Loss Aversion and the Transmission of Monetary Policy[*]

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Abstract

We propose a novel explanation of the evidence that output and prices react asymmetrically to monetary policy over the business cycle. We design a dynamic general equilibrium model in which households' utility depends on consumption deviations from a reference level below which loss aversion is displayed. In line with the theory developed by Kahneman and Tversky (1979), losses in consumption loom larger than gains. State-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption generate competing effects on output and inflation. The resulting state-dependent trade-off between inflation and output stabilization recommends stronger policy activism towards inflation during expansions.

JEL: E32; E42; E52; D03; D11.

Keywords: Asymmetry, Monetary Policy, Business Cycle, Loss Aversion.

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1 Introduction

Since the seminal work by Mitchell (1927), considerable effort has been devoted to the study of asymmetries in macroeconomic time series. Graham (1930), Keynes (1936) and Friedman and Schwartz (1963) have then stimulated a vast debate on the asymmetric effects of monetary policy. Widespread empirical evidence has been produced in support of the view that monetary policy exerts asymmetric effects on output and prices with respect to the economic conditions as well as the size and direction of the policy action.[1] Such effects have important implications not only for the way we think about the macroeconomy, but also for the conduct of economic policy.

In this paper we focus on asymmetries in the transmission of monetary policy over contractionary and expansionary phases of the business cycle. Contractions (expansions) are intended as periods in which the cycle moves from its peak (trough) to the trough (peak). In this respect, two major regularities have emerged (see, e.g., Weise, 1999; Peersman and Smets, 2005). On the one hand, monetary policy innovations have greater impact on output during negative stages of the cycles. On the other hand, changes in the monetary policy stance seem to exert stronger effects on prices during expansionary phases. These facts motivate our study. We put forward a novel potential explanation of the cyclical asymmetries in the transmission of monetary policy, based on households displaying loss-averse preferences in consumption. We show how embedding prospect theory (Kahneman and Tversky, 1979) in a popular macroeconomic framework can robustly account for the asymmetric effects of monetary policy on real activity and prices.

Reference-dependent preferences have received strong attention in the literature on asset pricing. Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001) have successfully employed prospect theory to explain the behavior of asset returns and resolve a number of quantitative asset pricing puzzles. A key feature of the model of Barberis, Huang, and Santos (2001) is that households care about gains and losses in financial wealth. By contrast, Koszegi and Rabin (2009) assume that households care about gains and losses in consumption. This assumption is empirically supported by Yogo (2008) and Rosenblatt-Wisch (2008).

The novelty of this paper is to embed reference-dependent utility in a dynamic general equilibrium framework where households' utility depends on both consumption and leisure. We set aside asset pricing implications and focus on the transmission of monetary policy on output and inflation. Our modeling strategy consists of assuming that consumers' utility partly depends on the deviation of their current consumption from the previous period's average consumption in the economy, which represents the habit reference level below which loss aversion is displayed. In line with the theory developed by

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[1] Some of the contributions in this strand of the literature are: Falk (1986), Cover (1992), Thoma (1994), Karras (1996), Acemoglu and Scott (1997), Weise (1999), Senda (2001), Ravn and Sola (2004), Dufrenot, Mignon, and Peguin-Feissolle (2004), Peersman and Smets (2005), Lo and Piger (2005).

Kahneman and Tversky (1979) and Tversky and Kahneman (1991), losses in consumption utility resonate more than gains.

Two key mechanisms are identified. First, during contractions changes in the real rate of interest exert stronger impact on output through an increase in the elasticity of intertemporal substitution between current and future consumption. This property of the elasticity of intertemporal substitution has been already explored in the literature on asset pricing (see, e.g., Yogo, 2008). Second, accounting for reference-dependent preferences in a general equilibrium setting implies a state-dependent marginal rate of substitution between consumption and leisure that can be related to firms' real marginal cost, so that equilibrium in the labor market holds. As a direct implication, the pass-through of real activity to prices depends on the level of consumption relative to its reference level and is lower during contractionary states as opposed to expansionary ones. Both features of the model are compatible with output being more adversely affected by monetary policy innovations during contractionary phases. Concurrently, inflation responses during negative growth cycles are somewhat insulated by the higher responsiveness of real activity through an increased degree of real rigidity in the labor market.

We explore the state-dependent trade-off that naturally arises in this context. In the expansionary state the Central Bank can attain a policy frontier which is otherwise unattainable during contractionary phases. The key implication is that reducing inflation variability by the same amount and from the same level in the two cyclical stages entails higher costs in terms of output variability during negative growth cycles. In line with recent evidence on asymmetries in the policy reaction function (Bec et al., 2002; Rabanal, 2004; Taylor and Davradakis, 2006; Cukierman and Muscatelli, 2008), the optimal policy imposes a stronger degree of reactiveness to the expected rate of inflation during expansionary states.

It is important to recall that the macroeconomic literature has proposed a variety of mechanisms acting from both the supply and the demand side of the economy and capable to account for different types of asymmetry. Among others, Lo and Piger (2005) and Peersman and Smets (2005) suggest that the balance-sheet channel is consistent with larger effects of monetary policy on output during unfavorable growth states. However, this mechanism implies an analogous amplification (attenuation) of monetary policy innovations on both prices and real activity during contractionary (expansionary) phases, thus contradicting the empirical evidence. Different potential factors have also been proposed to account for other sources of asymmetry in the monetary transmission, such as non-linearities in investment (Bertola and Caballero, 1994), patterns of entry and exit from a given market under uncertainty about profit perspectives (Dixit, 1989), nominal rigidities in the labor and goods market (Ball and Mankiw, 1995), learning and information aggregation (Chalkley and Lee, 1998), convex aggregate supply (Devereux and Siu,

2007), switches in consumer sentiment (De Grauwe, 2010).[2] However, a mechanism capable to account for the joint reaction of output and prices to a monetary policy innovation over different stages of the business cycle still has to emerge. We provide an alternative based on behavioral mechanisms that have found wide empirical and experimental support in the literature (Thaler, Tversky, Kahneman, and Schwartz, 1997).

The remainder of the paper is laid out as follows: Section 2 reviews some empirical evidence on the cyclical transmission of monetary policy that motivates our study; Section 3 details the theoretical framework we propose to account for these facts; Section 4 details the qualitative and quantitative features of the framework under scrutiny and provides intuition on the key mechanisms generating asymmetry in the responses of output and prices over different cyclical phases; Section 5 discusses the main policy implications of embedding loss-averse preferences over consumption in our general equilibrium setting; the last section concludes.

2 Asymmetries and Monetary Policy over the Cycle

There is widespread empirical and anecdotal evidence showing that monetary policy exerts asymmetric effects on output and prices with respect to the economic conditions, as well as the size and direction of the policy action. Three main regularities have emerged: (i) money does affect output strongly when monetary policy is restrictive and raises inflation when it is expansive; (ii) the effects of money on output is greater during the contractionary phases of business cycles and their impact on inflation is greater during expansionary phases; (iii) if prices are sticky, then only negative shocks affect output.

Our interest is on the effects of monetary policy on real activity and prices over different stages of the cycle. These are to be intended as phases of expansion and contraction in economic activity and are usually referred to as "growth cycles" (Zarnowitz, 1992).[3] Asymmetries in the responses of either output or prices (primarily the former) to monetary policy innovations have widely been documented in the existing literature, although the analysis of their joint behavior has been somewhat disregarded. Weise (1999) and Baumeister, Durinck, and Peersman (2008) represent noteworthy exceptions. Two main results emerge from these studies: (i) first, the response of output is greater during contractionary phases and significantly smaller during expansionary ones; (ii) second, despite the amplification of output responses during negative growth cycles, prices are invariantly affected by a monetary policy innovation over different stages of the cycle.

As remarked in Section 1, this evidence may be compatible with a number of mechanisms. In the remainder of the paper we show how a model of reference-dependent pref-

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[2]See Florio (2004) for a review of the literature.

[3]McKay and Reis (2008) have recently revived the interest in asymmetric fluctuations over different cyclical stages. They also consider the concept of growth cycles as opposed to classical cycles.

erences in consumption can provide a novel and alternative explanation of the observed asymmetries. Whereas this mechanism has drawn considerable attention in literature on asset pricing, which has been mainly aimed at reconciling the consumption-based CAPM with the empirical behavior of asset returns (see, e.g., Yogo, 2008), little effort has been made to explore its relevance in the macroeconomic literature. Some applications to price setting (Heidhues and Koszegi, 2005) and consumption theory (Koszegi and Rabin, 2009) have recently been proposed. However, to the best of our knowledge, we provide the first contribution exploring the role of loss aversion in consumption utility within a dynamic general equilibrium setting.

A further motivation argument for the present paper comes from the need to understand the connection between monetary policy-making and asymmetric responses of macroeconomic aggregates to changes in the policy stance. Once it is recognized that interest rate innovations exert asymmetric effects on output and prices over different stages of the cycle, it seems relevant to provide some guidance on how monetary policy should be designed to cope with such non-linearities. As a matter of fact, a number of papers have identified important asymmetries in the policy reaction function of various Central Banks. For instance, Rabanal (2004) shows that the U.S. monetary policy has been somewhat less active during expansions, attaching higher weight to inflation responses and reflecting stronger interest-rate inertia. Clarifying whether such empirical evidence is consistent with the theoretical policy prescriptions of the framework we put forward is an obvious goal for our study.

