The information content of real-time output gap estimates
An application to the euro area

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Abstract

The paper investigates real-time output gap estimates for the euro area obtained from various unobserved components (UOC) models. Based on a state space modelling framework, three criteria are used to evaluate real-time estimates, i.e. standard errors, unbiasedness and conditional inflation forecasts.

Real-time estimates from univariate moving average filters and from bivariate UOC models based on output and inflation are found to be rather uninformative. Extended models, which employ the information from cyclical indicators and factor inputs, however, improve substantially upon the former models in all criteria. The pessimism on the reliability of real-time output gap estimates expressed in earlier literature may therefore be overstated.

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1 Introduction

Empirical studies on monetary policy rules mostly employ output gap estimates, which make use of full-sample information to estimate historical gaps (e.g. Clarida et al., 1998; Rudebusch and Svensson, 2002). Policy makers, however, face a rather different situation, as they are confronted with estimates, which are based on more limited real-time information. Indeed, recent research has cast the reliability of real-time output gap estimates into doubt. Based on the analysis of public statements made by monetary authorities, Orphanides (2001) and Nelson and Nikolov (2001) have argued that gaps have been grossly mismeasured in real time during various historical episodes. Orphanides and van Norden (1999, 2001) have inspected subsequent revisions to real-time estimates from the recursive application of various univariate detrending methods. They found revisions to be large, in many occasions changing the real-time assessment in a qualitatively important way. Theoretical work has shown that high uncertainty of real-time estimates has strong implications for the weights to be attached to the output gap in optimal monetary policy rules (e.g. Ehrmann and Smets, 2002).

In this paper, I attempt to gain some insights on the information content of real-time estimates of the euro area gap and examine ways to improve the reliability of the estimates. I obtain real-time estimates from various unobserved components (UOC) models and moving average filters. Based on a state space modelling framework, I then apply three criteria to assess the statistical properties of the particular estimates. First, I address the uncertainty of real-time output gap estimates and evaluate their standard errors. Second, I propose a test for unbiasedness of real-time estimates based on the inspection of subsequent revisions to those estimates. Third, I examine the information content of output gap estimates for future inflation.

In applying these assessment criteria I attempt to partially escape the black-box nature of output gap estimates. It is well-known that output gap estimates often differ widely across methods (Canova, 1998). Many of these methods are based on purely statistical assumptions to identify the gap. As a consequence of a general lack of criteria to assess the particular estimates, the choice of methods is subject to considerable discretion. The uncertainties surrounding the real-time assessment of the gap may yet be significantly reduced from appropriate model selection procedures. In this sense, the paper relates to a few recent studies, which have focused on inflation forecasts and business cycle turning point predictions to evaluate output gap estimates (Camba-Mendez and Rodriguez-Palenzuela, 2001; Ross and Ubide, 2002). The state space modelling framework used in the paper provides various ways to extend on these papers.

In search for the kind of information to improve real-time estimates, the paper investigates various versions of UOC models. I start with univariate moving average filters, i.e. the widely used Hodrick-Prescott and bandpass filters, and with bivariate UOC models based on output and inflation, as proposed by Kuttner (1994) and Gordon (1997). I then inspect two multivariate extensions of the bivariate models, which also include capacity utilisation and factor inputs, i.e. the unemployment rate or total factor productivity. The main purpose of these extensions is to employ the co-movements of the gap with cyclical components in factor inputs and capacity utilisation and thereby to increase the amount of information in filtering.

The results indicate that increasing the amount of information in filtering may improve real-time estimates to a considerable extent. Real-time estimates from moving average filters and from bivariate UOC models turn out to be largely uninformative, being subject to large standard errors and significant biases. The large standard errors stem predominantly from filter uncertainty, resulting from the stochastic nature of the decomposition, whereas parameter uncertainty plays a lesser role. Filter uncertainty is yet sharply reduced for multivariate models, as the cyclical co-movements of output with capacity utilisation and factor inputs add important information to pinning down the real-time estimate. Overall, a model based on the output-capital ratio and total factor productivity performs best. Conditional inflation forecasts from the preferred model also compare favourably to forecasts from various other indicators confirming a significant role of the output gap in inflation dynamics. Unconditional inflation forecasts, however, remain quite uninformative owing to a limited predictability of output. The value of real-time output gap estimates as a leading indicator for future inflation thus appears to remain limited.

The paper is organised as follows. Section 2 reviews the state space modelling framework and discusses the various assessment criteria. Section 3 introduces the various UOC models. Section 4 presents output gap estimates for the euro area from those models and assesses their properties. Section 5 compares the inflation forecasting performance of the preferred model to various macroeconomic indicators. Section 6 concludes.

2 Criteria to assess output gap estimates

Variants of the following stylised aggregate demand - aggregate supply model have been widely used both for the analysis of monetary policy (e.g. Svensson and Rudebusch, 2002) and for estimating the gap (e.g. Kuttner, 1994, Smets and Gerlach, 1999).

$$y_t = y_t^* + y_t^C \quad (1)$$

$$\varphi(L) y_t^C = \varepsilon_t^y \quad (2)$$

$$\Delta\pi_t = \theta y_{t-1}^C + \varepsilon_t^\pi \quad (3)$$

L and Δ denote lag and difference operators, respectively.

From equation (1), output y_t (in logarithms) is decomposed into potential output y_t^* and the output gap y_t^C. Aggregate excess demand equation (2) describes the dynamics of the gap. Lag polynomial $\varphi(L) = 1 - \varphi_1 L - \varphi_2 L^2$ is restricted to have its roots outside the unit circle ensuring stationarity of the gap. Aggregate supply equation (3) relates the change in annual GDP deflator inflation, $\Delta\pi_t$, to the output gap, y_{t-1}^C, implying a permanent impact of the gap on inflation π_t. Crucially, the equation should be understood as a reduced form equation reflecting the joint impact of price mark-up and wage setting behaviour on domestic inflation (e.g. Layard et al., 1991:378ff).

Trend component y_t^* is modelled as a local linear trend (e.g. Harvey, 1989:45), i.e. a random walk with a stochastic slope term μ_t^y. The slope term is again specified as a random walk.

$$\begin{aligned} \Delta y_t^* &= \mu_{t-1}^y + \eta_t^y, & \eta_t^y &\sim \mathrm{NID}(0, \sigma_{\eta y}^2) \\ \Delta \mu_t^y &= \zeta_t^y, & \zeta_t^y &\sim \mathrm{NID}(0, \sigma_{\zeta y}^2) \end{aligned} \quad (4)$$

Level and slope innovations η_t^y and ζ_t^y are both Gaussian white noise with variances $\sigma_{\eta y}^2$ and $\sigma_{\zeta y}^2$. The local linear trend model contains a number of special cases. For $\sigma_{\zeta y}^2 = 0$ the slope is constant, $\mu_t \equiv \mu$, and the process reduces to a random walk with drift. The latter specification has been used by various authors (Kuttner, 1994; Smets and Gerlach, 1999). By setting $\mu_t \equiv 0$, the process may be further reduced to a random walk without drift. The specification of the model is completed by the assumption that innovations ε_t^y, ε_t^π, η_t^y, and ζ_t^y are normally independently distributed and mutually uncorrelated.

For estimation, the above UOC model is put in state space form, which comprises two equations. Measurement equation (5) relates observations $\mathbf{x}_t$ at time

t , $t = 1, \ldots, T$, to the state vector of unobserved components $\boldsymbol{\alpha}_t$. Transition equation (6) contains the stochastic laws of motion governing the dynamic behaviour of state vector $\boldsymbol{\alpha}_t$.

$$\mathbf{x}_t = Z\boldsymbol{\alpha}_t \tag{5}$$

$$\boldsymbol{\alpha}_t = T\boldsymbol{\alpha}_{t-1} + \mathbf{v}_t \tag{6}$$

The state space form for equations (1) to (4) may be specified as

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t^* \\ \mu_t^y \\ y_t^c \\ y_{t-1}^c \end{bmatrix}$$

$$\begin{bmatrix} \pi_t \\ y_t^* \\ \mu_t^y \\ y_t^c \\ y_{t-1}^c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \theta & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \varphi_1 & \varphi_2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1}^* \\ \mu_{t-1}^y \\ y_{t-1}^c \\ y_{t-2}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_t^\pi \\ \eta_t^y \\ \zeta_t^y \\ \varepsilon_t^y \\ 0 \end{bmatrix} .$$

Innovations $\mathbf{v}_t' = (\varepsilon_t^\pi, \eta_t^y, \zeta_t^y, \varepsilon_t^y, 0)'$ follow a multivariate Gaussian distribution with covariance matrix $\mathrm{cov}(\mathbf{v}_t) = \mathrm{diag}(\sigma_\pi^2, \sigma_{\eta y}^2, \sigma_{\zeta y}^2, \sigma_{\varepsilon y}^2, 0)$. Matrices T and $\mathrm{cov}(\mathbf{v}_t)$ depend on unknown hyperparameters $\xi = (\theta, \varphi_1, \varphi_2, \sigma_\pi^2, \sigma_{\eta y}^2, \sigma_{\zeta y}^2, \sigma_{\varepsilon y}^2)'$.

Given hyperparameters ξ together with assumptions on the distribution of initial state vector $\boldsymbol{\alpha}_1$ the Kalman filter (KF) recursion is initiated at time $t = 1$. The KF first, provides the likelihood function of model (5) and (6), which is used to obtain estimates $\widehat{\xi}$ of hyperparameters. Second, KF and smoother algorithms provide estimates of the state vector based on various information sets. Denote with $\mathbf{a}_{t|s} \equiv \mathbf{a}_{t|s}(\widehat{\xi})$ the estimate of $\boldsymbol{\alpha}_t$ based on observations up to time s, i.e. on information set $\mathcal{I}_s = \{\mathbf{x}_j\}_{j=1}^s$. Further, denote with $P_{t|s}(\widehat{\xi}) = \mathrm{cov}\left[(\mathbf{a}_{t|s} - \boldsymbol{\alpha}_t)|\widehat{\xi}\right]$ the mean squared error (MSE) of $\mathbf{a}_{t|s}$, conditional on estimates $\widehat{\xi}$. The KF recursion provides *real-time* estimates $\mathbf{a}_{t|t}$ of the state vector, based on past and current observations, together with mean squared errors $P_{t|t}(\widehat{\xi})$. Given the sequence of estimates $\{\mathbf{a}_{t|t}\}_{t=1}^s$ and $\{P_{t|t}(\widehat{\xi})\}_{t=1}^s$ up to a certain period s, smoother algorithms may then be applied to obtain, via backward recursions, *two-sided* estimates $\mathbf{a}_{t|s}$, together with $P_{t|s}(\widehat{\xi})$, for $t < s$. While the literature usually reports estimates

$\mathbf{a}_{t|T}$ based on full-sample information, policy makers apparently face real-time estimates $\mathbf{a}_{t|t}$.

Detailed discussions of state space models are provided, e.g. by Harvey (1989) and Gourieroux and Montfort (1989).

2.1 Filtering and information

It is a well-known feature of the filtering process that filter uncertainty declines, once the amount of information is expanded. This implies, first, that real-time estimates are subject to higher uncertainty compared to two-sided estimates. Second, gap estimates may potentially be improved by extending model (1) to (4) to include additional information.

Consider estimates $\mathbf{a}_{t|\mathcal{I}} = \mathbb{E}\left[\boldsymbol{\alpha}_t|\mathcal{I}\right]$ and $\mathbf{a}_{t|\mathcal{J}} = \mathbb{E}\left[\boldsymbol{\alpha}_t|\mathcal{J}\right]$ based on information sets $\mathcal{I}$ and $\mathcal{J}$, respectively, where $\mathcal{I} \subset \mathcal{J}$. Conditional on $\widehat{\xi}$, estimates represent best linear unbiased estimates of $\boldsymbol{\alpha}_t$ from the respective information set (e.g. Harvey, 1989:110ff). The property of best linear unbiasedness implies a zero expectation of revisions $\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}}$. Further, as shown in appendix A, revisions are orthogonal to estimate $\mathbf{a}_{t|\mathcal{I}}$ based on the smaller information set, $\mathbb{E}\left[(\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}})\mathbf{a}_{t|\mathcal{I}}\right] = 0$. As a consequence, MSE $P_{t|\mathcal{I}}(\widehat{\xi})$ can be decomposed into

$$P_{t|\mathcal{I}}(\widehat{\xi}) = P_{t|\mathcal{J}}(\widehat{\xi}) + \operatorname{cov}\left[\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}}|\widehat{\xi}\right] \tag{7}$$

and $P_{t|\mathcal{I}}(\widehat{\xi})$, hence, exceeds $P_{t|\mathcal{C}}(\widehat{\xi})$ by a positive semidefinite matrix.

