the property that policies that reduce the sacrifice ratio by relying on the announcement effects on inflation today of policies that produce unemployment in the future, have two weaknesses. First, they may be time-inconsistent. A commitment to inflict pain (increased unemployment) in the future, after

---

[6] The mainly forward-looking mixed model with $0 < \theta \leq \frac{1}{2}$ qualifies for a zero sacrifice ratio only on a technicality: costless disinflation is not possible in finite time, but only asymptotically.

the anti-inflationary benefits have been achieved in full (making the increase in unemployment unnecessary), is not credible. Second, even if credible, such policies would delay the achievement of the anti-inflationary gains relative to a policy that imposes unemployment earlier (and which therefore has a higher sacrifice ratio). Finally, the preferred model highlights the role of changes in the real exchange rate in the disinflation process. Even if successful disinflation does not alter the long-run real exchange rate, a policy of disinflation under a floating exchange rate and with perfect international financial capital mobility is likely to be associated with an initial sharp appreciation of the real exchange rate followed by a more gradual real depreciation back to the invariant long-run real exchange rate. Such a pattern of real exchange rate variations raises the sacrifice ratio relative to a policy that keeps the real exchange rate constant. The policy also speeds up the achievement of the lower inflation target. The preferred model permits a transparent characterisation of the policy maker's menu of choice.

References

[1] Ball, Laurence [1994], "What Determines the Sacrifice Ratio?", in *Monetary Policy*, N. G. Mankiw, ed., Chicago, University of Chicago Press, pp. 155-82.

[2] Ball, Laurence, Mankiw, N. Gregory and Romer, David [1988], "The New Keynesian Economics and the Output-Inflation Trade-Off", *Brookings Papers on Economic Activity*, Vol. 1, pp. 1-65.

[3] Blanchard, Olivier J. and Summers, Lawrence [1986], "Hysteresis and the European Unemployment Model", *NBER Macroeconomics Annual 1*: pp 15-78.

[4] Buiter, Willem H. [1981], "The Superiority of Contingent Rules over Fixed Rules in Models with Rational Expectations", *The Economic Journal*, 91, September, pp. 647-70. Reprinted in Willem H. Buiter, *Macroeconomic theory and stabilization policy*, Manchester University Press, 1989, pp. 77-104.

[5] Buiter, Willem H. [1985], "International Monetary Policy to Promote Economic Recovery", in C. van Ewijk and J.J. Klant (eds.), *Monetary Conditions for Economic Recovery*, Martinus Nijhof, Amsterdam. Reprinted as Chapter 6 in Willem H. Buiter, *International Macroeconomics*, Clarendon Press, Oxford, 1990, pp. 171-194.

[6] Buiter, Willem H. and Jewitt, Ian [1981], "Staggered wage setting with real wage relativities: variations on a theme of Taylor", *The Manchester School*, Vol. 49, pp. 211-28. Reprinted in Willem H. Buiter, *Macroeconomic theory and stabilization policy*, Manchester University Press, 1989, pp. 183-199.

[7] Buiter, Willem H. and Marcus H. Miller [1985], "Costs and benefits of an antiinflationary policy: questions and issues", in A. Argy and J. Neville, (eds), *Inflation and Unemployment: Theory, Experience and Policy Making*, London, George Allen and Unwin, reprinted in Willem H. Buiter, *Macroeconomic theory and stabilization policy*, Manchester University Press, 1989, pp. 200-227.

[8] Buiter, Willem H. and Clemens Grafe [2001], "No Pain no Gain", CEPR discussion paper

[9] Calvo, Guillermo A. [1983], "Staggered Prices in a Utility Maximising Framework", *Journal of Monetary Economics*, Vol 12, September, pp. 383-98.

[10] Calvo, Guillermo A., Celasum, Oya and Michael Kumhof [2003], "Inflation Inertia and Credible Disinflation-The Open Economy Case", NBER Working Paper 9557.

[11] Fuhrer, Jeffrey and Moore, George [1995], "Inflation Persistence", *Quarterly Journal of Economics*, Vol. 110(1), February, pp. 127-60.

[12] Gali, Jordi and Tomasso Manacelli [2002, ""Monetary Policy and Exchange Rate Volatility in a Small Open Economy", NBER working paper 8905

[13] Gregory, R.G. [1986], "Wages Policy and Unemployment in Australia", *Economica*, 53, (Supplement), pp. S53-S74.

[14] Lindbeck, Assar and Snower, Dennis [1988], *The Insider-Outsider Theory of Employment and Unemployment*. Cambridge: MIT Press.

[15] Mankiw, N. Gregory [2001], "The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment", *The Economic Journal*, 111, May, pp. C45-C61.

[16] Taylor, John B. [1980], "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy*, Vol. 88, pp. 1-22.

[17] Taylor, John B. [1998], "Staggered Price and Wage Setting in Macroeconomics", NBER Working Paper No. 6754.

Annex

From 8 it is clear that, since $\tilde{\pi}(t+1)$ will depend on $u(t+1)$ and on period $t+1$ expectations of $\tilde{\pi}(t+2)$, it is reasonable to postulate the following form for a solution that is both mathematically acceptable and economically meaningful:

$$\tilde{\pi}(t) = A_0\tilde{\pi}(t-1) + B_0\left[u(t) - u_N\right] + \sum_{i=1}^{\infty} B_i\left[E_t u(t+i) - u_N\right] \quad (32)$$

$$+ C_0\pi_\rho(t) + \sum_{i=1}^{\infty} C_i E_t \pi_\rho(t+i) + S(t)$$

The coefficients A_0, B_i, C_i, $i \geq 0$ and the process $S(t)$ are found by substituting the proposed solution into the model and equating coefficients between the resulting equation and the proposed solution.[7]. Everything on the right-hand-side of 32 except for $S(t)$ constitutes the 'minimal state, fundamental solution'. The term $S(t)$ denotes the remainder of the solution. It can either involve non-fundamental or 'sunspot' variables (that is, variables not present in 8) or non-minimal state representations involving the fundamental variables.