3   A Model of Reference-dependent Consumption Choices

This section sets out the structure of the model we put forward to explain asymmetries in the responses of output and prices to monetary innovations. The supply side is populated by monopolistically competitive firms producing intermediate goods and a perfectly competitive sector of production that sells a composite of consumption goods. As to the demand side, there is a continuum of atomistic consumers, indexed by $i \in [0, 1]$.

3.1   Demand Side

Households have preferences defined over leisure $(1 - N_{it})$, consumption (C_{it}) and gains and losses in consumption relative to its reference level (X_{it}). They maximize the expected present discounted value of their utility:

$$\mathcal{W}_{it} = E_t \sum_{s=0}^{\infty} \beta^s \left[U\left(C_{it+s}, X_{it+s}\right) - \chi \frac{N_{it+s}^{1+\eta}}{1+\eta} \right]; \quad \chi > 0, \tag{1}$$

where β is the intertemporal discount factor and η is the inverse of the Frisch elasticity of labor supply. Following Koszegi and Rabin (2006) and Yogo (2008), we consider a general class of reference-dependent preferences:[4]

$$U(C, X) = \alpha V(C) + (1 - \alpha) \Lambda(V(C) - V(X)); \quad \alpha \in [0, 1], \tag{2}$$

where $V(C)$ is a neoclassical utility function: this is assumed to be continuously differentiable, strictly increasing, and concave for all $C > 0$. $\Lambda(\cdot)$ is a gain-loss function (Kahneman and Tversky, 1979), that is, utility derived from the deviation of consumption utility $V(C)$ from its reference level, $V(X)$. Preferences that depend on a reference level of consumption have psychological foundations in hedonic adaptation (see Frederick and Loewenstein, 1999).

We assume that $\Lambda(\cdot)$ satisfies certain properties. Specifically: (i) $\Lambda(Z)$ is continuous for all Z's, twice differentiable for $Z \neq 0$ and $\Lambda(0) = 0$; (ii) $\Lambda(Z)$ is strictly increasing; (iii) $-\Lambda(-Z) > \Lambda(Z)$ and $\Lambda'(-Z) > \Lambda'(Z)$, $\forall Z > 0$; (iv) $\Lambda''(Z) \leq 0$ for $Z > 0$ and $\Lambda''(Z) \geq 0$ for $Z < 0$. Properties (i) and (ii) imply monotonicity, i.e. utility is strictly increasing in the magnitude of the gain. Property (iii) captures the notion of loss aversion, i.e. the impact of a loss is greater than that of an equally sized gain. In particular, the latter inequality is the strong-form of loss aversion of Wakker and Tversky (1993). Overall, these properties imply that the representative consumer becomes more sensitive to deviations from her relative consumption when she is in a bad state, compared to a good state. Finally, property (iv) is referred to as diminishing sensitivity, i.e. the marginal effect of a gain or a loss diminishes with its magnitude. This translates into a gain-loss function whose curvature approaches zero as $Z \to \pm\infty$.[5]

To account for these properties we use an exponential gain-loss utility (Köbberling and Wakker, 2005):

$$\Lambda(Z) = \begin{cases} \frac{1 - \exp(-\theta Z)}{\theta} & \text{iff} \quad Z \geq 0 \\ \\ -\lambda \frac{\left[1 - \exp\left(\frac{\theta}{\lambda} Z\right)\right]}{\theta} & \text{otherwise} \end{cases} ; \quad \theta \geq 0, \quad \lambda > 1, \tag{3}$$

where θ determines the degree of diminishing sensitivity and λ is a parameter that indexes the degree of loss aversion. Note that for $\theta = 0$ we obtain a linear gain-loss function. Otherwise, (3) retains the property to be smooth at the reference point.[6] To gain further

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[4]For the time being, and without loss of generality, we describe reference-dependent preferences by reporting variables without subscripts.

[5]This specification offers a parsimonious framework to think about risk-aversion and loss aversion. Risk-aversion refers to the curvature of consumption utility, which determines the household's behavior for large gambles. Loss aversion refers to the magnitude of marginal utility for losses relative to gains, which determines the household's behavior for small gambles.

[6]This property is particularly useful in the perspective of linearizing the model economy.

intuition on the structure of reference-dependent preferences over consumption, Figure 1 plots the exponential gain-loss function and its first order derivative for different values of Z (x-axis) and λ. As predicted by Kahneman and Tversky (1979), loss aversion reflects the widely observed behavior that agents are more sensitive to losses than gains, resulting in a gain-loss function that is steeper in the first case (see the left-hand panel of Figure 1).

Insert Figure 1 here

As to the reference consumption level, we assume that consumers evaluate the distance between consumption utility and a function of the average consumption in the previous period: $X_{it} = C_{t-1}^{\gamma}$, where $\gamma \in [0,1]$ indexes the importance of external habit formation. Therefore we follow Yogo (2008), embedding habit formation in a reference-dependent model.[7]

We assume that the ith consumer, whose labor is remunerated at the real wage W_t, enters period t with cash holdings M_{it}, B_{it-1} one-period nominal bonds that pay R_{t-1} gross interest $(1+i_{t-1})$. Moreover, she receives the flow of dividends from a continuum of monopolistically competitive producers (Γ_{it}) and a lump sum transfer from the monetary authority (T_{it}):

$$P_t C_{it} + B_{it} + M_{it+1} \leq M_{it} + R_{t-1}B_{it-1} + P_t W_t N_{it} + \Gamma_{it} + T_{it}, \qquad (4)$$

where $\Gamma_{it} = \int_0^1 D_{ijt}d_j$ and D_{ijt} denotes the dividends of firm j paid to the ith household.

Differentiating the Lagrangian with respect to individual consumption (C_{it}) and taking the consumption reference level as external to the ith household returns the following Euler equation:

$$E_t \left\{ \frac{U_C\left(C_{it}, X_{it}\right)|_{s_{it}}}{U_C\left(C_{it+1}, X_{it+1}\right)|_{s_{it+1}}} \right\} = \beta E_t \left\{ \frac{R_t}{\Pi_{t+1}} \right\}, \qquad (5)$$

where $\Pi_t = 1 + \pi_t$ denotes the gross rate of inflation and s_{it} is used to indicate that the marginal utility of consumption depends on the gain-loss profile. Therefore, U_C changes depending on whether consumption is above or below its reference level.

The expected marginal rate of substitution between C_{it} and N_{it} reads as:

$$\frac{\chi N_{it}^{\eta}}{U_C\left(C_{it}, X_{it}\right)|_{s_{it}}} = W_t. \qquad (6)$$

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[7] Since the work of Abel (1990), external habit formation has become known as "catching up with the Joneses." External habit formation in consumption is usually introduced to account for the empirical persistence in the consumption process (Smets and Wouters, 2007). Unlike internal habit formation, this mechanism implies that households fail to internalize the externality of their own consumption on the utility of other households.

Equations (5) and (6) are paramount to understand how cyclical asymmetries in the transmission of monetary policy may arise in our model. Equation (5) regulates the intertemporal substitution between current and future consumption. A closer look at this relationship allows us to provide an intuition on the key mechanism at work in the dynamics of consumption. Below the reference level the curvature of the gain-loss function is lower than that observed when consumption is above its reference level, thus implying a higher elasticity of intertemporal substitution. Consequently, the marginal impact of the (real) interest rate on current consumption is also higher, relative to the alternative scenario, which implies that an unforecasted monetary policy innovation will have a greater impact on current consumption during negative growth cycles. Concurrently, equation (6) governs the intratemporal substitution between consumption and leisure. For a given $|V(C) - V(X)|$, the marginal rate of substitution between labor and consumption is lower when $V(C) < V(X)$. Under these circumstances households are more willing to cut on their leisure so as to increase consumption in the same period, compared to what happens when $V(C) \geq V(X)$. This implies a convex labor supply schedule that, in a general equilibrium perspective, induces a lower elasticity of firms' real marginal cost w.r.t. consumption (output). As a result the impact of real activity on inflation is attenuated when consumption is below its reference level, compared to the alternative scenario.

3.2 Supply Side

The final good is produced by perfectly competitive firms and requires the assembly of a continuum of intermediate goods, indexed by $j \in [0, 1]$, via the following technology: $Y_t = \left(\int_0^1 (Y_{jt})^{1 - \frac{1}{\theta}} \, dj \right)^{\frac{\theta}{\theta - 1}}$. Profit maximization leads to the demand function:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t \quad \forall j, \tag{7}$$

where $P_t = \left(\int_0^1 (P_{jt})^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}}$ is the price index consistent with the final good producer earning null profits. Total production equals aggregate consumption.

A continuum of firms produce intermediate goods. Each firm employs labor under a constant returns to scale technology:

$$Y_{jt} = Z_t N_{jt}, \tag{8}$$

where Z_t is a log-stationary total factor productivity shifter.