The MSE of estimates $\mathbf{a}_{t|s}$ therefore declines as period s increases, because of $\mathcal{I}_t \subset \mathcal{I}_s$ for $t < s$. The higher uncertainty of real-time estimates is one potential source of the subsequent revisions to those estimates reported by Orphanides and van Norden (1999).

Uncertainty of both real-time and two-sided estimates may be also reduced by adding further information to model (1) to (4). Consider, for instance, survey data on capacity utilisation CU_t and assume that state-space form is extended by equation

$$(1 - \omega L)\,\mathrm{CU}_t = (b_0 + b_1 L) y_t^C + \varepsilon_t^C.$$

The equation contains two unobservable components, ε_t^C and y_t^C. The KF would attribute the prediction error in CU_t partially to y_t^C according to the estimated filtering rule and CU_t would thereby add information to estimates $\widehat{y}_{t|s}^C$. Intuitively, the efficiency gains would depend on the extent to which indicator CU_t is correlated with the output gap.

In this paper, I will examine the efficiency gains from adding further information to model (1) to (4). Coenen et al. (2001) propose to assess the gains from additional information by means of an entropy measure. The entropy measure, however, is limited to filter uncertainty, whereas estimates $\mathbf{a}_{t|s}$ may be affected as well from re-estimation of hyperparameters ξ for the extended models. I therefore will follow a more pragmatic approach in obtaining gap estimates from various extensions of model (1) to (4) based on re-estimation of hyperparameters for each model variant. I will use the reduction in the overall MSE, including parameter uncertainty as the principal measure of the uncertainty of the particular estimates. Further, I will examine the properties of revisions and the inflation forecasting performance of the particular models.

The remainder of this section discusses the various criteria to assess the properties of real-time estimates of the gap.

2.2 Standard errors

I follow a proposal of Hamilton (1986) to evaluate the impact of parameter uncertainty by Monte Carlo simulation. The method exploits the fact that the overall MSE can be decomposed into the contributions from filter and parameter uncertainty,

$$\operatorname{cov}\left[(\mathbf{a}_{t|s}(\widehat{\xi}) - \boldsymbol{\alpha}_t)\right] = P_{t|s}(\xi) + \operatorname{cov}\left[\mathbf{a}_{t|s}(\widehat{\xi}) - \mathbf{a}_{t|s}(\xi)\right]. \tag{8}$$

$P_{t|s}(\xi)$ represents filter uncertainty at true parameter values ξ, while the right hand term in equation (8) represents the impact of parameter uncertainty on estimates $\mathbf{a}_{t|s}(\widehat{\xi})$. Both are evaluated by Monte Carlo simulation, based on random draws $\xi^{(i)}$ from the asymptotic probability distribution of hyperparameters, $N(\widehat{\xi}, \widehat{\Sigma}_\xi)$, and evaluation of the expectational terms in equation (8) from $P_{t|s}(\xi^{(i)})$ and $\mathbf{a}_{t|s}(\xi^{(i)})$, respectively. The below results are based on 1000 random draws from the asymptotic distribution of hyperparameters as from a numerical estimate of matrix $\widehat{\Sigma}_\xi$.

2.3 Properties of revisions

Given the higher efficiency of two-sided estimates, several studies (Hahn and Rünstler, 1995; Orphanides and van Norden, 1999, 2001) have proposed to evaluate real-time estimates $\widehat{y}^c_{t|t}$ against full-sample estimates $\widehat{y}^c_{t|T}$ and to examine the size of revisions $\widehat{y}^c_{t|T} - \widehat{y}^c_{t|t}$. I will focus on revisions due to filter uncertainty. In such case, a systematic account of the sequence of revisions is given from the inspection of $\widehat{y}^c_{t|t+h}(\widehat{\xi}) - \widehat{y}^c_{t|t}(\widehat{\xi})$ for fixed $h > 0$, based on full-sample parameter estimates $\widehat{\xi}$.[1]

From equation (7) the MSE of revisions cov $\left[\widehat{y}^c_{t|t+h} - \widehat{y}^c_{t|t}|\widehat{\xi}\right]$ is a lower bound for MSE $P_{t|t}(\widehat{\xi})$ of real-time estimates. It can be estimated from the sample MSE of revisions. While this, in principle, adds little information to the inspection of $P_{t|t}(\widehat{\xi})$, it may be used to assess filter uncertainty of estimates from moving average filters. Those filters can in general not be put into state space form and, hence, $P_{t|s}(\widehat{\xi})$ is unobtainable. However, as argued in appendix A, equation (7) applies to linear filters.

In addition, the property $\mathbb{E}\left[(y^C_{t|t+h} - y^C_{t|t})y^C_{t|t}\right] = 0$, i.e. orthogonality of revisions to the real-time estimate, allows for a simple misspecification test for the unbiasedness of real-time against two-sided estimates. It has been conjectured that real-time output gaps give biased signals to economic policy possibly introducing a procyclical bias in the latter (Orphanides, 2001; Ross and Ubide, 2002). The test for unbiasedness of real-time against two-sided estimates may be based on a regression of revisions on real-time estimates.

$$\widehat{y}^c_{t|t+h} - \widehat{y}^c_{t|t} = b_0 + b_1 \widehat{y}^c_{t|t} + e_t \tag{9}$$

Unbiasedness of real-time estimates requires $b_0 = b_1 = 0$ and can be examined from standard χ^2-tests. Given the stylised fact of a length of business cycles of up to some 32 quarters, values of up to $h = 32$ are considered for this test. As residuals e_t are subject to serial correlation, hypothesis testing on b_0 and b_1 requires to adopt the Newey-West (1987) correction for covariance matrices (e.g. Clements and Hendry, 1998:57).

2.4 Inflation forecasts

From equations (1) to (3), potential output y^*_t is defined as the level of economic activity consistent with the absence of inflationary pressure from the utilisation of resources. In this sense, predictive power of the gap for inflation as from supply curve (3) is an essential precondition for the economic validity of gap estimates. Various studies have examined inflation forecasts from output gap estimates (e.g.

[1] It should perhaps be stressed that these revisions differ fundamentally from revisions due to recursive estimation of hyperparameters. The latter have been inspected by Camba-Mendez and Rodriguez Palenzuela (2001), while Orphanides and van Norden (1999, 2001) did not discriminate between the two sources of revisions.

Stock and Watson, 1999b; Camba-Mendez and Rodriguez-Palanzuela, 2001; Ross and Ubide, 2002).

I will inspect three types of forecasts, i.e. conditional and unconditional inflation forecasts and leading indicator regressions.

To examine the role of the output gap in explaining inflation dynamics, I inspect forecasts for inflation conditional on future observations of output and other real activity variables contained in the models. Partition $\mathbf{x}_t$ in equation (6) into $\mathbf{x}_t' = (\pi_t, \mathbf{x}_t^{(2)\prime})$. The conditional h-step ahead forecast of inflation is defined as the expectation of π_{t+h} conditional on past and current observations $\mathcal{I}_t = \{\mathbf{x}_s\}_{s=1}^t$ in conjunction with future observations on the subset $\left\{\mathbf{x}_{t+j}^{(2)}\right\}_{j=1}^h$. To derive the conditional forecast, an estimate of state vector $\boldsymbol{\alpha}_{t+h}$ based on this information set is required. It can be obtained from application of the KF to a modified state space form, where $\mathbf{x}_{t+j}$ is replaced by $\mathbf{x}_{t+j}^{(2)}$ over periods $t+j, \;\; j = 1, ..., h$ and the first row of matrix Z in equation (5) is eliminated, thereby ignoring future observations on inflation $\{\pi_{t+j}\}_{j=1}^h$ (Harvey, 1989:143f).[2]

To assess the information content of real-time output gap estimates as a leading indicator for future inflation, I will also examine unconditional forecasts and leading indicator regressions. The unconditional h-step ahead forecast, i.e. expectation $\mathbb{E}\left[\pi_{t+h}|\mathcal{I}_t\right]$ is obtained from iterating on measurement equation (6), $\mathbf{a}_{t+j|t} = T\mathbf{a}_{t|t+j-1}$, for $j = 1, \ldots, h$. Further, I will examine leading indicator regressions (e.g. Stock and Watson, 1999b)

$$\pi_{t+h} - \pi_t = \theta_h \widehat{y}_{t|t}^C + \sum_{i=0}^{k} c_i \Delta\pi_{t-i} + e_t \tag{10}$$

where the real-time gap estimate is used to forecast the h-step ahead change in inflation. Order of lags k is found from the Schwartz information criterion (SIC).

[2] Conditional forecasts from univariate filters are obtained in an equivalent manner, i.e. from sequences of estimates $\left\{\widehat{y}_{t+i|t+h}^C\right\}_{i=1}^h$ based on output observations up to period $t+h$.

Studies have based conditional inflation forecasts on two-step procedures, where either full-sample (Svensson and Gerlach, 2000) or sequences of one-sided gap estimates (Ross and Ubide, 2002) have been inserted into the supply curve. Both are based on improper information sets.

3 Models

I investigate two univariate moving average filters, two bivariate UOC models and two multivariate extensions of the latter. For all models, the supply is specified as

$$(1-\rho_1 L)\Delta\pi_t = \theta y^C_{t-k} + (1-\rho_2 L^4)\varepsilon^\pi_t. \tag{11}$$

which allows for a considerably more parsimonious specification than purely autoregressive dynamics. Lag k is chosen to maximise the likelihood in the particular models.

Studies often use the level of inflation as the dependent variable (e.g. Smets and Gerlach, 1999; Fabiani and Mestre, 2001). While standard Dickey-Fuller tests do not reject non-stationarity of inflation, it is argued that this reflects low power of these tests. I therefore conduct a test according to Leybourne and McCabe (1994), which takes stationarity as the null hypothesis. The test rejects stationarity at high significance levels.[3] This suggests that, at least for the data under consideration, the change in inflation should be used. This has the implication that output gap innovations ε^C_t have a permanent impact on inflation. I will yet also examine the significance of changes in the output gap in equation (11).

- *HP and BP filters*: I will apply the well-known HP (Hodrick and Prescott, 1980) and approximate bandpass (BP) (Baxter and King, 1999) filters with standard weights proposed in the literature, i.e. a smoothing parameter of $\lambda = 1600$ for the HP filter and cyclical frequency bounds of 2 to 32 quarters for the BP filter (Stock and Watson, 1999a). When it comes to estimating y^*_t in real-time, both filters become necessarily one-sided. It has been proposed to apply ARIMA extensions of the filters to obtain improved real-time estimates (e.g. Kaiser and Maravall, 2001). This amounts to extending the output series with ARIMA forecasts and application of the filters to the extended series. I will apply the ARIMA extension of both filters by extending the output series with forecasts of up to 12 quarters ahead generated from an ARIMA(2,1,0) model.

- *Model P* is solely based on supply curve

  $$(1-\rho_1 L)\Delta\pi_t = \theta\,[y_t - y^*_t] + (1-\rho_2 L^4)\varepsilon^\pi_t \tag{12}$$

  where y_t is predetermined and y^*_t is modelled as from equations (4). This model has been widely used to estimate the NAIRU (e.g. Gordon, 1997;

[3]The Leybournce McCabe (1994) test statistics for quarterly inflation rates amounts to .278, which compares to a 1% critical value of .216. The Augmented Dickey Fuller test statistics, in turn, is found with -1.20, which is insignificant at the 10% level.

Fabiani and Mestre, 2000), but may be applied to output as well. The model is of particular interest, because it shows the extent to which the supply curve by itself is successful in pinning down the gap estimate.

- *Model Y* represents the AD-AS model introduced in section 2. The model consists of equations (1), (2), (11) and (4).

I further investigate two multivariate extensions of model *Y*, which employ a Cobb-Douglas production function accounting framework,

$$\begin{aligned} y_t - k_t &= f_t + (1-\gamma)\,[e_t - k_t] \\ y_t^C &= f_t^C - (1-\gamma) u_t^C, \end{aligned}$$

where f_t, e_t, and k_t denote the logarithms of TFP, labour and the capital stock, respectively, and $\gamma = 0.35$ is imposed (see Wilman, 2002). Labour input $e_t = l_t - u_t$ may be further decomposed into the labour force l_t and the contribution from the unemployment rate $u_t = \ln(1 - \mathrm{UR}_t)$.[4] To find the output gap, the capital stock and the labour force are taken as predetermined, whereas u_t and f_t are, decomposed into trend and cyclical components.

