Discussion of "A Non-Parametric Model-Based Approach to Uncertainty and Risk Analysis of Macroeconomic Forecasts" by Claudia Miani and Stefano Siviero

Malte Knüppel

Deutsche Bundesbank

November 2009

Very Brief Summary of the Paper

- Aim: Providing (possibly) skewed density forecasts of important variables (growth of GDP, consumption deflator,...) based on (possibly) skewed densities of future shocks and exogenous variables (oil prices, foreign demand,...)
- Approach: Stochastic simulation of macro-econometric model with asymmetric bootstrap procedure
  - Why bootstrap? Shocks retain empirical covariances without imposing distributional assumptions
  - Why asymmetric bootstrap? In order to impose skewness of shocks
  - Why stochastic simulation? Model is non-linear $\rightarrow$ analytical investigation of asymmetries infeasible
- Interesting features and results:
  - New way to create asymmetric bootstrap shocks
  - Asymmetry of shocks hardly carries over to variables of interest, above all in the long run

Questions and Comments

- Sometimes more information would be helpful
  - What are the statistical properties of the asymmetric density s if the original density f is normal (mean, variance, skewness...)?
  - How are the rejection sampling techniques implemented, and how are asymmetries in the residuals treated?
  - In the application, what are the statistical properties of the variables of interest (skewness, p^*, variance...) and what would they be if the shocks were symmetric?
  - How pronounced is the non-linearity of the model?
  - The process of scaling the variances of the variables of interest is not completely clear to me. Is the variance of shocks not regarded as risk factors set to zero? In this case, the results would be biased towards asymmetry, and the non-linearity of the model could cause further problems.
- The Bank of England's approach may be less formal than stated in the paper

A Potential Problem

- Consider simple model

$$z = x + y$$

where x and y are correlated, normally distributed variables with covariance matrix

$$\Omega_1 = \begin{bmatrix} var(x) & cov(x, y) \\ cov(x, y) & var(y) \end{bmatrix}$$

Then draws can be simulated using the Cholesky decomposition

$$\Omega_1 = L_1 \times L_1',$$

where L_1 is lower triangular, by using

$$\begin{bmatrix} x \\ y \end{bmatrix} = L_1 \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \begin{bmatrix} l_{1,11} & 0 \\ l_{1,21} & l_{1,22} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix}$$

where ε_x and ε_y have an $N(0, 1)$ distribution.

A Potential Problem

- Draws can just as well be simulated based on a different order of x and y (Order 1 used before: first x, then y. Order 2 used here: first y, then x.), where

$$\Omega_2 = \begin{bmatrix} var(y) & cov(x,y) \\ cov(x,y) & var(x) \end{bmatrix}.$$

Then draws can again be simulated using the Cholesky decomposition

$$\Omega_2 = L_2 \times L_2',$$

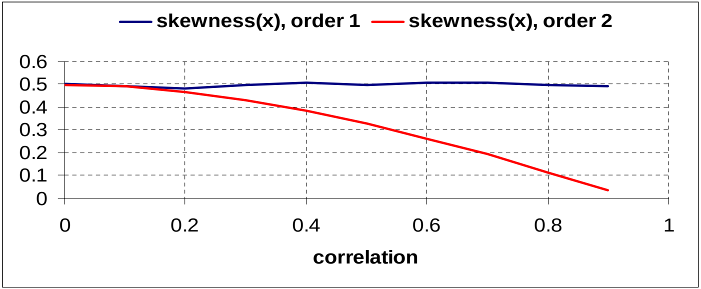
where L_2 is lower triangular, as

$$\begin{bmatrix} y \\ x \end{bmatrix} = L_2 \times \begin{bmatrix} \varepsilon_y \\ \varepsilon_x \end{bmatrix} = \begin{bmatrix} l_{2,11} & 0 \\ l_{2,21} & l_{2,22} \end{bmatrix} \times \begin{bmatrix} \varepsilon_y \\ \varepsilon_x \end{bmatrix}$$

Both ways of drawing yield normally distributed variables with the same variances of x and y and the same correlations of x and y.

A Potential Problem

- For correlated variables with asymmetric distributions, the order matters! (see Ferreira & Steel, Statistica Sinisca, 2007; Pinheiro & Esteves, 2008)
- The covariance matrix of x and y does not depend on the order, but their other moments do.
- Consider case $var(x) = var(\varepsilon_x) = 1$, $cov(x, y) = \rho, \varepsilon_y \sim N(0, 1)$ $\varepsilon_x \sim tpn$, $skew(\varepsilon_x) \approx 0.5$, $mode(\varepsilon_x) = 0$



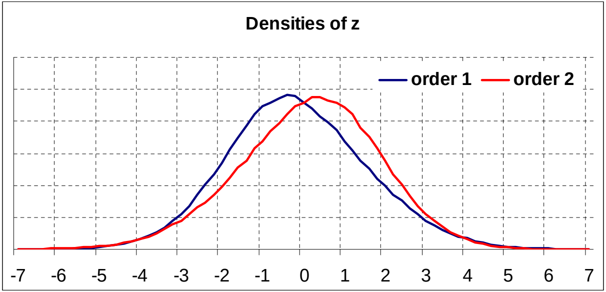
A Potential Problem

- Case considered before not very interesting. Forecaster is not interested in properties of ε_x and ε_y, but wants certain properties for x and y $\rightarrow$ Forecaster plays around with ε_x and ε_y until x and y have desired properties.
- Consider case with all variances $= 1$, $cov(x, y) = 0.5$,
  $mode\,(\varepsilon_x) = mode\,(\varepsilon_y) = 0$
  Desired skewness: $skew\,(x) \approx 0.5$, $skew\,(y) \approx -0.5$
- Aim can be achieved using *tpn*-distributed ε's,
  order 1 and: $skew\,(\varepsilon_x) \approx 0.5$ $skew\,(\varepsilon_y) \approx -0.9$
  order 2 and: $skew\,(\varepsilon_x) \approx 0.9$ $skew\,(\varepsilon_y) \approx -0.5$
- But what happens with other moments and with z?

	$mean\,(x)$	$mean\,(y)$	$mean(z)$	$var(z)$	$skew(z)$
order 1	0.52	−0.67	−0.15	3.00	0.21
order 2	0.67	−0.52	0.15	3.00	−0.21

A Potential Problem

- Although variances, correlation and asymmetries of x and y are as desired in both cases (order 1 and order 2), the properties of the variable of interest z strongly depend on the order. This is quite an annoying result for risk analysis.



- Note: In some cases it might be impossible to get desired correlation and asymmetries for x and y. Imagine $\rho = 0.99$, $skew\left(x\right) = c$, $skew\left(y\right) = -c$ with c being large.

Suggestions

- Study correlations of shocks. Are they large and significant?
- If shocks supposed to be asymmetric are not just weakly correlated,...
  - 1st possibility: try to find reasonable order of shocks (difficult)
  - 2nd possibility: try out all possible orders and check robustness of results (Pinheiro & Esteves, 2008; enormous workload)
- If correlations of shocks are small, ignore problem.
  - 1st possibility: Impose asymmetry on ε's, do not check asymmetry of correlated shocks.
  - 2nd possibility: Play around with ε's to get desired asymmetry of correlated shocks.
  - 3rd possibility: Directly generate independent asymmetric shocks, i.e. ignore correlations.

  If correlations are small, each possibility should yield similar results. 1st and 3rd possibility are easier to implement.

  $\rightarrow$ Both could be used and results compared (robustness check)