Following Christiano, Eichenbaum, and Evans (2005), we assume that firms set prices according to a variant of the Calvo (1983) specification. The probability that a firm can re-optimize its price in each period is $1 - \omega$. Firms that cannot re-optimize simply follow

a lagged-inflation indexation rule:

$$P_{jt} = \Pi_t^{\vartheta} P_{jt-1}; \quad \vartheta \in [0,1]. \tag{9}$$

Optimizing firms maximize their expected stream of future profits, subject to (7) and (8). The cost minimization problem is specified as follows:

$$\min_{N_{jt}} W_t N_{jt} + \Phi_{jt} \left[Y_{jt} - Z_t N_{jt} \right]. \tag{10}$$

The resulting real marginal cost is $RMC_t = W_t/Z_t$.

3.3   The Monetary Authority

The government sets the nominal rate of interest in accordance with a standard instrumental rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{r_R} \left[\left(\frac{\Pi_t}{\Pi} \right)^{r_\Pi} \left(\frac{Y_t}{Y} \right)^{r_Y} \right]^{1-r_R} \exp\left(\mu_t \right), \tag{11}$$

where μ_t is an $iid(0,1)$ monetary innovation.[8] We assume that the government adheres to this rule via open market operations, which are financed by means of money transfers to the households, such that any deficits are equal to zero, i.e. $T_t = B_t - R_{t-1}B_{t-1}$.

Assuming a symmetric policy function to stabilize output and inflation represents a convenient way to close the model and focus on the effects of introducing reference-dependent preferences into an otherwise standard framework. In Section 5 we focus on the monetary policy implications of this modeling assumption and formulate some policy prescriptions that account for the presence of reference-dependent preferences.

3.4   Model Solution

In the framework set out above households' utility is reference-dependent, i.e. its functional form depends on whether individual consumption is above or below the reference level (which is itself determined by aggregate past consumption). Furthermore, we assume that agents do not actually observe the reference level, X_t, but form beliefs on their relative position. In this respect, our assumption is similar in spirit to Veronesi (2004), who assumes that consumers do not observe their stock of habits, but possess a probability distribution on it.[9]

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[8]In the remainder, variables without time subscript denote the steady state values of their time-indexed counterparts.

[9]Veronesi (2004) introduces the concept of "beliefs-dependent" utility function (see also Geaneakoplos et al., 1989; Caplin and Leahy, 2001; Yariv, 2002) in a pure exchange economy, emphasizing the role of aversion to "state-uncertainty", which naturally arises in this context. In our case we consider beliefs-dependent preferences with neutrality to state-uncertainty. This allows us to partial out complications

Given the intertemporal dimension of households' maximization problem, we need to specify how agents' beliefs are formed, as these will determine the relevant arm of the reference-dependent utility function. We assume that agents' beliefs about their position with respect to the stock of habits, i.e. on whether they are going to be above or below the reference consumption level in future periods, evolve as an invariant Markov chain.[10] This assumption also allows us to overcome the endogeneity problem arising from the fact that at any given period t agents evaluate their utility based on a state-dependent function and have to select a future consumption plan that depends on their reference consumption level, which is itself determined in the same period.

Within this setting we can cast the model in the form of a Markov Switching Rational Expectations (MSRE) model (see Farmer, Waggoner and Zha, 2008, 2009). To this end, we linearize the key equations in the neighborhood of $C/X = 1$ and aggregate across individuals, assuming that in a symmetric equilibrium households make identical consumption-saving decisions.[11] Conditional on their relative consumption in each period, consumers' beliefs evolve in accordance with the following transition matrix:

$$Q = \begin{bmatrix} q_{AA} & q_{AB} \\ q_{BA} & q_{BB} \end{bmatrix}, \tag{12}$$

where A (B) stands for above (below) the reference level, $q_{AB} = \Pr\left(s_{t+1} = A \mid s_t = B\right) = 1 - q_{AA}$ and $q_{BA} = 1 - q_{BB}$. Equilibrium dynamics depends on agents' beliefs about their relative position with respect to the stock of habits. This implies that certain parameters can vary depending on the perceived "consumption state". As agents' beliefs evolve in accordance with (12), we have a standard MSRE model and it can be shown that the Minimal State Variable (MSV) solution is a Markov Switching VAR (see Farmer, Waggoner and Zha, 2008, 2009).

Specifically, the model is solved under the assumption that the representative agent correctly observes in which regime she is in the current period. Therefore, uncertainty pertains solely to the states she is going to be in future periods. The dynamics is one where agents weight the behavioral equations for the probabilities of regarding themselves

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that may arise from aversion to state-uncertainty.

[10] A natural extension would be to consider an endogenous mechanism of beliefs switching, so as to check whether the monetary transmission is altered and how the Central Bank should respond to smooth fluctuations in real activity and prices in such a scenario. This could be done, for instance, along the lines of Davig and Leeper (2008). At this stage of the analysis, however, we are mainly interested in showing how the adoption of reference-dependent preferences in a model of external habit formation allows us to reproduce the asymmetric reaction of prices and quantities to a monetary innovation. In this respect, the general principles examined in Section 3.1 remain valid irrespective of the switching mechanism.

[11] The difference between log-variables under sticky prices and their linearized steady state is denoted by lower case letters. We also assume, without loss of generality, logarithmic consumption utility. For further details, see Appendix A, where we report the linearized conditions for each of the four states taken separately. In Appendix B we report the model under the representation used to solve it through the generalized MSV approach developed by Farmer, Waggoner and Zha (2008, 2009), which is based on the canonical VAR representation of Sims (2002).

in any of the "states" of the economy. Therefore, a measurement equation relates the model to the observables. This can be written as $Y_t = \sum_i \wp(s_t = i | I_{t-1}) Y_{it}$, where $\wp(s_t = i | I_{t-1})$ is the probability of being at state i in period t given the information set in the previous period. The latter is recursively updated based on the Bayes' rule (see, e.g., Liu, Waggoner and Zha, 2010). The probability associated with each state can be thought as a set of beliefs that the representative agent associates with each state at the moment of making her consumption choice, so that the utility function is effectively beliefs-dependent (see Veronesi, 2004).

4 Asymmetries in the Transmission of Monetary Policy

4.1 Some Qualitative Insights

As discussed in Section 3.1, embedding reference-dependent preferences over consumption in a general equilibrium framework induces two major changes in the key equations describing the dynamics of real activity and prices: (i) first, the elasticity of intertemporal substitution is state-dependent, being higher (lower) in contraction (expansion); (ii) second, a state-dependent marginal rate of substitution between consumption and leisure induces an attenuation of the impact of real activity on firms' pricing behavior during contractions. The first property has been already explored by Yogo (2008) within an asset pricing framework. The second property is intimately related to the role of reference-dependent preferences in a general equilibrium setting. A globally convex aggregate supply function can be envisaged in this context, which retains the property to be steeper (flatter) during expansionary (contractionary) episodes. Analogous functional forms have been explored in the literature on the Phillips curve, emphasizing the role of large shocks relative to small ones for firms' pricing behavior.[12] In this respect, the existence of menu costs can rationalize a convex aggregate supply schedule. Nonetheless, it is interesting to note how embedding reference-dependent preferences on consumption in a general equilibrium setting allows us to provide a microfoundation that emphasizes the role of state-dependent degrees of real rigidity in the labor market equilibrium allocation, rather than nominal rigidities.

It is useful to explore these intuitions further by inspecting the linearized relationships describing the behavior of demand and supply across different states. Specifically, we compare the responses of output and inflation to a monetary policy shock when agents "naively" expect the economy to permanently stay in either expansion or contraction. This amounts to impose, just for the time being, $q_{AA} = q_{BB} = 1$. Liu, Waggoner,

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[12] See Laxton, Rose, and Tambakis (1999) for a review of the literature and the analysis of the monetary policy implications of assuming a convex aggregate supply.

and Zha (2009) refer to the difference in the dynamic responses under the model with naive vs. sophisticated consumers as the "expectation effect." As explained by Davig and Leeper (2007), such expectation effect generally plays an important role in the presence of autocorrelated disturbances, which is not the case of the monetary policy innovation we envisage. To provide some useful analytical insight we also set $\gamma = 0$, which corresponds to a case in which households consider the deviation of their consumption utility from the utility accruing from a constant consumption reference level. In this case we implicitly look at cyclical variations in output rather than at expansions/contractions in these cyclical movements. However, analogous implications carry over to the full model. We also assume an instrumental rule whereby the Central Bank responds solely to contemporaneous inflation and the output gap (i.e., $r_R = 0$) and no indexation in the pricing process (i.e., $\vartheta = 0$). The system of equations describing the model economy reads as:

$$y_t = E_t y_{t+1} - \phi(\xi_t)\left(i_t - E_t\pi_{t+1}\right), \tag{13}$$

$$\pi_t = \beta E_t\pi_{t+1} + \psi\left(\xi_t\right)y_t - \kappa\left(1+\eta\right)z_t, \tag{14}$$

$$i_t = r_\Pi\pi_t + r_Y y_t + \mu_t, \tag{15}$$

where ξ_t indexes the state-dependent parameters and $\kappa = \left(1-\omega\beta\right)\left(1-\omega\right)\omega^{-1}$. The system (13)-(15) admits the following solution under rational expectations:

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \mathbf{A}\left(\xi_t\right)\begin{bmatrix} z_t \\ \mu_t \end{bmatrix}, \tag{16}$$

where $\mathbf{A}\left(\xi_t\right)$ is a state-dependent 3×2 matrix of parameters that determines the marginal impact of the shocks on the vector of state and control variables. Let us now compare the response (on impact) of output and inflation to the monetary policy shock (μ_t) over expansions and contractions. As to output:

$$a_{12}\left(\xi_t^E\right) = -\frac{1}{r_Y + \kappa r_\Pi\left(\eta + 1 + \left(1-\alpha\right)\theta\right) + 1 + \left(1-\alpha\right)\theta}, \tag{17}$$

$$a_{12}\left(\xi_t^C\right) = -\frac{1}{r_Y + \kappa r_\Pi\left(\eta + 1 - \left(1-\alpha\right)\frac{\theta}{\lambda}\right) + 1 - \left(1-\alpha\right)\frac{\theta}{\lambda}}, \tag{18}$$

where ξ_t^E (ξ_t^C) denotes the expansionary (contractionary) state. As to inflation:

$$a_{22}\left(\xi_t^E\right) = -\frac{\kappa\left(\eta + 1 + \left(1-\alpha\right)\theta\right)}{r_Y + \kappa r_\Pi\left(\eta + 1 + \left(1-\alpha\right)\theta\right) + 1 + \left(1-\alpha\right)\theta}, \tag{19}$$

$$a_{22}\left(\xi_t^C\right) = -\frac{\kappa\left(\eta + 1 - \left(1-\alpha\right)\frac{\theta}{\lambda}\right)}{r_Y + \kappa r_\Pi\left(\eta + 1 - \left(1-\alpha\right)\frac{\theta}{\lambda}\right) + 1 - \left(1-\alpha\right)\frac{\theta}{\lambda}}. \tag{20}$$

These results readily provide us with some implications about the responses of output

and inflation to a monetary innovation in the two states. These are summarized in the following propositions.

Proposition 1 *In the model described by equations (13)-(15) under $q_{AA} = q_{BB} = 1$ (i.e., naive agents) the absolute response of output to a monetary innovation is greater under ξ_t^C than under ξ_t^E.*

The proof to this proposition is easily obtained by showing that

$$\left| a_{12} \left(\xi_t^C \right) \right| > \left| a_{12} \left(\xi_t^E \right) \right|,$$

which is always true for $\lambda > -1$ (recall that $\lambda > 1$ by definition). As hinted in Section 3.1, this result is solely driven by the emergence of state-dependent degrees of intertemporal substitution in consumption.

Proposition 2 *In the model described by equations (13)-(15) under $q_{AA} = q_{BB} = 1$ (i.e., naive agents) the inflation response conditional to the output gap response in the contractionary state is greater under ξ_t^E than under ξ_t^C.*

To prove this statement it is sufficient to show that:

$$\left| \frac{a_{22} \left(\xi_t^C \right)}{a_{12} \left(\xi_t^C \right)} \right| < \left| \frac{a_{22} \left(\xi_t^E \right)}{a_{12} \left(\xi_t^E \right)} \right|, \tag{21}$$

which is always true for $\lambda > -1$.

Proposition 3 *In the model described by equations (13)-(15) under $q_{AA} = q_{BB} = 1$ (i.e., naive agents) the absolute response of inflation to a monetary innovation is greater under ξ_t^E than under ξ_t^C if and only if $r_Y > \eta$.*

This statement can be proved by imposing $\left| a_{22} \left(\xi_t^C \right) \right| < \left| a_{22} \left(\xi_t^E \right) \right|$.

Whereas the result in Proposition 2 is exclusively driven by the emergence of a state-dependent labor supply schedule, the unconditional inflation response examined in Proposition 3 is determined by the interplay between the state-dependent elasticity of intertemporal substitution and the convex labour supply function. This point will be discussed in further detail in the remainder of this section. In the meanwhile, note that imposing $\lambda = -1$ implies that $\Lambda(\cdot)$ is no longer reference-dependent, as its functional form is the same regardless of the value of $V(C)$ relative to $V(X)$. Under these circumstances $a_{12} \left(\xi_t^C \right) = a_{12} \left(\xi_t^E \right)$ and $a_{22} \left(\xi_t^C \right) = a_{22} \left(\xi_t^E \right)$. Indeed, the responses of output and inflation are also the same for $\alpha = 1$, which amounts to ruling out reference-dependent preferences.

Assuming reference-dependent preferences implies state-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption that potentially generate competing effects on the responses of output and inflation following a monetary innovation. Specifically, the IS schedule displays a higher elasticity of intertemporal substitution in contractions as compared to expansions, as $\phi(\xi_t^C) > \phi(\xi_t^E)$. This result is in line with the analysis of Yogo (2008) and solely depends on the introduction of loss aversion in households' preferences. As to the state-dependent NKPC, it is straightforward to show that $\psi\left(\xi_t^C\right) < \psi\left(\xi_t^E\right)$ in Equation (14), meaning that the elasticity of inflation to the output gap is lower in contractions. This inequality is nothing but (21), which shows that the conditional response of inflation to the monetary innovation is always lower in contractions. However, this may not be enough to generate an attenuation in the absolute response of inflation to a monetary innovation. To see why this is the case note that in contractions lower real rigidity in the NKPC is counteracted by a greater amplification in the response of output, as predicted by Proposition 1. Therefore, the overall impact of a monetary shock on inflation depends on the relative magnitude of these competing forces. We have shown that $\left|a_{22}\left(\xi_t^C\right)\right| < \left|a_{22}\left(\xi_t^E\right)\right|$ does not hold unless the systematic policy response to the output gap (r_Y) is strong enough to overcome the amplification effect induced by the inverse of the Frisch elasticity of labour supply (η).[13]

To assess the amplification/attenuation effects induced by reference-dependent preferences on output and inflation over different cyclical phases it is useful to inspect the following ratios, for different values of λ:[14]

$$ M_y = \frac{a_{12}\left(\xi_t^C\right)}{a_{12}\left(\xi_t^E\right)}; \quad M_\pi = \frac{a_{22}\left(\xi_t^C\right)}{a_{22}\left(\xi_t^E\right)}; \quad M_{\pi|y} = \frac{a_{22}\left(\xi_t^C\right)/a_{22}\left(\xi_t^E\right)}{a_{12}\left(\xi_t^C\right)/a_{12}\left(\xi_t^E\right)}. \tag{22} $$

We set $r_Y = 0.5$, $r_\Pi = 1.5$, $\alpha = 0.5$, $\theta = 1$, $\kappa = 0.09$. As to η, we consider three values, namely 0, 0.5 and 1, so as to allow for $r_Y \lessgtr \eta$ and compare the extent of amplification/attenuation of inflation responses to the policy shock. Figure 2 reports numerical values for M_y, M_π and $M_{\pi|y}$. The amplification induced by reference-dependent preferences on output responses in contraction monotonically declines in the level of λ.[15] However, loss aversion induces substantial amplification of output responses in the

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[13]It must be stressed that, should the Central Bank be a pure inflation targeter (i.e., $r_Y = 0$), $r_Y > \eta$ is never attained. However, it is useful to note that for $r_Y > 0$ this condition is not unlikely to hold, if we consider the values of η that have generally been calibrated. Recall that η measures the elasticity of the marginal disutility of labor with respect to hours worked. Rotemberg and Woodford (1998) report evidence of low values of this elasticity, generally between 0.25 and 0.4, while McCallum (2001) suggests values closer to the lower bound. Kimball and Shapiro (2008) estimate this elasticity to be equal to 1, and stress that such a value is higher than most estimates previously obtained. According to Eusepi and Preston (2009) the elasticity is about 0.25. Notice also that $\eta = 0$, i.e., linear disutility of labor, would be consistent with the concept of "indivisible labor" which implicitly incorporates the extensive margin of labor adjustment in the baseline RBC setting (Hansen, 1985; Rogerson, 1988).

[14]It is also worth pointing out that envisaging different values of λ for a given value of θ amounts to impose different degrees of diminishing sensitivity above and below the reference level.

[15]To provide an intuition on why this happens it is useful to inspect Figure 1, where the curvature of

contractionary state, with values of $\left|a_{12}\left(\xi_t^C\right)\right|$ that can be from 106% to 44% greater than $\left|a_{12}\left(\xi_t^E\right)\right|$, for $\lambda \in [1,5]$.

Insert Figure 2 here

As to inflation, for $\eta = 0$ the attenuation effect experienced during contractionary phases increases in λ, ranging from 15% to 32% over the same interval for λ. For $\eta = 1$, the multiplier decreases over the range of values for λ. Within this interval inflation responses are only slightly greater in contractions, with an amplification that ranges from 18% to 8%. Note that the absolute magnitude of the amplification/attenuation effect on inflation responses for various parameter values and different cyclical phases is much lower compared to that induced on output responses by loss aversion. As a result, inflation responses to policy innovations during negative growth cycles are somewhat insulated from the strong amplification in the response of real activity through reduced real rigidity in the labor market equilibrium. The third panel of Figure 2 also reports the ratio between the responses of inflation in contraction and expansion, conditional to the output responses. Clearly, as implied by Proposition 2, conditional inflation responses are always much stronger in expansion.