In estimation, one of the three cyclical components can be skipped. Model *Y-UR* is based on output and the unemployment rate, whereas model *YK-F* uses the output-capital ratio and total factor productivity (TFP).

- *Model Y-UR* comprises equations (1), (2), (11) and (4), extended by a decomposition of u_t into its trend and cyclical components. The cyclical component u_t^C is related to the output gap. A further equation relates mean-adjusted survey data on capacity utilisation CU_t to current and lagged values of the unemployment cycle.

$$\begin{aligned} u_t &= u_t^* + u_t^C \\ (1-\phi L) u_t^C &= (a_0 + a_1 L) y_t^C + \varepsilon_t^u \\ (1-\omega L)\,\mathrm{CU}_t &= (b_0 + b_1 L) u_t^C + \varepsilon_t^C \end{aligned} \qquad (13)$$

The trend component u_t^* is again modelled as a local linear trend (4). Innovations (ε_t^y, ε_t^π, $\eta_t^y, \zeta_t^y, \varepsilon_t^u, \varepsilon_t^C, \eta_t^{ur}, \zeta_t^{ur}$) are assumed to be normally independently distributed and mutually uncorrelated. Models based on output and the unemployment rate, yet without capacity utilisation, have been investigated by various authors (Clark, 1987; Hahn and Rünstler, 1995; Apel and Jansson, 1999), including applications to the euro area (Fabiani and Mestre, 2001; Ross and Ubide, 2002).

[4] Note the approximation $\ln(1 - \mathrm{UR}_t) \approx -\mathrm{UR}_t$.

- *Model YK-F* uses the output-capital ratio in place of output plus total factor productivity (TFP). Equation (1) is replaced by

$$y_t - k_t = (y_t - k_t)^* + y_t^C \tag{14}$$

The trend component $(y_t - k_t)^*$ is again modelled as from equations (4), but restricted to follow a random walk without drift. Indeed, $y_t - k_t$ should not exhibit systematic drift, although it appears to have been subject to significant level shifts in recent decades (e.g. Blanchard, 1998). The model further comprises equations (2) and (11), together with equations (15) to obtain a decomposition of TFP and an to relate capacity utilisation to the TFP cycle.

$$\begin{aligned} f_t &= f_t^* + f_t^C \\ (1 - \phi L) f_t^C &= (a_0 + a_1 L) y_t^C + \varepsilon_t^f \\ (1 - \omega L)\,\mathrm{CU}_t &= (b_0 + b_1 L) f_t^C + \varepsilon_t^C \end{aligned} \tag{15}$$

The TFP trend component f_t^* is again modelled as a local linear trend, while the TFP cycle is related to the output gap. Innovations (ε_t^y, ε_t^π, $\eta_t^y, \varepsilon_t^u, \varepsilon_t^C, \eta_t^f, \zeta_t^f$) are assumed to be normally independently distributed and mutually uncorrelated.

Models *Y-UR* and *Y-KF* mimic the production function approach as widely used by policy institutions. Applications typically use univariate detrending methods to obtain cyclical components in the particular factor inputs. The HP filter is often used to obtain the TFP cycle, while various methods, including the *HP* filter and model *P* are used to estimate the NAIRU (e.g. Elmeskov, 1993; Giorno et al., 1995; de Masi, 1997; Bolt and Els, 1998; McMorrow and Röger, 2001). In the multivariate approach, the models are put into one single state space form with the observation vector in equation (5) for e.g. *YK-F* given by $\mathbf{x}_t = (\pi_t, y_t, f_t, \mathrm{CU}_t)'$. The information contained in the cyclical co-movements of the gap with factor inputs and CU_t is therefore employed in estimation.

4 Real-time output gap estimates for the euro area

The empirical analysis is based on quarterly data for the euro area from 1970Q1 to 2000Q4.[5] The main data are shown in Fig. 1. They display familiar facts about the longer-term evolution of factor contributions to output growth. In particular, the unemployment rate and the capital-output ratio have been subject to pronounced and apparently closely related level shifts in the 1970s and 1980s. Short-run variations in output growth predominantly reflect variations in TFP growth.

While national accounts data are subject to subsequent revisions, the data set is actually based on final releases of euro area national accounts data. The construction of real-time data vintages for the euro area is practically infeasible. Orphanides and van Norden (1999, 2001) report that data revisions explain a rather small fraction of overall revisions to real-time gap estimates for the US and Canada. Delays in data releases are no major constraint in real-time application of the models, as national accounts data are available with a delay of about one quarter, which then allows also to update capital stock and TFP data, whereas all other data are available with a shorter delay.

Table 1a reports parameter estimates for the various UOC models, whereas Table 1b presents various diagnostics. Coefficient θ of the output gap in supply curve (11) is significant at the 1% level for all models (Table 1b).[6] The output gap appears at lag 1 for *HP*, *BP* and *Y*, but at lag zero for the other models. While estimates of coefficient θ differ considerably across models, ranging from .05 to .20, this is partially attributable to differences in the size of the estimated output gaps. The long-run elasticity of the change in inflation with respect to the output gap amounts to $\theta(1-\rho_1)^{-1}$, which, for instance, from models *Y-UR* and *YK-F* is estimated with about 0.09. An output gap of one percent prevailing over a period of one year would therefore lead to a cumulated increase in annual GDP deflator inflation of some 0.35 percentage point.

I also examined various extensions of supply curve (11). First, changes in the output gap, Δy^C_{t-k}, were generally insignificant. While Δy^C_{t-1} was found significant

[5]The data are taken from the ECB area wide model. These data can be downloaded from the ECB website (http://www.ecb.int/); the data file is associated with Fagan et al., 2001). Capacity utilisation data from 1986 onwards are taken from the European Commission survey. For earlier periods they have been constructed as GDP-weighted averages of available national data.

[6]For *HP* and *BP* filters, estimates of supply curve (3) shown in Table 1a are based on two-sided filters. It is worth noting that for the one-sided *BP* filter coefficient θ becomes insignificant.

Figure 1: Data

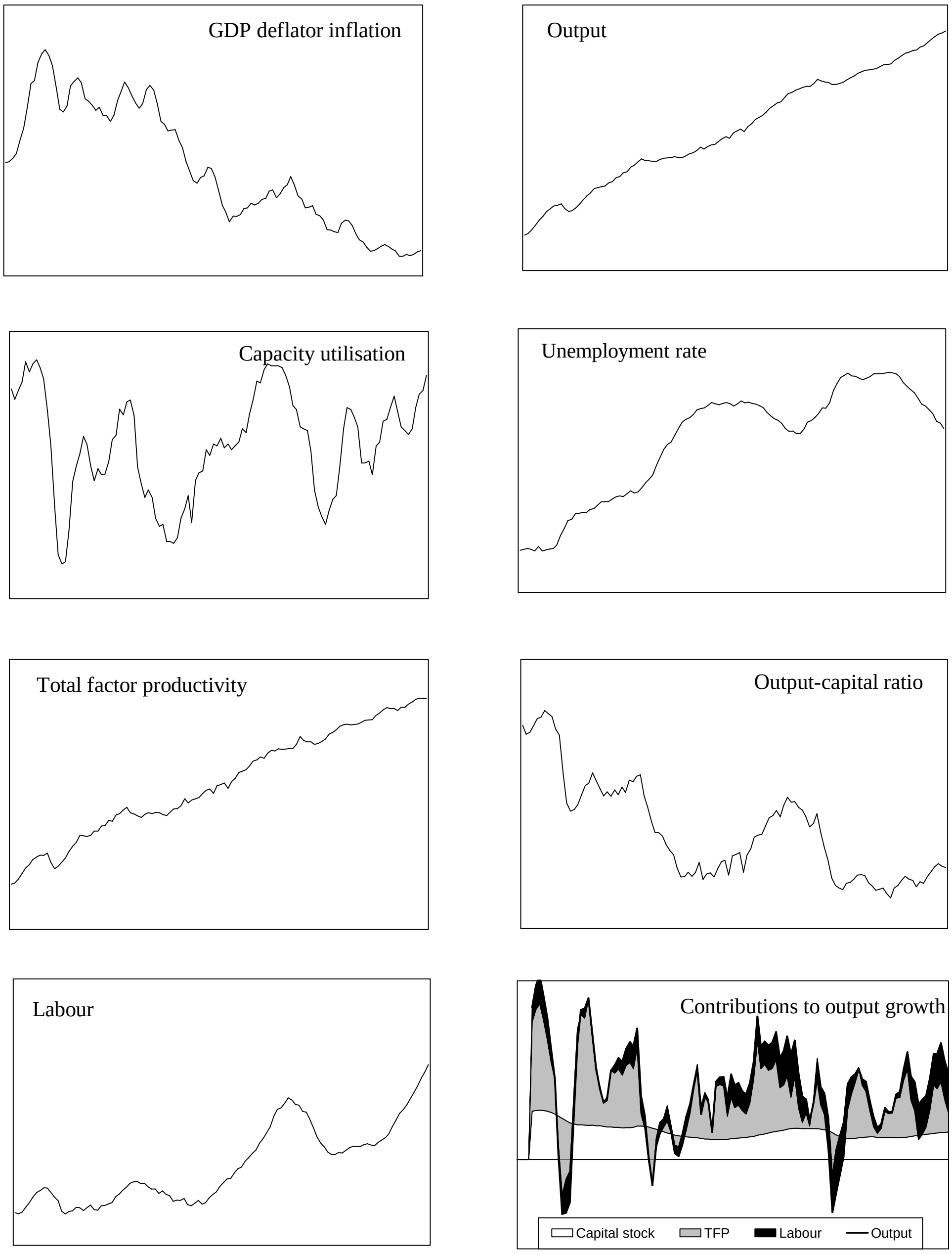


Table 1a: Parameter estimates

		HP	BP	P	Y	Y-UR	YK-F
Supply curve	θ	.2007	.1366	.0510	.1589	.0543	.0503
		(.063)	(.092)	(.010)	(.063)	(.019)	(.008)
	ρ_1	.4825	.4738	.4260	.2811	.4335	.4222
		(.086)	(.089)	(.073)	(.089)	(.096)	(.048)
	ρ_2	-.4817	-.4435	-.9500	-.9376	.7512	-.7909
		(.091)	(.092)		(.080)	(.082)	(.072)
	σ_ξ	.2397	.2483	.2583	.2468	.2795	.2793
				(.019)	(.019)	(.021)	(.019)
Cyclical dynamics	φ_1				1.5973	1.5612	1.2017
					(.10)	(.012)	(.025)
	φ_2				-.6999	-.6074	-.2374
					(.10)	(.018)	(.019)
	σ_ε^y				.2716	.3782	.5687
					(.074)	(.029)	(.035)
Capacity utilisation	ω					.7185	.4798
						(.028)	(.027)
	b_0					-7.370	.5029
						(1.08)	(.094)
	b_1					7.152	.6533
						(1.04)	(.130)
	σ_ε^C					.5277	.5400
						(.039)	(.038)
TFP / UR cycle	ϕ					.7692	.8594
						(.033)	(.018)
	a_0					-.1212	.9342
						(.018)	(.024)
	a_1						-.9068
							(.024)
	$\sigma_\varepsilon^{u/f}$					.0000	.0500
						(.020)	
Trend innovations	σ_η^y			.5560	.4574	.3672	.1208
				(.285)	(.050)	(.036)	(.012)
	σ_ζ^y			.0171	.0529	.0311	
				(.020)	(.032)	(.014)	
	$\sigma_\eta^{u/f}$					.1063	.0000
						(.014)	(.021)
	$\sigma_\zeta^{u/f}$					.0290	.0591
						(.008)	(.011)

Table 1b: Diagnostics

Inflation	HP	BP	P	Y	Y-UR	YK-F
R^2	.533	.505	.552	.558	.587	.626
Q(24)	27.61	29.19	21.36	23.85	17.94	20.26
JB	.38	.34	.25	.87	2.28	1.13
GDP	HP	BP	P	Y	Y-UR	YK-F
R^2				-.036	-.018	.061
Q(24)				26.62	24.45	24.80
JB				*6.11	4.98	3.17
UR / TFP					Y-UR	YK-F
R^2					.68	-.10
Q(24)					**50.08	28.09
JB					1.38	1.85

Likelihood ratio tests	HP	BP	P	Y	Y-UR	YK-F
$LR_1 : \theta = 0$	**10.18	**8.41		**7.24	**19.44	**9.12
$LR_2 : b_0 = b_1 = 0$					**70.64	**35.82
$LR_3 : a_0 = a_1 = 0$					**78.26	**304.46
$LR_4 : \beta = 0$	.62	.10	.02	**7.16	1.94	.11
$LR_5 : \mu = 0$	**24.10	**30.69		.52	.03	3.60

R^2 denotes the percentage of explained variance for the change in respective series. Q(24) and JB denote the Ljung-Box statistics for serial correlation in prediction errors over 24 lags and the Jarque-Bera test for normality of prediction errors. Those statistics are evaluated over the period of 1980Q1 to 2000Q4. LR_1 to LR_5 denote likelihood ratio tests on the specification of the supply curve and on cyclical dynamics. $LR_{4,5}$ test for the significance of output gap changes (β) and constant μ in supply curve (11). Both coefficients are not contained in the supply curve. For tests $LR_{2,3}$ equations for CU_t, u_t^C and f_t^C have been extended to include second-order autoregressive lags. Asterisks * and ** indicate significance at 5% and 1% levels, respectively.

for model Y (coefficient β in Table 1b), its inclusion did not improve the properties of gap estimates. Second, while various external cost factors (oil prices, nominal effective exchange rate) were found significant, their exclusion had negligible impacts on the below results. Third, I tested for the presence of a constant term μ in equation (11). The constant is insignificant for all models, apart from *HP* and *BP* filters (Table 1b). As argued by Adams and Coe (1990), given the absence of deterministic drift in inflation and hence a zero mean of $\Delta\pi_t$, the insignificance of the constant confirms a zero mean of the gap.