The undetermined coefficients are solved for from:

$$A_0 = [1 - (1-\theta)A_0]^{-1}\theta \quad (33)$$

$$B_j = -\left\{[1 - (1-\theta)A_0]^{-1}(1-\theta)\right\}^j [1 - (1-\theta)A_0]^{-1}\beta, \quad j \geq 0 \quad (34)$$

$$C_j = \left\{[1 - (1-\theta)A_0]^{-1}(1-\theta)\right\}^j [1 - (1-\theta)A_0]^{-1}\delta, \quad , j \geq 0 \quad (35)$$

$$E_t S_{t+1} = [1 - (1-\theta)A_0](1-\theta)^{-1}S_t \quad (36)$$

The two solutions for A_0 and the corresponding solutions for the other coefficients and processes are given in 37 and 38:[8]

$$
\begin{array}{|l|}
\hline
\hline
A_0 = \frac{\theta}{1-\theta} \\
\hline
B_i = -(1-\theta)^{-1}\beta, \ i \geq 0 \\
\hline
C_i = (1-\theta)^{-1}\delta, \ i \geq 0 \\
\hline
E_t S_{t+1} = S_t \\
\hline
\hline
\end{array}
\quad (37)
$$

---

[7] Use is made of the 'Law of Iterated Projections', $E_t E_{t+j}(x) = E_t(x)$, $j \geq 0$, (the earlier expectation of a later expectation is the earlier expectation). This is an immediate application of a fundamental property of conditional expectations, provided that the information set conditioning expectations at an earlier date is not richer than the information set at a later date. We assume for simplicity that the current state is known, that is, $E_t\tilde{\pi}(t) = \tilde{\pi}(t)$.

[8] $\frac{\theta}{1-\theta}$ and 1 are the two roots of the homogenous equation of 8 $(1-\theta)E_t\tilde{\pi}(t+1) - \tilde{\pi}(t) + \theta\tilde{\pi}(t-1) = 0$.

$$\begin{array}{|l|}
\hline
A_0 = 1 \\
\hline
B_i = -\left(\frac{1-\theta}{\theta}\right)^i \theta^{-1}\beta, \ \ i \geq 0 \\
\hline
C_i = (\frac{1-\theta}{\theta})^i \theta^{-1}\delta, \ \ i \geq 0 \\
\hline
E_t S_{t+1} = \left(\frac{\theta}{1-\theta}\right) S_t \\
\hline
\end{array}
\tag{38}$$

We have a linear rational expectations models with constant coefficients whose homogeneous part is a second-degree difference equation. When there are two roots, one of which is weakly stable (modulus less than or equal to 1) and one of which is unstable (modulus greater than 1), we would choose the stable one to drive the predetermined state variable. In equation 8, except when $\theta = 0,$ the rate of inflation, $\tilde{\pi}$, is a predetermined state variable. We want the homogeneous part of our proposed solution in equation 32, that is, $\tilde{\pi}(t) = A_0\tilde{\pi}(t-1)$, to be non-explosive.

For $A_0 = \frac{\theta}{1-\theta}$, A_0 increases monotonically with θ over the interval $0 \leq \theta \leq 1$. Also, $\theta > \frac{1}{2}$ implies $A_0 > 1$, and indeed $\lim_{\theta\uparrow 1}\left(\frac{\theta}{1-\theta}\right) = \infty$. We therefore use $A_0 = \frac{\theta}{1-\theta}$ for $\theta \leq \frac{1}{2}$ and $A_0 = 1$ for $\theta \geq \frac{1}{2}$.

For $\theta \leq \frac{1}{2}$, that is, for the mainly forward-looking model, the solution looks as follows:

$$\tilde{\pi}(t) = \left[\frac{\theta}{1-\theta}\right]\tilde{\pi}(t-1) - (1-\theta)^{-1}\beta\sum_{i=0}^{\infty} E_t[u(t+i) - u_N] \tag{39}$$

$$+ (1-\theta)^{-1}\delta\sum_{i=0}^{\infty} E_t\pi_\rho(t+i) + S_t$$

$$S(t) = E_t S(t+1) \tag{40}$$

For $\theta \geq \frac{1}{2}$, that is, for mainly backward-looking model, the solution looks as follows:

$$\tilde{\pi}(t) = \tilde{\pi}(t-1) - \theta^{-1}\beta\sum_{i=0}^{\infty}\left[\frac{1-\theta}{\theta}\right]^i E_t[u(t+i) - u_N] \tag{41}$$

$$+ \theta^{-1}\delta\sum_{i=0}^{\infty}\left[\frac{1-\theta}{\theta}\right]^i E_t\pi_\rho(t+i) + S(t)$$

$$\begin{aligned}
S(t) &\equiv 0 \text{ for } \theta > \frac{1}{2} \\
&= E_t S(t+1) \text{ for } \theta = \frac{1}{2}
\end{aligned}
\tag{42}$$