4.2   Quantitative Analysis

To quantify the differential impact of monetary policy over contractions and expansions we compute numerical solutions to the linearized model economy. To this end, we need to assign some parameter values. We calibrate the model at a quarterly frequency. We set $\beta = 0.99$ and $\theta = 1$. The inverse of the Frisch elasticity of labor supply (η) is set to 0.25, while χ is such that steady state consumption equals one. As to the weight of habit formation, $\gamma = 0.9$. We do not have any direct empirical reference in the literature on dynamic general equilibrium models about the parameter indexing the degree of loss aversion. We set it in accordance with the value suggested by Tversky and Kahneman (1992), i.e. $\lambda = 2.25$, which is also in line with the estimate of Yogo (2008). Nominal rigidity is such that $\omega = 0.66$ and $\vartheta = 0.5$. As to the policy reaction function, we set $r_R = 0.7$, $r_Y = 0.5$ and $r_\Pi = 1.5$. We assume a purely transitory monetary policy shock, with $\sigma^\mu = 0.02$.

As to the transition matrix, Q, we use the CBO official output gap series and compute the average duration between the bottom of a recession and the top of an expansion.[16] Reverse arguments apply to compute the average duration of contractionary phases. The probabilities of switching across states are then retrieved as the inverse of the duration

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the left-hand of $\Lambda\left(\cdot\right)$ diminishes in λ, coming close to zero for high values of this parameter.

[16]The turning points are identified through the filter put forward by Bry and Boschan (1971).

of each phase: $q_{AB} = 0.135$ and $q_{BA} = 0.163$, respectively.[17] These values translate into strong persistence of both expansionary and contractionary states, whose probability is captured by the main diagonal elements of Q.

We are now ready to assess the impact of a monetary policy contraction on output and inflation. Figure 3 reports the responses to a monetary policy shock. In each graph, the solid line corresponds to the model under the expansionary state, while the dashed line refers to the contractionary one. A note of caution is warranted at this point. The graphs are responses of the system to a monetary policy shock conditional on other shocks being set at time zero. It is a convention to report variables in log-deviation from their steady state level. As such, a contractionary monetary policy shock inevitably implies a negative output response under both regimes, and not just in the contractionary one. However, in a more general setting we could envisage a composition of shocks hitting the economy, so that the monetary shock alone is not enough to influence the realization of a certain state. Therefore, a rising rate of interest can cause a contraction in economic activity without this being in contrast with the specification of the model.

<center>Insert Figure 3 here</center>

It is clear that output responses are greater during contractionary phases as opposed to expansionary ones, a result in line with the analysis of Section 4.1, which confirms the robustness of the mechanism at work in generating asymmetric responses of real activity between positive and negative growth cycles. As to the response of inflation during contractions, this is insulated from the higher responsiveness of real activity: overall, we cannot appreciate much difference in inflation responses over different states.[18] Thus, the model can qualitatively replicate the evidence discussed in Section 2.

5 Implications for Monetary Policy

Once it is recognized that monetary policy exerts an asymmetric impact on output and inflation, it is of obvious importance to explore how the policy maker should behave to properly account for these facts. To evaluate the policy implications of embedding reference-dependent preferences into a dynamic general equilibrium context it is useful to think about a scenario in which the monetary authority acts discretionally and takes the

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[17]We also explore the responses to a monetary contraction under the assumption that agents naively believe that a given state will last forever. These results are available, upon request, from the authors. However, as explained above, a transitory policy innovation implies no major difference in the dynamic responses under the model with naive vs. sophisticated consumers.

[18]This result is robust to alternative values of η relative to r_Y. For $r_Y > \eta$ we commonly observe a weak attenuation of inflation responses to monetary shocks taking place during contractions. Otherwise, we appreciate no major difference in the response of inflation between contractions and expansions for alternative parameterizations.

perceived expansionary and contractionary stages as given.[19] This is done for two main reasons. First, from a practical viewpoint, working under discretion allows us to envisage a sequence of static optimization problems. In this perspective, the Central Bank does not need to consider the probability of switching across different states. Second, from an institutional viewpoint it is hard to think about a Central Bank that makes any strictly binding commitment on its future policy action (Clarida, Gali, and Gertler, 1999).

To simplify the discussion we employ a purely forward-looking system. Once again, this amounts to set $\gamma = \vartheta = 0$, thus allowing for: (i) a gain-loss function in which deviations of consumption from a constant reference level ($X = 1$) are weighted; (ii) no indexation in price-setting. In each period the monetary authority chooses y_t and π_t to maximize

$$L_t = -\frac{1}{2} \left[\varrho y_t^2 + \pi_t^2 \right] + F_t, \tag{23}$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \psi\left(\xi_t\right) y_t + u_t, \tag{24}$$

taking $F_t = -\frac{1}{2} E_t \sum_{i=1}^{\infty} \beta^i \left[\pi_{t+i}^2 + \varrho y_{t+i}^2 \right]$ as given and where u_t is a stationary AR(1) exogenous cost shifter with autoregressive parameter ρ^u.[20] This shock is included to account for a meaningful policy trade-off between inflation and output stabilization. The solution to this problem returns the well known relationship:

$$y_t = -\frac{\psi\left(\xi_t\right)}{\varrho} \pi_t, \tag{25}$$

which means that whenever inflation is above the target the Central Bank should contract output below capacity (thus implementing a *leaning against the wind* policy). However, allowing for loss-averse preferences determines state-dependent degrees of real rigidity, captured by $\psi\left(\xi_t\right)$, that alter the nature of the trade-off between output and inflation stabilization depending on the perceived deviation of consumption from its reference level:

$$\psi\left(\xi_t^E\right) = \kappa\left(\eta + 1 + (1 - \alpha)\theta\right), \tag{26}$$

$$\psi\left(\xi_t^C\right) = \kappa\left(\eta + 1 - (1 - \alpha)\frac{\theta}{\lambda}\right). \tag{27}$$

It is evident how, for a given level of above-the-target inflation, the Central Bank does not need to contract output in the contractionary state as much as it should do in

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[19] In the setup we envisage the Central Bank does not exert any control on which regime is in place at any given point in time. In this respect, policy interventions are only "modest" in their scope (see Leeper and Zha, 2003). By contrast, consumption regimes solely depend on agents' beliefs, which are assumed to evolve exogenously. As a direct consequence the architecture of the model economy is practically insulated from the Lucas' critique.

[20] The welfare criterion expressed in (23) is widely used to capture the stabilization objective of the central bank over the target variables π_t and y_t. However, under reference-dependent utility we could envisage a state-depenedent welfare criterion that mimics households' preferences.

the expansionary one. Recall that in the contractionary state the real interest rate has a much greater impact on real activity, while inducing only moderate effects on inflation. Within this setting we can derive the optimal state-dependent interest rate rule under discretion:

$$i_t = r_\Pi\left(\xi_t\right) E_t \pi_{t+1}, \tag{28}$$

where

$$r_\Pi\left(\xi_t^E\right) = 1 + \frac{\kappa\left(1 - \rho^u\right)\left(\eta + 1 + \left(1 - \alpha\right)\theta\right)}{\frac{\varrho\rho^u}{1 + (1-\alpha)\theta}}, \tag{29}$$

$$r_\Pi\left(\xi_t^C\right) = 1 + \frac{\kappa\left(1 - \rho^u\right)\left(\eta + 1 - \left(1 - \alpha\right)\frac{\theta}{\lambda}\right)}{\frac{\varrho\rho^u}{1 - (1-\alpha)\frac{\theta}{\lambda}}}, \tag{30}$$

and it can be shown that

$$r_\Pi\left(\xi_t^E\right) > r_\Pi\left(\xi_t^C\right). \tag{31}$$

Therefore, the optimal policy under discretion imposes a higher degree of reactiveness to the expected rate of inflation in the expansionary state as compared to the contractionary one.

A useful way to illustrate the trade-off between inflation and output stabilization implied by the model is to construct the corresponding efficient policy frontier. Combining the IS with the aggregate supply schedule and the optimal policy under discretion returns the locus of points that characterize how the unconditional variances of output and inflation vary with Central Bank preferences, as indexed by ϱ. Figure 4 portrays the efficient policy frontiers for the two alternative scenarios under the calibration considered in the previous section.

Insert Figure 4 here

The graph clearly shows that in the expansionary state monetary policy can reach a policy frontier which is instead infeasible under the contractionary one. During contractions an aggressive monetary stance on inflation, i.e. a policy that aims at completely offsetting fluctuations in inflation, incurs into relatively higher costs in terms of output volatility. However, attaching increasing importance to output volatility gradually leads to similar costs in terms of inflation volatility across different states. A perhaps more important observation is that decreasing output variability by the same amount in contractions and expansions entails a lower increase in inflation volatility in the first case. This can be readily noticed by picking a point on both frontiers at the same level of σ_y, thus moving down along each locus so as to attain an analogous reduction in output variability: the relative increase in σ_π, which is measured on the x-axis, is greater under expansions than contractions. Therefore, pursuing a decrease in output variability

as a policy objective should be rather done during economic slowdowns, so has to exploit a lower pass-through from output to inflation and trigger lower pressures in terms of inflation volatility, provided that the Central Bank aims at remaining on the policy frontier.