Turning to cyclical dynamics, estimates of lag polynomial $\varphi(L)$ generally give rise to pairs of conjugate complex roots with modulus close to one resulting in highly persistent cyclical responses to innovations ε_t^y. Parameters a_i and b_i, which account for cyclical co-movements as from (equations (13) and (15) are highly significant (Table 1b). Overall, the diagnostics of prediction errors is satisfactory. However, the Ljung-Box statistics indicates significant autocorrelation left in prediction errors for the unemployment rate in model *Y-UR*, which could not be removed by including higher order lags in cyclical dynamics.

4.1 Gap estimates

Real-time and two-sided output gap estimates from the various models are shown in Fig. 2. There arise large discrepancies among estimates from the various models, in particular as regards real-time estimates. In line with the findings of Orphanides and van Norden (1999, 2001), real-time estimates from different models would haven given rather different signals on the cyclical stance, in particular in the periods of 1982-1987 and 1993-1997. In the former period, model Y and univariate filters provide considerably smaller negative estimates than the other models. The differences among the particular estimates are also reflected in considerable differences in the standard deviations of both real-time and two-sided estimates. The first panel of Table 2a reports the standard deviations of output gap estimates, evaluated over the period of 1980Q1 to 2000Q4. For real-time estimates, standard deviations range from 0.64 (*BP*) to 2.28 (*P*). Given these differences among real-time estimates, the question arises whether the various assessment criteria introduced in section 2 provide ways to identify the more appropriate models.

Beforehand it is yet also of interest to have a closer look at the estimates from model *YK-F*. Estimates of trend and cyclical components of the particular factor inputs are shown in Fig 3. The unemployment rate contribution to the output gap tends to be somewhat larger compared to the one of TFP, while lagging behind the latter. The output-capital ratio shows a smooth downward trend, which tends to stabilise in the second half of the 1990s. Similarly, the rise in the trend unemployment rate comes to a halt in the early 1990s and shows signs of a reversal thereafter. Fabiani and Mestre (2001) obtained similar results on the evolution of the euro area trend unemployment rate.

Fig 2: Output gap estimates

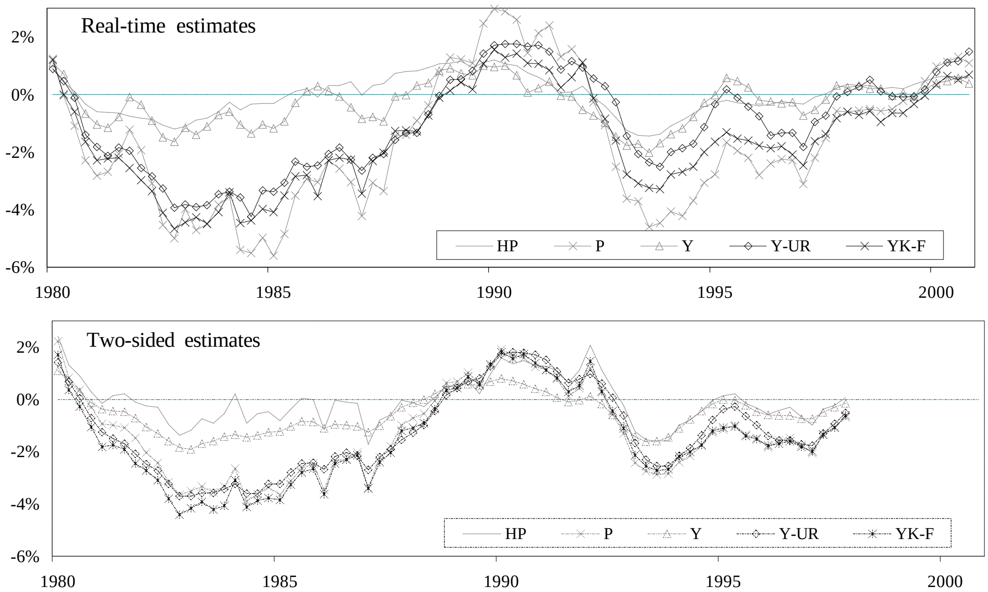
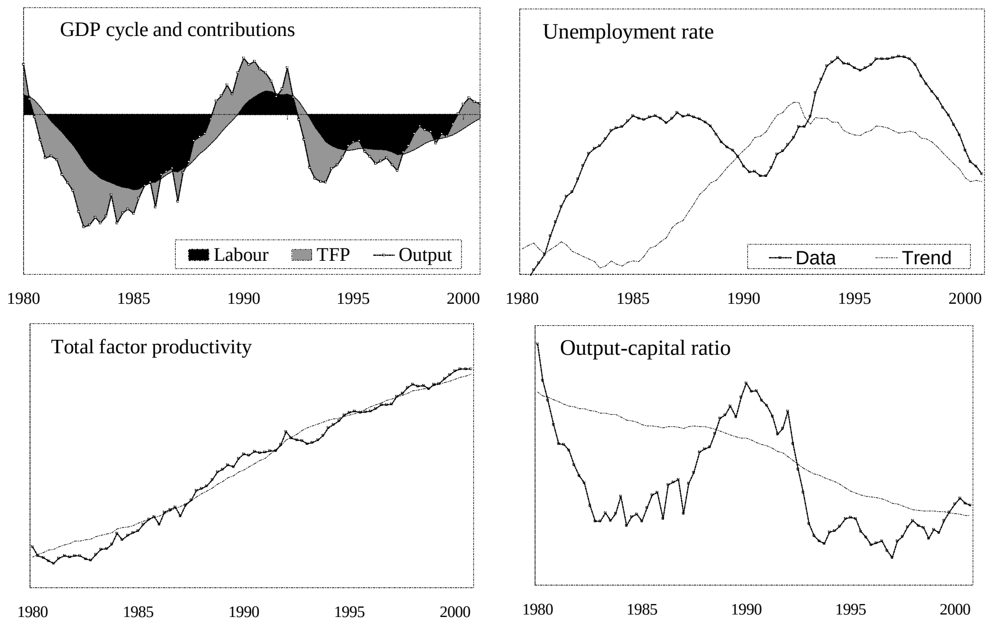


Fig 3: Estimates from model YK-F



4.2 Standard errors

Standard errors of real-time and two-sided output gap estimates, evaluated at the last observation, are shown in Table 2a. In addition to the overall standard error, evaluated at 2000Q4, the table also shows the contributions from filter and parameter uncertainty as from equation (8). Two main features of the results are of interest.

Table 2a: Standard errors

Standard deviations (*100)	HP	BP	P	Y	Y-UR	YK-F
Two-sided estimates $\left(\widehat{y}^C_{t\|t+12}\right)$	.88	.72	1.63	.76	1.60	1.72
Real-time estimate $\left(\widehat{y}^C_{t\|t}\right)$	.68	.64	2.28	.75	1.67	1.70
	(.77)	(.74)	(1.40)	(1.02)	(1.04)	(.99)
Standard errors (*100)			**P**	**Y**	**Y-UR**	**YK-F**
Real-time estimate $\left(\widehat{y}^C_{t\|t}\right)$			1.65	.88	.62	.41
			(1.01)	(1.16)	(.38)	(.24)
Filter uncertainty			1.60	.83	.56	.35
Parameter uncertainty			.37	.27	.27	.22
Two-sided estimates $\left(\widehat{y}^C_{t\|t+12}\right)$			.91	.66	.53	.34
			(0.56)	(.87)	(.33)	(.20)
Filter uncertainty			.87	.47	.42	.26
Parameter uncertainty			.25	.46	.33	.22

Numbers in brackets denote ratios to standard deviations of two-sided estimates.

First, real-time output gap estimates from multivariate UOC models are subject to considerably smaller standard errors compared to bivariate models, predominantly as a consequence of lower filter uncertainty. The differences appear particularly pronounced when compared to the standard deviations of the output gap estimates. Real-time estimates from models *YK-F* and *Y-UR* are subject to standard errors of 0.41 and 0.62 percentage point, respectively, of less than half the size of the standard deviations of output gap estimates. In contrast, the standard errors of real-time estimates from bivariate models P and Y amount to 1.65 and 0.88, respectively, of about the same size as the standard deviations of two-sided estimates. Indeed, real-time output gap estimates from both models would hardly be found significantly different from zero at any point of time in the sample period.

Second, standard errors of two-sided estimates then decline compared to those of real-time estimates as a consequence of lower filter uncertainty, whereas the impact of parameter uncertainty remains broadly unchanged. However, the reduction in filter uncertainty is considerably larger for bivariate models. For model P, in particular the overall standard error of two-sided estimates is nearly halved. The gains from the multivariate extensions thus appear to apply in particular to real-time estimates. This result highlights the importance of inspecting real-time estimates when it comes to evaluating the particular methods for policy purposes.

Table 2b: Standard errors under restricted parameter estimates

With supply curve	P	Y	Y-UR	YK-F
Real-time estimate $(\widehat{y}^C_{t\|t})$	2.18	1.09	.74	.40
Filter uncertainty	2.03	1.01	.65	.38
Parameter uncertainty	.81	.41	.35	.15
Two-sided estimates $(\widehat{y}^C_{t\|t+12})$	1.27	.70	.62	.31
Filter uncertainty	1.18	.47	.46	.27
Parameter uncertainty	.48	.52	.41	.16

Without supply curve	Y	Y-UR	YK-F
Real-time estimate $(\widehat{y}^C_{t\|t})$	1.97	1.98	1.46
Filter uncertainty	1.94	1.92	1.44
Parameter uncertainty	.37	.44	.25
Two-sided estimates $(\widehat{y}^C_{t\|t+12})$	1.77	1.88	1.44
Filter uncertainty	1.75	1.83	1.40
Parameter uncertainty	.31	.43	.29

Estimates under $\sigma_{\eta y}$=0.2$\times$ 10^{-2}.

Tables 2b further reports the results of two experiments on the impact of a restriction on parameter $\sigma_{\eta y}$ and on the information content of the supply curve in pinning down real-time estimates. One may, first, conjecture that the volatility of the trend component, i.e. the standard deviations $\sigma_{\eta y}$ of output level innovations η^y_t may affect filter uncertainty. Various applications, in particular of model P, have imposed restrictions on parameter $\sigma_{\eta y}$ to reduce the volatility of the output trend component (e.g. Gordon, 1997, Fabiani and Mestre, 2001). Indeed, estimates of $\sigma_{\eta y}$ differ considerably across models, ranging from 0.12×10^{-2} for model

YK-F to 0.56 $\times 10^{-2}$ for model *P* and are generally larger for bivariate models. To examine the impact of these differences, I re-estimated all models under the identical restriction of $\sigma_{\eta y} = 0.2 \times 10^{-2}$. As shown in Table 2b, this does not alter the above findings in any significant way.

Second, although model *P* provides inferior results, supply curve (11) nevertheless turns to play an important role in pinning down both real-time and two-sided estimates. The lower panel of Table 2b reports the standard errors of output gap estimates from variants of the models, where the supply curve has been omitted. In all cases, the omission of the supply curve results in a substantial increase in filter uncertainty of both real-time and two-sided estimates. The reason for the large increase in uncertainty, in particular as regards models *YK-F* and *Y-UR*, is the high persistence of the gap. As the moduli of the roots of $\varphi(L)$ are estimated with very close to one, the filter faces the task of decomposing output into a non-stationary trend and a highly persistent cycle. In this situation, the supply curve apparently adds important information to pinning down the level of potential output.

4.3 Properties of revisions

The upper panel of Table 3 shows the sample root mean squared error (RMSE) of revisions $\widehat{y}^c_{t|t+h}(\widehat{\xi}) - \widehat{y}^c_{t|t}(\widehat{\xi})$, evaluated over the period of 1980Q1 to 2000Q4.[7]

For *HP* and *BP* filters, revisions are found to be large, in particular when compared to the standard deviations of output gap estimates. The RMSE of 32-quarter ahead revisions comes close to the standard deviation of two-sided estimates and even exceeds the one of the real-time estimate. In contrast, for bivariate models the RMSE of revisions amounts to some 60% of the standard deviation of two-sided estimate. For models *Y-UR* and *YK-F* this declines further to a mere 20%. Generally, with the exception of *HP* and *BP* filters, the predominant part of revisions to real-time estimates takes place within the first 12 quarters. Revisions are shown in Fig 4.

The results of test (9) for unbiasedness of real-time estimates $\widehat{y}^c_{t|t}$ are reported in the lower panel of Table 3. Unbiasedness is rejected for all models, but the bias is

[7] This paper employs the Diffuse Kalman filter (deJong, 1991). The filter imposes a diffuse prior on non-stationary elements of initial state vector $\boldsymbol{\alpha}_0$, while stationary elements are initialised with their unconditional distribution. As a consequence, matrices $P_{t|t}(\widehat{\xi})$ are time-varying in the early part of the sample before gradually converging towards steady state values. Hence, estimates $\mathbf{a}_{t|t}$ from the early part of the sample should be omitted when evaluating the properties of real-time estimates.

smaller for multivariate models. For bivariate UOC models, coefficient b_1 in equation (9) is generally significantly negative, attaining values of some -0.2 to -0.3. While *HP* and *BP* filters pass the test with respect to $\widehat{y}^c_{t|t+12}$, real-time estimates are heavily biased against 32-quarter ahead estimates $\widehat{y}^c_{t|t+32}$. Coefficient b_1 is comparatively small, although significant for model *Y-UR* while it is insignificant for *YK-F*. For model *YK-F*, in turn, constant b_0 is significant owing to a systematic, albeit small, under-estimation of the output gap in real-time. This appears to be an unpleasant consequence of the assumption that the output-capital ratio trend follows a random walk without drift, as the estimates do not fully track in real time the downward trend in the output-capital ratio in the earlier part of the sample.

Table 3: Revisions: RMSE and bias tests

RMSE(*100)		HP	BP	P	Y	Y-UR	YK-F
$\widehat{y}^C_{t\|t+12} - \widehat{y}^C_{t\|t}$		.64	.61	1.12	.46	.28	.36
		(.73)	(.90)	(.68)	(.61)	(.18)	(.20)
$\widehat{y}^C_{t\|t+32} - \widehat{y}^C_{t\|t}$		.82	.75	1.09	.48	.31	.38
		(.93)	(1.04)	(.60)	(.63)	(.19)	(.19)

Bias tests		HP	BP	P	Y	Y-UR	YK-F
$\widehat{y}^C_{t\|t+12} - \widehat{y}^C_{t\|t}$	b_0	.001	.001	-.000	-.002	-.000	.002
		(.47)	(.45)	(0.00)	(2.09)	(.88)	(7.66)
	b_1	-.093	-.110	-.330	-.206	-.054	-.006
		(.53)	(.62)	(8.70)	(2.37)	(2.85)	(.31)
$\widehat{y}^C_{t+32\|t} - \widehat{y}^C_{t\|t}$	b_0	.001	.002	-.000	-.002	-.000	.003
		(.57)	(.93)	(0.58)	(2.16)	(.47)	(9.21)
b_1	b_1	-.269	-.317	-.328	-.162	-.076	.002
		(2.34)	(2.80)	(10.06)	(3.29)	(6.51)	(.36)

RMSE: numbers in brackets represent ratios to the std dev of respective two-sided estimates. Bias tests (9) : numbers in brackets denote t-values.

It should be recalled that ARIMA extensions are used for both *HP* and *BP* filters. Application of the filters without the ARIMA extension results in a deterioration of the properties of real-time estimates. I have also experimented with imposing smoother trends on *HP* and *BP* filters, i.e. higher values of smoothing parameter λ on the *HP* filter and the exclusion of high frequencies in the *BP* filter, with broadly unchanged results. van Norden (2002) shows that the gain function of the one-sided BP filter is subject to substantial compression, even if the ARIMA extension is applied.

4.4 Inflation forecasts

Tables 4a and 4b report the root mean squared errors (RMSEs) of forecasts for annual GDP deflator inflation relative to the RMSEs of random walk forecasts. The table reports results for 4, 8, and 12 quarter ahead forecasts. In addition to in-sample forecasts from 1980Q1 to 2000Q4, based on full-sample parameter estimates, I also inspect forecasts from 1991Q1 onwards, which are based on recursive estimates of parameters $\widehat{\xi}$.

Within-sample conditional forecasts from multivariate UOC models improve substantially on the random walk (RW) forecast at all horizons. However, only model *YK-F* also provides informative out-of-sample forecasts. The RMSE of the latter improves by some 30 - 40% on the random walk forecast.[8] Bivariate UOC models and moving average filters again perform worse than multivariate models with conditional forecasts, at best, improving only slightly on the random walk forecast within the sample, while falling short of the latter in out-of-sample forecasts. The particularly high RMSEs of forecasts from model *P* may be explained from the fact that estimates $y^*_{t|t}$ are not updated once future output observations are added, as the latter are exogenous to the model.

Fig. 5 shows 4 and 12 quarter ahead changes in inflation together with the respective conditional forecasts. Multivariate models, although less successful in predicting short-term movements in inflation during the 1980s, track longer-term inflation developments pretty well. Model *Y-UR* yet tends to overpredict inflation in the 1990s. Model *Y* and the *HP* filter largely fail in explaining the disinflation of the 1980s. The worse forecasting performance of model *Y-UR* in the 1990s may be related to some misspecification of the latter, as indicated by the significant autocorrelation in unemployment rate prediction errors (Table 1b). Fabiani and Mestre (2000) report difficulties in obtaining plausible euro area NAIRU estimates from bivariate UOC models. Overall, TFP may thus be more informative than the unemployment rate for the purpose of estimating the euro area output gap.[9]

[8] From Diebold-Mariano tests (Harvey et al., 1997), the improvements in the conditional forecasts from *Y-KF* on the RW forecasts and on forecasts from other models are in most cases significant at the 5% level.

[9] In this context it is worth noting that replacement of the output-capital ratio with output in model *YK-F* results only in a small deterioriation of the properties of estimates.

Table 4a: In-sample inflation forecasts (1980Q1 to 2000Q4)

RRMSE	HP	BP	P	Y	Y-UR	YK-F
Conditional						
4	.90	1.08	1.03	.93	.73	.63
8	1.03	1.28	1.33	.91	.66	.49
12	1.07	1.30	1.64	.98	.63	.46
Unconditional						
4				.94	.80	.73
8				.98	.84	.76
12				1.06	.88	.87
LI regressions						
4	.76	.90	.74	.85	.70	.64
8	.85	1.10	.85	.93	.86	.67
12	.96	1.18	.89	.98	.88	.70

Table 4b: Recursive inflation forecasts (1991Q1 to 2000Q4)

RRMSE	HP	BP	P	Y	Y-UR	YK-F
Conditional						
4	1.12	1.18	1.74	1.09	1.11	.73
8	.1.27	1.36	2.78	1.18	1.22	.64
12	1.23	1.29	3.32	1.21	1.12	.56
Unconditional						
4				1.03	1.21	.90
8				1.05	1.60	1.21
12				.99	1.60	1.33
LI regressions						
4	.99	1.07	1.23	.90	1.06	.84
8	1.24	1.14	1.49	1.07	1.38	1.09
12	1.26	1.12	1.39	.99	1.25	.97

RRMSE denotes the root mean squared error of the inflation forecast relative to the RMSE of the random walk forecast. RMSEs of in-sample random walk forecasts are given by 0.0099, 0.0124, and 0.0159 for 4, 8, and 12 steps ahead, respectively. RMSEs of out-of-sample random walk forecasts are given by 0.0065, 0.0081, and 0.0110 for 4, 8, and 12 steps ahead, respectively.

Fig 4: Revisions to output gap estimates

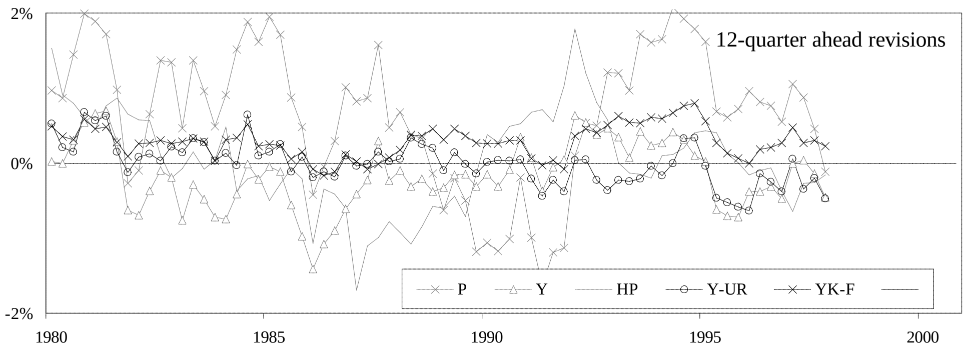
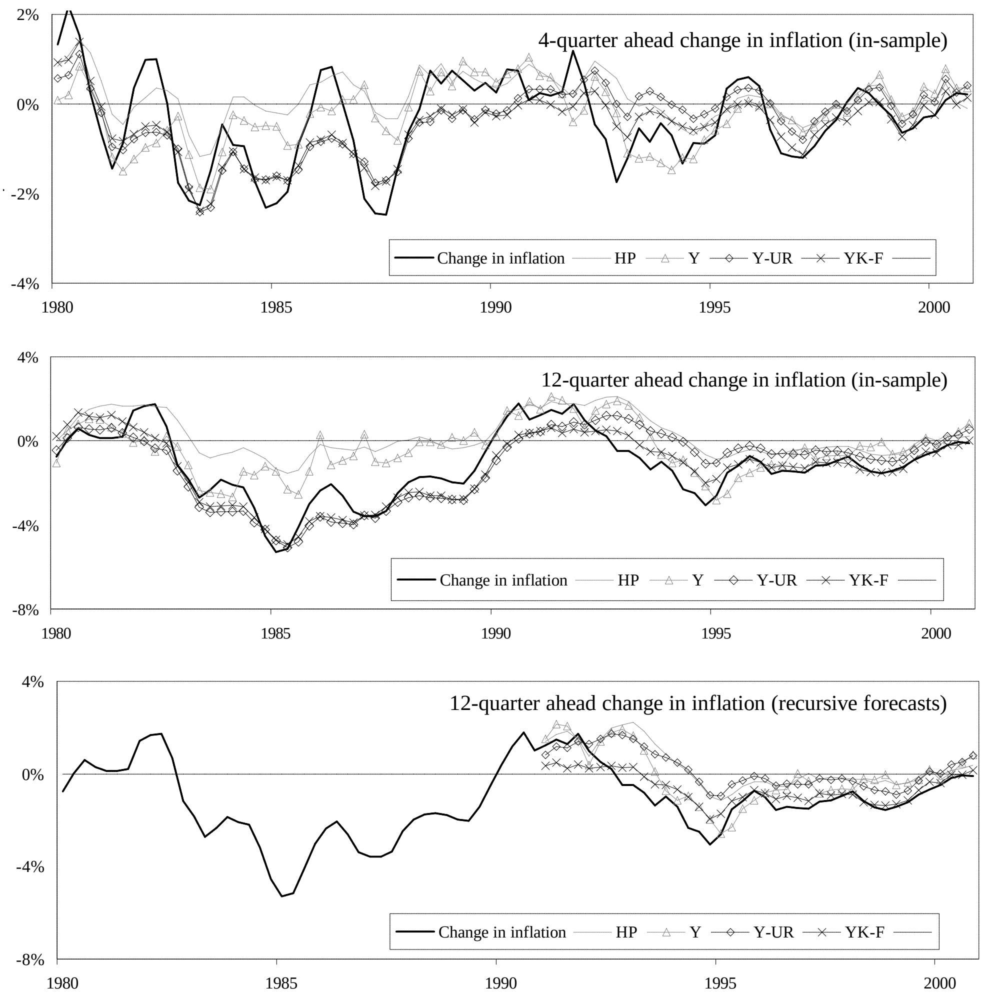


Fig 5: Conditional inflation forecasts



The results for unconditional forecasts and the leading indicator regressions are less appealing and the leading indicator properties of real-time estimates for future inflation appear to remain quite limited for all models. Within the sample, unconditional forecasts and leading indicator regressions from models *Y-UR* and *YK-F* show some improvement on the random walk forecast. Recursive out-of-sample forecasts, however, contain little information and only model *YK-F* beats the random walk forecast at a horizon of one year. The poor out-of-sample forecasting performance apparently emerges as a result of poor forecasts for the output gap. Indeed, leading indicator regressions tend to give a lower RMSE compared to unconditional forecasts.