Overall, these results suggest an alternative interpretation of the empirical evidence showing that the response coefficients in the Taylor rules adopted by various monetary authorities have changed over time and, in particular, have varied in connection with changes in the economic conditions.[21] Rabanal (2004), for instance, shows that the U.S. monetary policy has been somewhat less active during expansions, attaching higher weight to inflation responses and reflecting stronger interest-rate inertia. By contrast, during contractions the Federal Reserve has shifted its relative focus on controlling output growth. While a natural interpretation of shifts in the policy action is to attribute them to changes in the preferences of the policy maker, we show that state-dependent behavior in monetary policy making could be explained by changes in the preferences of the public.

6   Concluding Remarks

This paper provides a novel explanation of the vast empirical evidence showing that output and prices react asymmetrically to monetary policy innovations over contractionary and expansionary phases of the business cycle. It is a well-established finding that monetary policy has stronger effects on the GDP during contractions, as compared to expansions. As to prices, their response is generally not statistically different across different stages of the cycle.

These facts are consistent with a dynamic general equilibrium model featuring external habit formation in consumption in which we embed reference-dependent preferences. Specifically, households' utility partly depends on deviations of their consumption from a reference level below which switching to different preferences takes place. In line with the theory developed by Kahneman and Tversky (1979) losses in consumption utility loom larger than gains. This implies state-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption that generate competing effects on the responses of output and inflation following a monetary innovation. The model is solved by imposing that agents have imperfect observability of the stock of habits and their relative position with respect to this endogenous reference point. We also assume that agents' beliefs evolve as an invariant Markov chain, which allows us to deal with a Markov Switching Rational Expectations (MSRE) model (see Farmer, Waggoner and Zha, 2008, 2009). The qualitative and quantitative analyses return predictions that are

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[21]See, e.g., Bec, Salem, and Collard (2002), Rabanal (2004), Sims and Zha (2006), Assenmacher-Wesche (2006), Taylor and Davradakis (2006), Cukierman and Muscatelli (2008), Alcidi, Flamini, and Fracasso (2010).

in line with the empirical evidence. Output responses to a monetary contraction are greater in contractions as compared to expansions. Despite the amplification of output responses during negative growth cycles, inflation responses are insulated through lower real rigidity induced by reference-dependent preferences, so that we cannot detect major differences in inflation responses over different cyclical phases.

To the best of our knowledge, this is the first application of reference-dependent preferences over consumption in a dynamic general equilibrium perspective. This modeling device has already proven to be a useful extension of the consumption-based asset pricing model (CAPM) in various studies (e.g., Barberis and Thaler, 2003). Importantly, we show how loss aversion in consumption utility can also be useful at reconciling the otherwise standard DSGE workhorse with the widespread evidence on the asymmetric effects of monetary policy over different stages of the business cycle. We explore the state-dependent trade-off between inflation and output stabilization that naturally arises in this context. Optimal policy under discretion imposes a higher degree of reactiveness to the expected rate of inflation in the expansionary state, the reason being that during contractions an aggressive monetary stance on inflation, i.e. a policy that aims at completely offsetting fluctuations in inflation, incurs into relatively higher costs in terms of output volatility. In line with recent evidence on asymmetries in the policy reaction function (Bec et al., 2002; Rabanal, 2004; Taylor and Davradakis, 2006), the optimal policy imposes a stronger degree of reactiveness to the expected rate of inflation in the expansionary state.

It must be stressed that we have focused on a necessarily simplified model in which the mechanism of switching between different states follows an exogenous process. Within this setup we can provide a clear intuition of the key intertemporal and intratemporal mechanisms at work under reference-dependent utility in a general equilibrium framework, as well as their monetary policy implications. Allowing for an endogenous mechanism of switching should represent an interesting extension to this framework (Davig and Leeper, 2008). In this respect preemptive policy behavior, as emphasized by Bernanke (2004), might have lasting effects on expectation formation and the management of public beliefs would be a core part of the policy action. Moreover, as discussed by Andolfatto and Gomme (2003), beliefs over the actual and future monetary policy stance should play an important role in propagating various shocks and inducing varying degrees of persistence in the dynamics of nominal and real variables.

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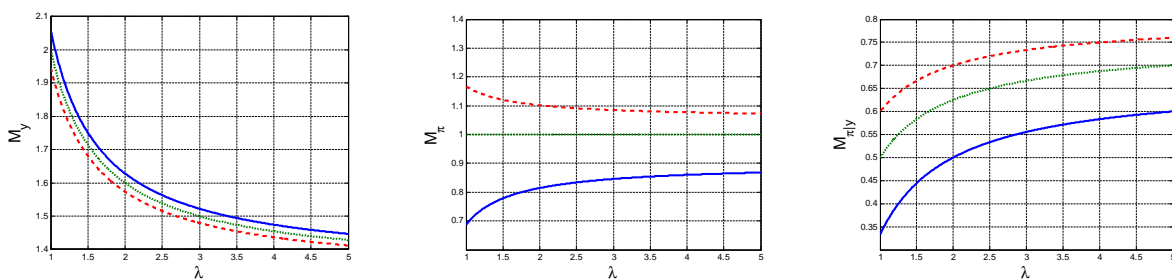
Figures

Figure 1. Reference-dependent preferences.

$\Lambda(Z)$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\Lambda^{'}(Z)$
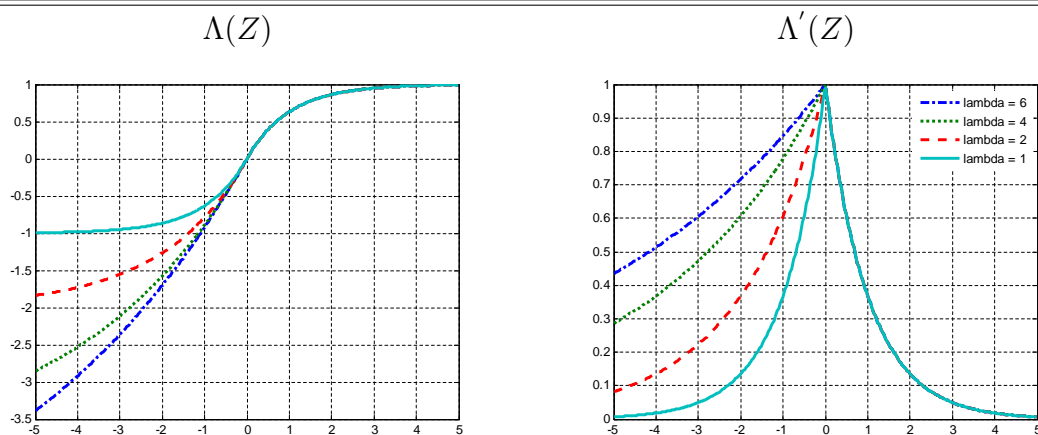
Notes. Gain-loss function (LHS panel) and its first-order derivative (RHS panel), for $\theta = 1$ and different values of λ.

Figure 2. M_y, M_π and $M_{\pi|y}$ for different values of λ.

Notes. We use the continuos line for $\eta = 0$, while the dotted and dashed lines refer to $\eta = 0.5$ and $\eta = 1$, respectively.
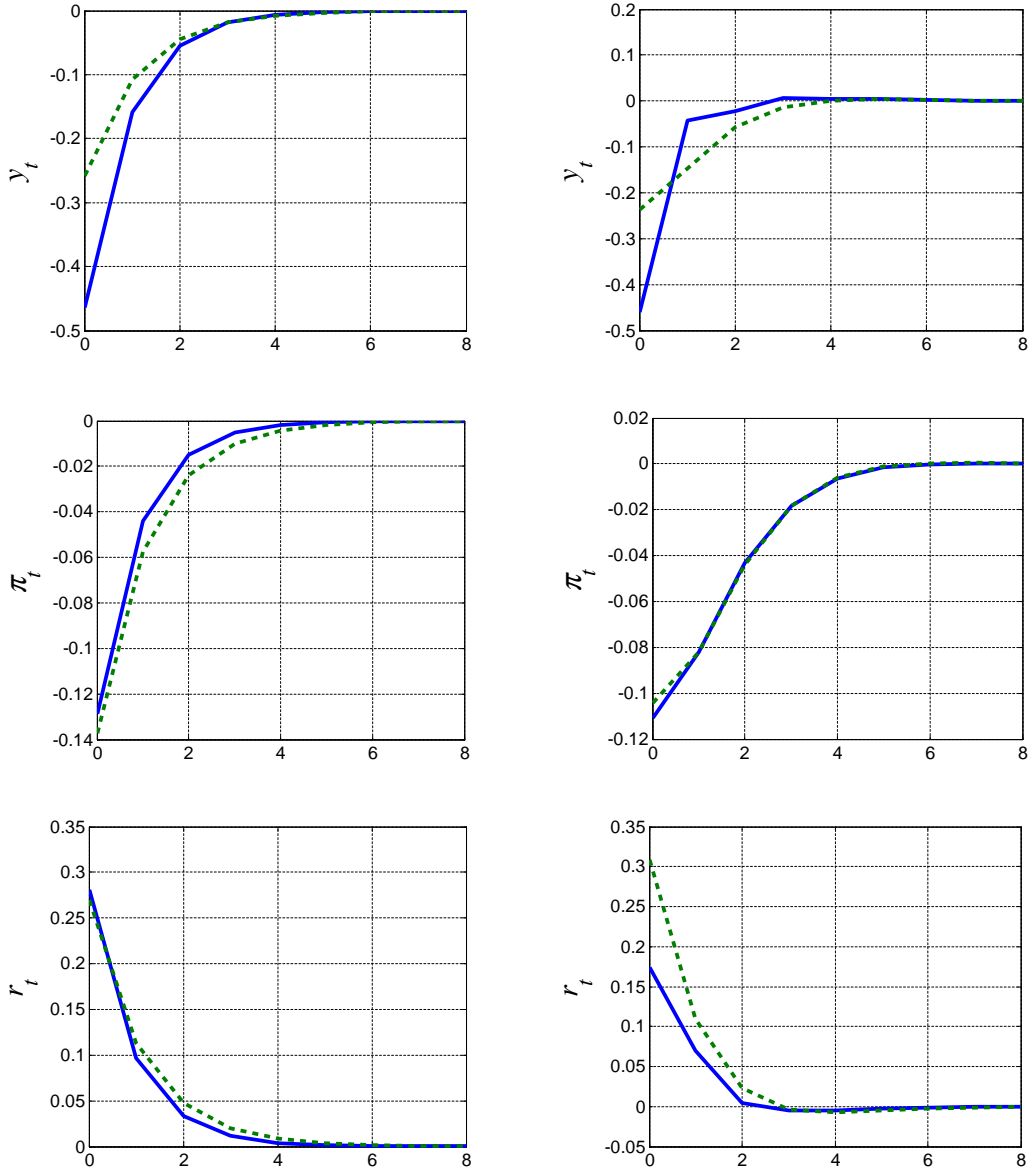
Figure 3. Impulse responses to a monetary policy shock (MS-DSGE model).