5 Inflation forecasts: comparison with other indicators

This section compares inflation forecasts from model *YK-F* with those from various other indicators proposed in earlier work. I briefly examine two questions.

First, do inflation forecasts from model *YK-F* encompass forecasts based on standard regression equations including real activity variables? It has been argued, in particular, that the unemployment rate and capacity utilisation have high predictive power for inflation in the US (e.g. Staiger et al., 1996; Stock and Watson, 1999b). I will examine various real activity variables used in the above UOC models.

Second, how do inflation forecasts from model *YK-F* compare to forecasts from monetary and financial variables? Various studies report good leading indicator properties for inflation of money M3 growth, interest rates and real money gap (RMG) indicators (Gerlach and Svensson, 2000; Trecroci and Vega, 2000; Nicoletti Altimari, 2001). The RMG has been defined as the deviation $m_t - m_t^*$ of real money balances m_t from their long-run equilibrium value m_t^*, *evaluated at potential output.* Hence, RMG indicators themselves require an output gap estimate. I will examine RMG indicators constructed along the lines of Nicoletti Altimari (2001), based on the long-run money demand equation from Brand and Cassola (2000). I will use real-time estimates from either the *HP* filter or model *YK-F*.[10]

Again, I inspect both conditional forecasts and leading indicator regressions.

[10]The Brand-Cassola (2000) long-run money demand equation, $m_t = d_0 + d_1 y_t + d_2 l_t$ uses long-term rates l_t as a measure for the opportuntity cost of holding money. To calculate RMG-YK and RMG-HP indicators m_t^*, real-time estimates $\widehat{y}_{t|t}^*$ from model *YK-F* or the *HP* filter, respectively, are inserted for y_t.

5.1 Conditional forecasts

Table 5a reports the results for conditional forecasts from the various indicators, based on equation

$$(1-\rho_1 L)\Delta\pi_t = \mu + \theta_x(L)x_t + (1-\rho_2 L^4)\xi_t + e_t \tag{16}$$

Lengths of lag polynomials $\theta_x(L)$ were selected from the Akaike information criterion (AIC), which in many cases yielded better results than the SIC. In case of recursive out-of-sample forecasts, lag length estimation was also done recursively. Table 5a again shows the RMSE relative to the random walk forecast of conditional in-sample and recursive out-of-sample forecasts. Forecasts from model *YK-F* correspond to those reported in section 4.4. Table 5a also reports results from a shorter estimation period starting in 1980Q1, as data for money M3 are available only from 1980Q1.

To examine whether the particular indicators add information to forecasts from model *YK-F*, the table also reports forecast encompassing regressions

$$\Delta\pi_{t+h} = \lambda\pi^{(C)}_{t+h|t} + (1-\lambda)\pi^{(x)}_{t+h|t} \tag{17}$$

where $\pi^{(C)}_{t+h|t}$ and $\pi^{(x)}_{t+h|t}$ denote forecasts from model *YK-F* and from indicator x_t, respectively. If $\lambda = 1$, then forecasts from indicator x_t add no information to forecasts from model *YK-F*. If $\lambda = 0$, then forecasts from model *YK-F* add no information to forecasts from indicator x_t.

The results point to a superior performance of model *YK-F* compared to the various indicators, in particular as regards out-of-sample inflation forecasts. Most of the indicators are found to Granger-cause inflation. With respect to in-sample forecasts, some indicators, notably capacity utilisation, interest rates, and RMG indicators, show RMSEs close to those from model *YK-F*. Recursive out-of-sample forecasts yet in many cases fall grossly short of those from model *YK-F* and coefficient λ from forecast encompassing regression (17) is generally estimated with close to one. The major exceptions are GDP growth, when based on the full-range estimation period and capacity utilisation, when based on the shorter estimation period.

The bad out-of-sample performance of the various indicators may be related to instabilities in estimates of constant μ in equation (16). Table 5a shows a test for the stability of constant μ, i.e. Andrews' (1993) extension of the Chow breakpoint test, which allows for an unknown break point. For many indicators, estimates of

Table 5a: Conditional forecasts from alternative indicators

	Lags	Granger-causality	Stability test max	Stability test at	RRMSE In-sample (1980Q1 - 2000Q4) 4	8	12	RRMSE Out-of-sample (1991Q1 - 2000Q4) 4	8	12	Encompassing test Lambda Out-of-sample 4	8	12
Model YK-CF			0.55	90Q2	0.63	0.49	0.46	0.73	0.64	0.56			
Estimation starting at 1973Q1													
None			3.53	81Q4	0.94	0.99	0.94	1.01	0.94	0.86	1.43	1.40	1.41
Capacity utilisation	0	**8.59	**16.44	83Q3	0.81	0.78	0.77	1.14	1.57	1.90	0.93	1.02	0.90
UR (level)	0	*2.76	**8.73	85Q1	0.92	0.96	0.90	1.46	1.90	2.07	0.88	0.92	0.90
UR (change)	4	8.49	**8.82	81Q4	0.88	0.84	0.87	1.01	0.89	0.78	1.18	1.18	1.18
GDP growth	3	*8.97	1.85	81Q4	0.85	0.85	0.82	0.95	0.69	0.69	1.25	0.56	0.61
Long-term rate	3	*8.99	2.11	91Q4	0.73	0.61	0.52	1.28	1.46	1.38	1.09	1.02	1.03
Short-term rate	4	**32.67	0.69	87Q3	0.68	0.59	0.51	1.13	1.18	1.04	0.80	0.74	0.73
Spread	4	**20.04	2.10	81Q4	0.92	0.93	0.82	1.94	2.11	2.42	1.02	0.98	0.97
Estimation starting at 1980Q1													
None			4.98	87Q3	0.95	1.12	0.97	1.40	1.88	1.97	0.95	0.93	0.97
Capacity utilisation	0	**17.31	2.33	86Q1	0.63	0.46	0.39	0.75	0.75	0.97	0.58	0.49	0.98
UR (level)	0	0.75	**16.15	88Q1	0.92	1.02	0.97	1.38	1.81	1.89	0.90	0.96	0.98
UR (change)	1	2.20	3.05	87Q3	0.86	0.91	0.86	1.30	1.67	1.81	0.88	0.88	0.92
GDP growth	2	**8.71	3.26	88Q1	0.85	0.89	0.86	1.36	1.74	1.88	0.89	0.88	0.92
Long-term rate	3	**14.49	**8.07	88Q1	0.72	0.67	0.54	1.31	1.76	1.91	0.94	1.05	1.06
Short-term rate	3	*7.88	**8.96	88Q1	0.75	0.64	0.64	1.34	1.84	1.91	0.99	0.97	1.07
Spread	1	3.21	4.92	87Q3	0.91	1.02	0.98	1.43	1.83	1.82	1.08	1.01	0.99
Money M3 growth	3	*8.59	**10.90	87Q3	1.02	1.31	1.19	0.98	1.00	1.77	1.16	0.94	1.07
RMG (YK-CF)	1	**13.75	6.01	89Q4	0.72	0.73	0.61	1.45	1.56	1.65	1.28	1.37	1.08
RMG (HP)	1	**11.91	**7.55	87Q3	0.75	0.84	0.78	1.48	1.69	1.15	0.95	0.99	1.03

* and ** denote significance at 10% and 5% levels, respectively.
Critical values for the stability test are 6.05 and 7.51 for 10% and 5% significance levels, respectively (Andrews, 1992, Table 1)

μ are unstable with breakpoints at around either 1983 or 1988, related to either the beginning or the end of the 1980s disinflation period. Apparently, these instabilities arise as a consequence of the incapability of the various indicators to explain the downward trend in inflation in the 1980s, which is spuriously accounted for by μ. Such instability occurs notably for capacity utilisation and the unemployment rate. In contrast, the *YK-F* gap estimates appear to sufficiently explain the decline in inflation in this period in the absence of a constant term in supply curve (11).

To check the robustness of these findings, I have experimented with purely autoregressive versions of equation (16), using inflation either in levels or first differences. The above conclusions remain unchanged. If inflation was used in levels, autoregressive lag polynomials invariably displayed a root of very close to one, reflecting the nonstationarity of inflation in the sample and resulting in estimates very close to the specifications in first differences.

5.2 Leading indicator regressions

I follow Stock and Watson (1999b) and Nicoletti Altimari (2001) in examining the leading indicator properties of the various indicators from regressions

$$\begin{aligned}\pi_{t+h} &= \mu + \theta_x(L)x_t + \sum_{i=0}^{k} c_i \pi_{t-i} + e_t \\ \pi_{t+h} - \pi_t &= \mu + \theta_x(L)x_t + \sum_{i=0}^{k} c_i \Delta\pi_{t-i} + e_t,\end{aligned}$$

where inflation appears either in levels or first differences.

Table 5b reports the results for recursive out-of-sample forecasts from 1991Q1 to 2000Q4. Overall, the results depend heavily on the forecast horizon and on whether inflation in levels or first differences is used. Differences in root mean squared errors among forecasts from the various indicators are small. Inflation forecasts from model *YK-F* tend to slightly outperform those from other indicators at a horizon of one year, whereas they fall short of most other indicators at higher horizons.

From the level equation, as shown by Nicoletti Altimari (2001) money M3 growth and RMG indicators show the best performance at longer forecast horizons. For

Table 5b: Leading indicator regressions (recursive)

	RRMSE Level equation (1980Q1 - 2000Q4)			RRMSE Difference equation (1991Q1 - 2000Q4)		
	4	8	12	4	8	12
Model YK-CF	0.80	0.87	0.84	0.79	0.85	0.85
None	1.03	0.92	0.87	0.96	0.84	0.93
Capacity utilisation	0.84	0.85	0.79	0.82	0.92	0.82
UR (level)	0.84	0.76	0.72	0.89	0.81	0.71
UR (change)	0.96	0.84	0.76	0.83	0.76	0.79
GDP (growth)	0.99	0.80	0.80	0.81	0.86	0.76
Long-term rate	0.90	0.77	0.81	0.72	0.79	0.75
Short-term rate	1.05	0.88	0.77	0.88	0.89	0.80
Spread	1.01	0.87	0.79	0.88	0.86	0.90
Money M3 growth	0.97	0.70	0.67	1.10	0.83	1.10
RMG (YK-CF)	0.94	0.74	0.66	0.92	0.74	0.79
RMG (HP)	1.17	0.88	0.86	1.17	0.68	0.82

Estimation period starts at 1980Q1.

those indicators, the discouraging findings for conditional out-of-sample forecasts, notably the instabilities in the estimates of the constant, stand in some contrast to the results from leading indicator regressions. This raises some questions on the robustness of the latter results, in particular in view of the near non-stationarity of inflation in the present sample, the exploration of which yet going beyond the scope of this paper. Results from the difference equation are quite mixed. While most of the indicators beat the random walk forecast at all horizons, none of them does so sharply.