Forward Looking Model   Baseline Calibration

Notes. The responses are calculated by Montecarlo methods, using a procedure similar to that embodied in the generalized impulse response function (Koop et al., 1996; Liu et al., 2009). The dashed (continuous) line corresponds to the responses under the expansionary (contractionary) state. The forward looking model refers to the configuration with forward looking IS and NKPC and an instrumental rule with interest rate smoothing.
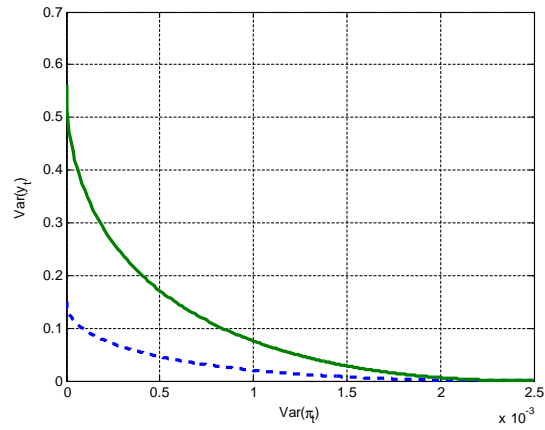
Figure 4. Efficient policy frontiers.

Notes. The dashed (continuous) line corresponds to the locus of different combinations of σ_π and σ_y, for varying values of ϱ in the expansionary (contractionary) state.

APPENDIX A: Log-linear State-Dependent System.

The IS Curve

We start by reporting the Euler equation consistent with each of the four cases:

$$\beta E_t\left\{\frac{R_t}{1+\pi_{t+1}}\right\}=\begin{cases} E_t\left\{\frac{\alpha C_{it}^{-1}+(1-\alpha)C_{it}^{-1}\exp(-\theta\ln H_{it})}{\alpha C_{it+1}^{-1}+(1-\alpha)C_{it+1}^{-1}\exp(-\theta\ln H_{it+1})}\right\} & \text{iff } \{s_{it}=A\}\cap\{s_{it+1}=A\} \\[4mm] E_t\left\{\frac{\alpha C_{it}^{-1}+(1-\alpha)C_{it}^{-1}\exp(-\theta\ln H_{it})}{\alpha C_{it+1}^{-1}+(1-\alpha)C_{it+1}^{-1}\exp\left(\frac{\theta}{\lambda}\ln H_{it+1}\right)}\right\} & \text{iff } \{s_{it}=A\}\cap\{s_{it+1}=B\} \\[4mm] E_t\left\{\frac{\alpha C_{it}^{-1}+(1-\alpha)C_{it}^{-1}\exp\left(\frac{\theta}{\lambda}\ln H_{it}\right)}{\alpha C_{it+1}^{-1}+(1-\alpha)C_{it+1}^{-1}\exp(-\theta\ln H_{it+1})}\right\} & \text{iff } \{s_{it}=B\}\cap\{s_{it+1}=A\} \\[4mm] E_t\left\{\frac{\alpha C_{it}^{-1}+(1-\alpha)C_{it}^{-1}\exp\left(\frac{\theta}{\lambda}\ln H_{it}\right)}{\alpha C_{it+1}^{-1}+(1-\alpha)C_{it+1}^{-1}\exp\left(\frac{\theta}{\lambda}\ln H_{it+1}\right)}\right\} & \text{iff } \{s_{it}=B\}\cap\{s_{it+1}=B\} \end{cases},$$

where $H_{it}=C_{it}/X_{it}$. After log-linearizing around $H=1$ we obtain the following state-dependent system of linearized IS curves:

$$y_t=\begin{cases} \frac{1+(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)\theta}E_ty_{t+1}+\frac{(1-\alpha)\gamma\theta}{1+(1-\alpha)(1+\gamma)\theta}y_{t-1}-\frac{i_t-E_t\pi_{t+1}}{1+(1-\alpha)(1+\gamma)\theta} & \text{iff } \xi_t=\{A,A\} \\[4mm] \frac{1-(1-\alpha)\frac{\theta}{\lambda}}{1+(1-\alpha)\theta\left(1-\frac{\gamma}{\lambda}\right)}E_ty_{t+1}+\frac{(1-\alpha)\theta\gamma}{1+(1-\alpha)\theta\left(1-\frac{\gamma}{\lambda}\right)}y_{t-1}-\frac{i_t-E_t\pi_{t+1}}{1+(1-\alpha)\theta\left(1-\frac{\gamma}{\lambda}\right)} & \text{iff } \xi_t=\{A,B\} \\[4mm] \frac{1+(1-\alpha)\theta}{1-(1-\alpha)\theta\left(\frac{1}{\lambda}-\gamma\right)}E_ty_{t+1}-\frac{(1-\alpha)\frac{\theta}{\lambda}\gamma}{1-(1-\alpha)\theta\left(\frac{1}{\lambda}-\gamma\right)}y_{t-1}-\frac{i_t-E_t\pi_{t+1}}{1-(1-\alpha)\theta\left(\frac{1}{\lambda}-\gamma\right)} & \text{iff } \xi_t=\{B,A\} \\[4mm] \frac{1-(1-\alpha)\frac{\theta}{\lambda}}{1-(1-\alpha)(1+\gamma)\frac{\theta}{\lambda}}E_ty_{t+1}-\frac{(1-\alpha)\gamma\frac{\theta}{\lambda}}{1-(1-\alpha)(1+\gamma)\frac{\theta}{\lambda}}y_{t-1}-\frac{i_t-E_t\pi_{t+1}}{1-(1-\alpha)(1+\gamma)\frac{\theta}{\lambda}} & \text{iff } \xi_t=\{B,B\} \end{cases},$$

where we have aggregated across individuals (imposing homogeneity) and used the goods market clearing condition, $Y_t=C_t$.

When it comes to linearize the model economy in the neighborhood of $C/X=1$, it is important to note that $\Lambda'(Z)$ presents an ordinary double point at $Z=0$. As such, $\Lambda'(Z)$ is not purely differentiable in that point, as also implied by property (i). Therefore, standard linear approximation techniques such as the Taylor expansion do not immediately apply in this case. However, we can resort to a first-order approximation of $\Lambda'(Z)$ by computing an affine global underestimator, thus determining the subgradients of the marginal utility function at $Z=0$. A subgradient determines a support hyperplane to the graph of the function under scrutiny. In such a case the corresponding subdifferential is a direct generalization of the differentiable case. For a convex and non necessarily

differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, the subdifferential at x_0 is defined as $\partial f(x_0) = \{g \in \mathbb{R} : \; f(x) \geq f(x_0) + \langle g, x - x_0 \rangle\}$. Thus, $g \in f(x_0)$ is subgradient in x_0.[22] In our case it is straightforward to notice that at $Z = 0$ there will be a single subgradient for each branch of the function under scrutiny. To gain intuition on this, we can re-write the marginal utility as $\Lambda'(Z) = \min\{\Lambda'_A(Z), \Lambda'_B(Z)\}$ for $Z \in \mathbb{R}$, where $\Lambda'_A(Z)$ and $\Lambda'_B(Z)$ are the functions that encompass the arms of marginal utility corresponding to $Z > 0$ and $Z < 0$, respectively. These functions are both convex. It is also easy to see that $\Lambda'_B(Z) > \Lambda'_A(Z)$ for $Z \in \mathbb{R}^+$ and $\Lambda'_B(Z) < \Lambda'_A(Z)$ for $Z \in \mathbb{R}^-$. Hence, our approach corresponds to a piecewise linear approximation in the neighborhood of $Z = 0$. Notice also that assuming a smooth gain-loss function $\Lambda(Z)$ at $Z = 0$ allows us to obtain a continuous first derivative function, which improves the approximation around the point $Z = 0$, compared to what would happen, say, with a linear gain-loss function, which implies a discontinuity at $\Lambda'(0)$.