6 Conclusions

While the application of some of the criteria used in this paper requires a state space modelling framework, the findings also have implications for other methods. First, they suggest that univariate detrending methods face the fundamental difficulty that the information contained in output dynamics by itself is simply insufficient for obtaining informative real-time estimates. Second, real-time estimates may yet be substantially improved by employing the information contained in cyclical co-movements of output with factor inputs and stationary business cycle indicators. This finding has also implications for the production function approach as implemented by many institutions from the application of univariate detrending techniques to factor inputs. Third, the information from inflation as from a supply curve is essential as well. Fourth, while the results of conditional inflation forecasts suggest that the output gap has been an important determinant of inflation dynamics in recent decades, its leading indicator properties for future inflation nevertheless remain limited.

Overall, model selection based on the application of appropriate assessment criteria may reduce the uncertainties surrounding the real-time assessment of the gap to a substantial extent. From a policy point of view, these uncertainties nevertheless remain an important limitation. Any real-time assessment of the gap should therefore be embedded into broader based conjunctural analysis. Future research may further investigate the question whether gap estimates add genuine information to such analysis. It may be a useful strategy to assess the properties of conditional inflation forecasts from various models in regular terms and to put a larger weight on estimates, which produced the better forecasts in the recent past.

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Appendix A: Properties of revisions

Proposition 1 *Consider state space form (5) and (6) and assume that ξ is known. Consider two estimates $\mathbf{a}_{t|\mathcal{I}} = \mathbb{E}\left[\boldsymbol{\alpha}_t|\mathcal{I}\right]$ and $\mathbf{a}_{t|\mathcal{J}} = \mathbb{E}\left[\boldsymbol{\alpha}_t|\mathcal{J}\right]$ based on information sets $\mathcal{I}$ and $\mathcal{J}$, respectively, where $\mathcal{I} \subset \mathcal{J}$. It holds*

$$\mathbb{E}\left[\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}}\right] = \mathbf{0} \quad \textit{and} \quad \mathbb{E}\left[(\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}})|\mathbf{a}_{t|\mathcal{I}}\right] = \mathbf{0}$$

Further

$$\text{cov}\left[\mathbf{a}_{t|\mathcal{I}} - \boldsymbol{\alpha}_t\right] = \text{cov}\left[\mathbf{a}_{t|\mathcal{J}} - \boldsymbol{\alpha}_t\right] + \text{cov}\left[\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}}\right]$$

Proof. Conditional on hyperparameters ξ, estimates $\mathbf{a}_{t|()}$ represent best linear unbiased estimates, i.e. orthogonal projections of $\boldsymbol{\alpha}_t$ on the information set (Harvey, 1989: 110ff). The equivalence of conditional expectations and orthogonal projections in case of linear Gaussian processes is well known. In particular, for two Gaussian random variables z_1 and z_2 it holds that $z_1 - \mathbb{E}(z_1|z_2)$ is independent from z_2 (e.g. Gourieroux and Monfort, 1989(2):481f).

The first property, $\mathbb{E}\left[\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}}\right] = 0$ is trivial. Further, from the law of iterated expectations (Gourieroux and Monfort, 1989(2):463)

$$\mathbb{E}(\mathbf{a}_{t|\mathcal{J}}|\mathcal{I}) = \mathbb{E}\left[\mathbb{E}(\boldsymbol{\alpha}_t|\mathcal{J})|\mathcal{I}\right] = \mathbb{E}(\boldsymbol{\alpha}_t|\mathcal{I}) = \mathbf{a}_{t|\mathcal{I}}$$

and, hence, $\mathbb{E}(\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}})|\mathcal{I}) = \mathbf{0}$. Estimate $\mathbf{a}_{t|\mathcal{I}}$ can therefore be interpreted as the orthogonal projection of $\mathbf{a}_{t|\mathcal{J}}$ on $\mathcal{I}$. Finally, as $\mathbf{a}_{t|\mathcal{I}}$ lies in the space spanned by $\mathcal{I}$ it holds

$$\mathbb{E}\left[(\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}})|\mathbf{a}_{t|\mathcal{I}}\right] = \mathbb{E}\left[\mathbb{E}(\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}})|\mathcal{I})|\mathbf{a}_{t|\mathcal{I}})\right] = \mathbb{E}(\mathbf{0}|\mathbf{a}_{t|\mathcal{I}}) = \mathbf{0}$$

To prove equation (7) it is sufficient to show $\mathbb{E}\left[(\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}})(\mathbf{a}_{t|\mathcal{J}} - \boldsymbol{\alpha}_t)'\right] = 0$. Equivalently to above, this follows from $\mathbb{E}\left[(\mathbf{a}_{t|\mathcal{J}} - \boldsymbol{\alpha}_t)|\mathcal{J}\right] = 0$, whereas revisions $\mathbf{a}_{t|\mathcal{J}} - \mathbf{a}_{t|\mathcal{I}}$ lie in the space spanned by $\mathcal{J}$. ■

In case of unknown parameters ξ being estimated by maximum likelihood, the above orthogonality conditions hold asymptotically, as estimates $\widehat{\xi}$ converge in probability towards true parameter values. The above orthogonality conditions also apply to moving average filters in the sense that the latter can be understood as optimal Wiener filters for y_t^C. For the problem of estimating y_t^C from $y_t = y_t^* + y_t^C$, where both y_t^* and y_t^C represent certain linear Gaussian processes, the Wiener filter provides best linear unbiased estimates $y_{t|s}^C$.

Hence, proposition 1 applies. Harvey and Jäger (1993) have shown, for instance, that the HP filter represents the Wiener filter of a UOC model consisting of a local linear trend y_t^* under the restriction of $\sigma_{\eta y} = 0$ and white noise y_t^C. For a discussion of the Wiener filter and its relationship to the Kalman filter see Priestley (1981:775ff).

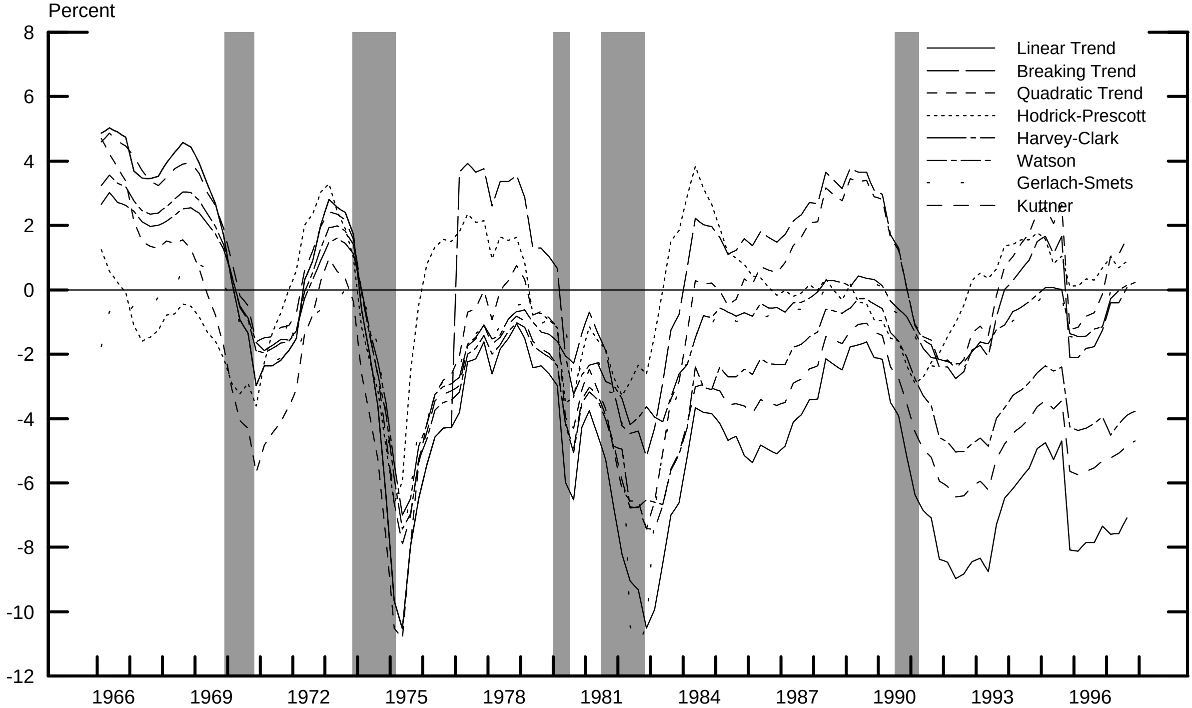
Bias test (9) can be conducted for any stationary element of the state vector. Under H_0, OLS estimates $\widehat{\mathbf{b}}$ converge in distribution to $N(\mathbf{0}, \Omega)$, and the statistics $\widehat{\mathbf{b}}'\widehat{\Omega}^{-1}\widehat{\mathbf{b}}$, where $\widehat{\Omega}$ is a consistent estimate of Ω, is asymptotically Chi-squared distributed (Gourieroux and Montfort, 1989(2): 153f). Residuals u_t display serial correlation, and an autocorrelation-consistent estimator of Ω should hence be used (e.g. Newey and West, 1987) to conduct the test.

Finally, revisions $\mathbf{a}_{t|s} - \mathbf{a}_{t|t}$ can be related to prediction errors. Using $\mathbf{a}_{t+1|t} = \mathbf{a}_{t+1|t+1} + K_{t+1}\boldsymbol{\upsilon}_{t+1}$ the backward recursion starting at $t+h-1$ can be written as a first difference equation in revisions $\mathbf{a}_{t|s} - \mathbf{a}_{t|t}$ for fixed s,

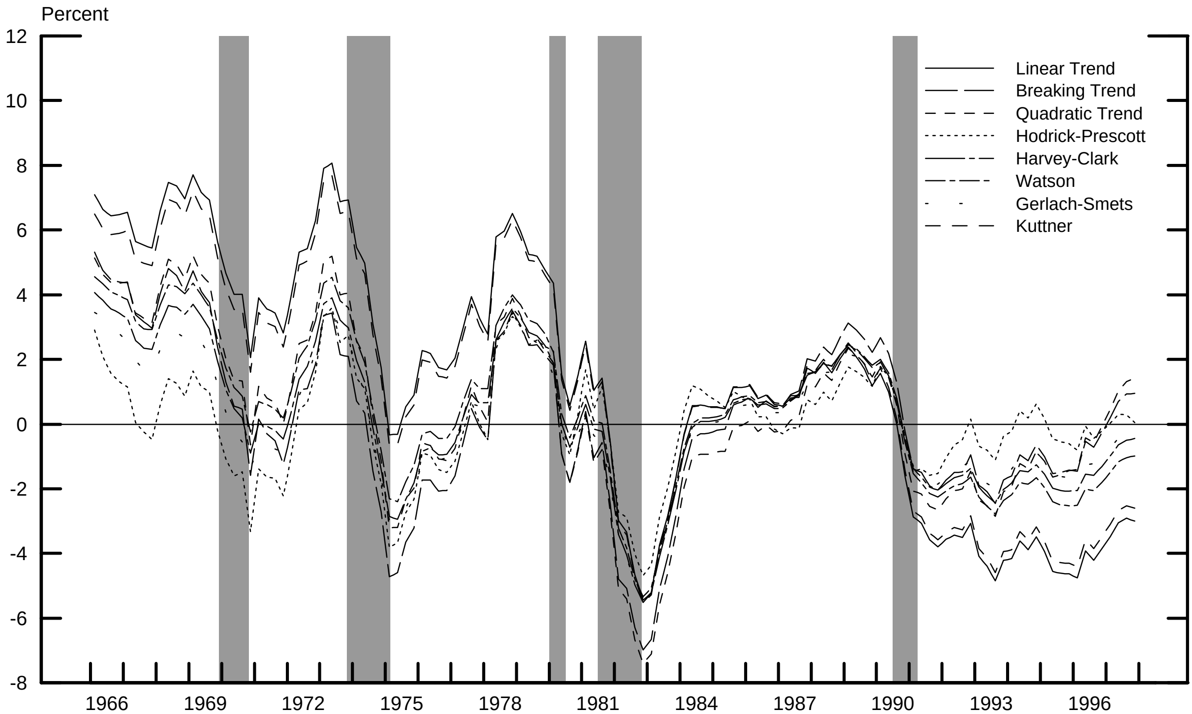
$$\mathbf{a}_{t|s} - \mathbf{a}_{t|t} = C_t(\mathbf{a}_{t+1|s} - \mathbf{a}_{t+1|t+1}) + C_t K_{t+1}\boldsymbol{\upsilon}_{t+1}$$

where $C_t = P_{t|t}T'P_{t-1|t}^{-1}$, K_{t+1} is the Kalman gain and $\boldsymbol{\upsilon}_{t+i} = \mathbf{x}_{t+i} - \mathbf{x}_{t+i|t+i-1}$ denotes the one-step ahead prediction error. Once the Kalman filter approaches the steady state revisions can be expressed as $\mathbf{a}_{t|t+h} - \mathbf{a}_{t|t} = \sum_{i=1}^{h} \overline{C}^i \overline{K} \boldsymbol{\upsilon}_{t+i}$, where $\overline{C}$ and $\overline{K}$ denote steady state values of C_t and K_t, respectively. It further holds $\mathbb{E}(\mathbf{a}_{t|t}\boldsymbol{\upsilon}'_{t+i}) = 0$ for $i > 0$ (Gourieroux and Montfort, 1989(1):482).