Inflation Dynamics

After applying some trivial algebra we retrieve a log-linearized expression for the real marginal cost:

$$
rmc_t = \begin{cases} (\eta + 1 + (1 - \alpha)\theta)\, c_t - (1 - \alpha)\theta\gamma c_{t-1} - (1 + \eta)\, z_t & \text{iff} \;\; s_t = A \\[2em] \left(\eta + 1 - (1 - \alpha)\frac{\theta}{\lambda}\right) c_t + (1 - \alpha)\frac{\theta}{\lambda}\gamma c_{t-1} - (1 + \eta)\, z_t & \text{otherwise} \end{cases}.
$$

Thus the piecewise linear NKPC reads as:

$$
\pi_t = \varphi^f E_t \pi_{t+1} + \varphi^b \pi_{t-1} + \kappa \begin{cases} (\eta + 1 + (1 - \alpha)\theta)\, y_t - (1 - \alpha)\theta\gamma y_{t-1} - (1 + \eta)\, z_t & \text{iff} \;\; s_t = A \\[2em] \left(\eta + 1 - (1 - \alpha)\frac{\theta}{\lambda}\right) y_t + (1 - \alpha)\frac{\theta}{\lambda}\gamma y_{t-1} - (1 + \eta)\, z_t & \text{otherwise} \end{cases},
$$

where:

$$
\begin{aligned}
\varphi^f &= \frac{\beta}{1 + \beta\vartheta}, \\
\varphi^b &= \frac{\vartheta}{1 + \beta\vartheta}, \\
\kappa &= \frac{(1 - \omega\beta)(1 - \omega)}{\omega(1 + \beta\vartheta)}.
\end{aligned}
$$

---

[22] See Hiriart-Urruty and Lemaréchal (2001).

APPENDIX B: Setting the Model into the Form of Farmer, Waggoner and Zha (2008, 2009).

Importantly, dealing with reference-dependent habits necessarily implies that the parameters in the equilibrium system depend on states at time t and $t-1$. Thus, as in Liu, Waggoner, and Zha (2009), we need to define a composite regime that accounts for all possible realizations of states in t and $t-1$:

$$\xi_t = \{s_t, s_{t-1}\} = \{(A,A), (A,B), (B,A), (B,B)\}. \tag{32}$$

which implies four states, as reported in extensive form in Appendix A: an expansionary one, a contractionary one, and two turning points. In the remainder we pose stronger emphasis on the behavior of the model economy in expansions and contractions. To this end, we will refer to the expansionary state as that indexed by $\xi_t^E = \{A, A\}$, while the contractionary one is indicated by $\xi_t^C = \{B, B\}$.

We can write the model reported above into canonical form in the following compact form:

$$\underset{n_1 \times n}{A(\xi_t)} \underset{n \times 1}{x_t} = \underset{n_1 \times n}{B(\xi_t)} \underset{n \times 1}{x_{t-1}} + \underset{n_1 \times m}{\Psi} \underset{m \times 1}{\varepsilon_t},$$

where

$$x_t = [y_t, \pi_t, i_t, z_t, u_t, \mu_t, E_t y_{t+1}, E_t \pi_{t+1}]',$$

is the vector of variables to solve for and u_t denotes a log-stationary cost-push shock. Also,

$$\varepsilon_t = [\varepsilon_t^z, \varepsilon_t^u, \varepsilon_t^\mu]',$$

is a vector of fundamental shocks. The following notation applies:

$$
\begin{aligned}
n &= \quad \text{number of all variables for each regime (8),} \\
m &= \quad \text{number of fundamental shocks (3),} \\
n_1 &= \quad \text{number of equations in each regime (4).}
\end{aligned}
$$

After writing the model in the following form we can report the matrices A_{ξ_t} and B_{ξ_t} containing parameters that are possibly regime dependent:

$$
\begin{aligned}
y_t &= \phi_1(\xi_t) E_t y_{t+1} + \phi_2(\xi_t) y_{t-1} - \phi_3(\xi_t)(i_t - E_t \pi_{t+1}), \\
\pi_t &= \varphi^f E_t \pi_{t+1} + \varphi^b \pi_{t-1} + \psi_1(\xi_t) y_t + \psi_2(\xi_t) y_{t-1} - (1+\eta) \kappa z_t + u_t, \\
i_t &= r_R i_{t-1} + (1-r_R) r_\Pi \pi_t + (1-r_R) r_Y y_t + \mu_t,
\end{aligned}
$$

where:

$$\phi_1\left(\xi_t = 1\right) = \frac{1 + (1-\alpha)\theta}{1 + (1-\alpha)(1+\gamma)\theta}; \quad \phi_1\left(\xi_t = 2\right) = \frac{1 - (1-\alpha)\frac{\theta}{\lambda}}{1 + (1-\alpha)\theta\left(1 - \frac{\gamma}{\lambda}\right)},$$

$$\phi_1\left(\xi_t = 3\right) = \frac{1 + (1-\alpha)\theta}{1 - (1-\alpha)\theta\left(\frac{1}{\lambda} - \gamma\right)}; \quad \phi_1\left(\xi_t = 4\right) = \frac{1 - (1-\alpha)\frac{\theta}{\lambda}}{1 - (1-\alpha)(1+\gamma)\frac{\theta}{\lambda}},$$

$$\phi_2\left(\xi_t = 1\right) = \frac{(1-\alpha)\gamma\theta}{1 + (1-\alpha)(1+\gamma)\theta}; \quad \phi_2\left(\xi_t = 2\right) = \frac{(1-\alpha)\theta\gamma}{1 + (1-\alpha)\theta\left(1 - \frac{\gamma}{\lambda}\right)},$$

$$\phi_2\left(\xi_t = 3\right) = -\frac{(1-\alpha)\frac{\theta}{\lambda}\gamma}{1 - (1-\alpha)\theta\left(\frac{1}{\lambda} - \gamma\right)}; \quad \phi_2\left(\xi_t = 4\right) = -\frac{(1-\alpha)\gamma\frac{\theta}{\lambda}}{1 - (1-\alpha)(1+\gamma)\frac{\theta}{\lambda}},$$

$$\phi_3\left(\xi_t = 1\right) = \frac{1}{1 + (1-\alpha)(1+\gamma)\theta}; \quad \phi_3\left(\xi_t = 2\right) = \frac{1}{1 + (1-\alpha)\theta\left(1 - \frac{\gamma}{\lambda}\right)},$$

$$\phi_3\left(\xi_t = 3\right) = \frac{1}{1 - (1-\alpha)\theta\left(\frac{1}{\lambda} - \gamma\right)}; \quad \phi_3\left(\xi_t = 4\right) = \frac{1}{1 - (1-\alpha)(1+\gamma)\frac{\theta}{\lambda}},$$

$$\psi_1\left(\xi_t = 1\right) = \psi_1\left(\xi_t = 2\right) = \kappa\left(\eta + 1 + (1-\alpha)\theta\right),$$

$$\psi_1\left(\xi_t = 3\right) = \psi_1\left(\xi_t = 4\right) = \kappa\left(\eta + 1 - (1-\alpha)\frac{\theta}{\lambda}\right),$$

$$\psi_2\left(\xi_t = 1\right) = \psi_2\left(\xi_t = 2\right) = -(1-\alpha)\theta\gamma\kappa,$$

$$\psi_2\left(\xi_t = 3\right) = \psi_2\left(\xi_t = 4\right) = (1-\alpha)\frac{\theta}{\lambda}\gamma\kappa.$$

Consequently:

$$
\underset{6\times 8}{A\left(\xi_t\right)} = \begin{bmatrix}
1 & 0 & \phi_3\left(\xi_t\right) & 0 & 0 & 0 & -\phi_1\left(\xi_t\right) & -\phi_3\left(\xi_t\right) \\
-\psi_1\left(\xi_t\right) & 1 & 0 & \left(1+\eta\right)\kappa & -1 & 0 & 0 & -\varphi^f \\
-\left(1-r_R\right)r_Y & -\left(1-r_R\right)r_\Pi & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix},
$$

$$
\underset{6\times 8}{B\left(\xi_t\right)} = \begin{bmatrix}
\phi_2\left(\xi_t\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi_2\left(\xi_t\right) & \varphi^b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_R & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho^z & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho^u & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho^\mu & 0 & 0
\end{bmatrix},
$$

$$
\underset{6\times 3}{\Psi} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\sigma^z & 0 & 0 \\
0 & \sigma^u & 0 \\
0 & 0 & \sigma^\mu
\end{bmatrix}.
$$

Following Farmer, Waggoner, and Zha (2008), we can now expand the system under each regime into an expanded linear system to obtain the MSV solution.