Real-Time Estimates of the Business Cycle



Final Estimates of the Business Cycle



The information content of real-time output gap estimates

Too many methods

- High model uncertainty
- Lack of evaluation criteria

Large revisions to real-time estimates

- Revisions to initial data releases
- Parameter uncertainty
- *One-sided filters!!!*

1 Plan of this paper

Search for evaluation criteria

- UOC models useful
- *Standard errors & inflation forecasts*

Search for reliable estimates

- Multivariate UOC models
- Extend information in filtering
- Exploit high coherence at business cycle

Model P (Fabiani and Mestre, 2000)

$$\Delta\pi_t = \rho\Delta\pi_{t-1} + \theta\left[y_t - y_t^*\right] + \varepsilon_t^\pi \qquad \text{(AS)}$$

$$y_t^* \ldots \text{ stochastic trend}$$

Model Y (Smets and Gerlach, 1999)

$$\begin{aligned} y_t &= y_t^* + y_t^C \\ y_t^C &= \varphi_1 y_{t-1}^C + \varphi_2 y_{t-2}^C + \varepsilon_t^y && \text{(AD)} \\ \Delta\pi_t &= \rho\Delta\pi_{t-1} + \theta y_{t-1}^C + \varepsilon_t^\pi && \text{(AS)} \end{aligned}$$

Model Y-UR (Fabiani and Mestre, 2001): **Y** plus

$$\begin{aligned} u_t &= u_t^* + u_t^C \\ u_t^C &= \phi u_{t-1}^C + a_0 y_t^C - a_1 y_{t-1}^C + \varepsilon_t^u \end{aligned}$$

$$C_t = \omega C_{t-1} + b_0 u_t^C - b_1 u_{t-1}^C + \varepsilon_t^C$$

Production function approach

$$
\begin{aligned}
y_t &= f_t + \alpha k_t + (1-\alpha)\left[L_t - u_t\right] \\
y_t^C &= f_t^C - (1-\alpha) u_t^C
\end{aligned}
$$

Model YK-F

$$
\begin{aligned}
y_t - k_t &= (y_t - k_t)^* + y_t^C \\
&\quad \text{(AD)} \\
&\quad \text{(AS)}
\end{aligned}
$$

$$
\begin{aligned}
f_t &= f_t^* + f_t^C \\
f_t^C &= \phi f_{t-1}^C + a_0 y_t^C - a_1 y_{t-1}^C + \varepsilon_t^f
\end{aligned}
$$

$$
C_t = \omega C_{t-1} + b_0 f_t^C - b_1 f_{t-1}^C + \varepsilon_t^C
$$

Principles

- Put into single state-space form
- Estimate jointly
- $\Rightarrow$ *Inflation and indicators add info in filtering!*

Kalman filter output

1. Estimate parameters $\xi = (\theta, \varphi_1, \varphi_2, \rho, ...)$
2. Estimates $\widehat{y}^C_{t|t+s}(\widehat{\xi})$ based on data up to $t+s$
3. Filter uncertainty $P_{t|t+s}(\widehat{\xi}) = \text{cov}\left[(\widehat{y}^C_{t|t+s} - y^C_t)|\widehat{\xi}\right]$
4. Parameter uncertainty (Hamilton, 1986)

Data (1970 Q1 - 2000 Q4)

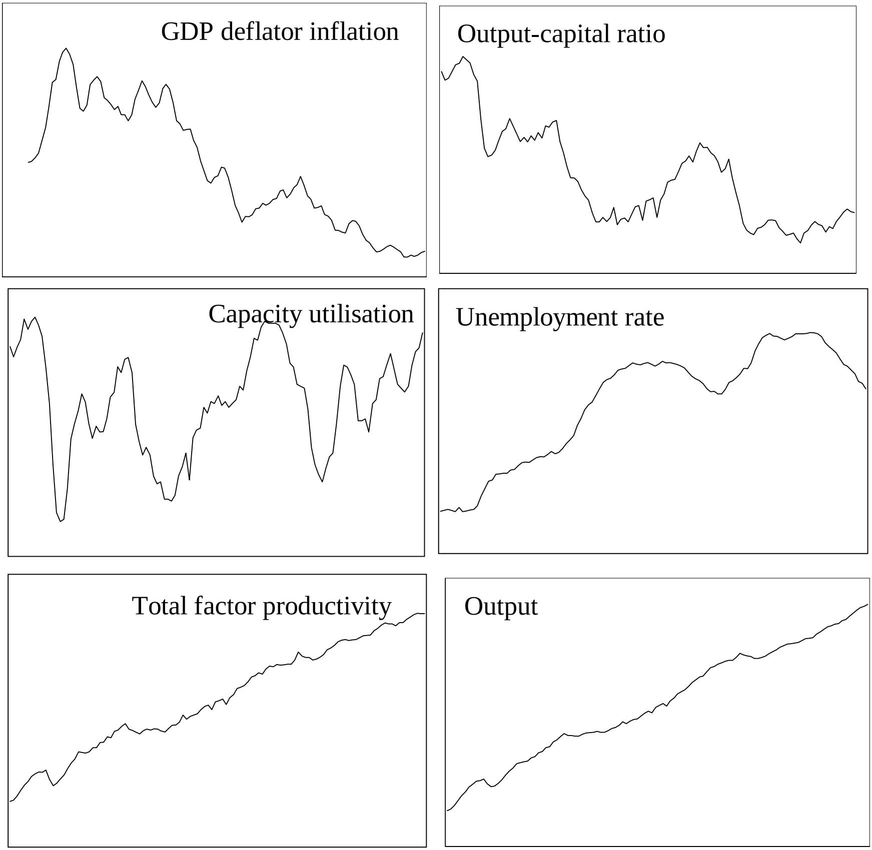


Fig 1: Output gap estimates

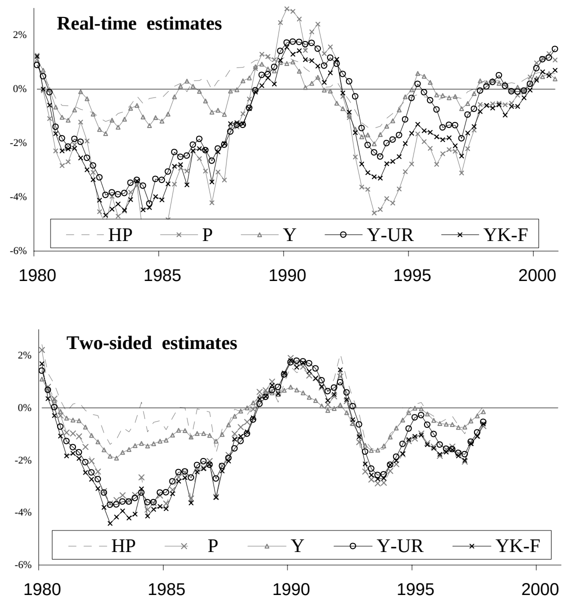
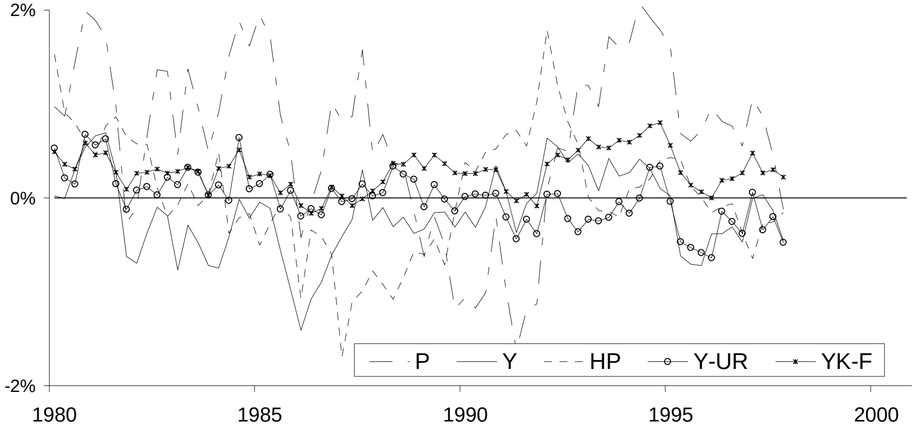


Fig 2: Revisions (12 quarter ahead)



Revisions to real-time estimates

Standard deviations
lower bound for filter uncertainty

	HP		P	Y	Y-UR	YK-F
Two-sided estimate	.88		1.63	.88	1.60	1.72
Revisions	.82		1.09	.48	.31	.38

Misspecification test for unbiasedness

$$\widehat{y}^C_{t|t+s} - \widehat{y}^C_{t|t} = b_0 + b_1 \widehat{y}^C_{t|t} + e_t$$

Unbiasedness requires $b_0 = b_1 = 0$.

32 quarters	HP		P	Y	Y-UR	YK-F
b_0	.001		-.001	-.002	.000	**.003**
b_1	**-.269**		**-.328**	**-.162**	**-.076**	.002

Significant estimates are in bold

Standard errors

$$\text{cov}\left[\left(\widehat{y}_{t|s}(\widehat{\xi}) - y_t^C\right)\right] = P_{t|s}(\xi) + \text{cov}\left[\widehat{y}_{t|s}(\widehat{\xi}) - \widehat{y}_{t|s}(\xi)\right].$$

	P	Y	Y-UR	YK-F
Standard dev	**1.63**	**.88**	**1.60**	**1.72**
Uncertainty: Standard errors				
Real-time	**1.65**	**.88**	**.62**	**.41**
Filter uncert.	1.60	.83	.56	.35
Parameter uncert.	.37	.27	.27	.22
Two-sided (t + 12)	**.91**	**.66**	**.53**	**.34**
Filter uncert.	.87	.47	.42	.26
Parameter uncert.	.25	.46	.33	.22

(in percentage points)

Inflation forecasts

(In-sample 1980Q1-2000Q4)

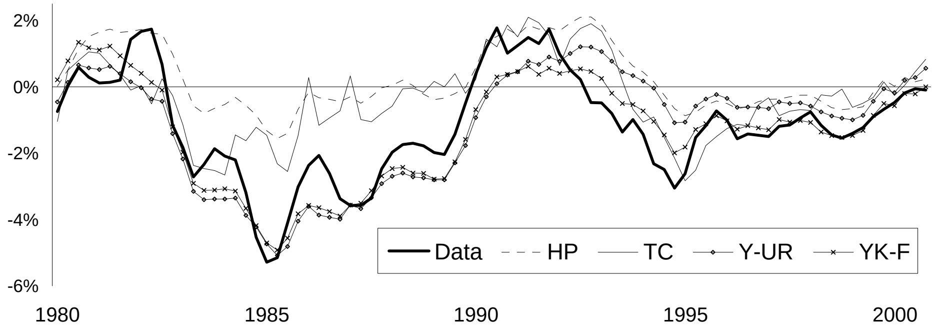
$$\text{LI}: \pi_{t+s} - \pi_t = \theta \widehat{y}^C_{t|t} + \sum_{i=0}^{k} c_i \Delta \pi_{t-i} + u_t$$

	HP	P	Y	Y-UR	YK-F
Conditional					
4	.90	1.03	.93	.73	.63
8	1.03	1.33	.91	.66	.49
12	1.07	1.64	.98	.63	.46
LI regressions					
4	.76	.74	.85	.70	.64
8	.85	.85	.93	.86	.67
12	.96	.89	.98	.88	.70

Root mean squared error relative to naive forecast

Conditional inflation forecasts (in-sample)

12-quarter ahead change in inflation



Conclusions

Multivariate models

- Lower filter uncertainty from using indicators
- Improved conditional inflation forecasts
- Bad leading indicator for inflation

Economic content of estimates?

Application

- Model uncertainty remains important
- Aim at model-based estimates
- Use many models
- Combine with other information