

Is Central Bank Transparency Desirable?

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Abstract

I analyse central bank transparency when the central bank's objective function is its private information. Non-transparency exists when the public does not observe the action of the central bank and an unobservable component of the inflation-control error keeps the public from using its observation of inflation to infer the action, and hence, the central bank's objective. The degree of transparency is defined as the fraction of the inflation-control error that is observable. This notion is similar to that of Cukierman and Meltzer [9], Faust and Svensson [13], [14] and others. I find a number of results; some are different than what previous authors have found and others are novel.

I demonstrate that non-transparent central banks with private information inflate less than central banks in a regime with perfect information. Moreover, in contrast to transparent central banks with private information, non-transparent banks with private information respond optimally to shocks.

Increased transparency lowers planned inflation, but surprisingly, it can worsen the public's ability to infer the central bank's objective function. I find that, no matter what their preferences, central banks and societies are made better off by more transparency. I further demonstrate that the transparent regime is not the same as the non-transparent regime when non-transparency goes to zero. I show that planned inflation is not necessarily lower in the transparent regime than in the non-transparent regime. However, numerical results suggest that all central banks and societies are better off in the transparent regime.

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1 Introduction

The past decade has seen a restructuring of many of the world's central banks. One apparent intent of much of this legislation was to increase central bank transparency. A striking result is that now most industrialised countries' central banks make monetary policy by choosing an overnight night lending rate, or other short-term interest rate, and then immediately and publicly announcing their choice. For many central banks this is a sharp departure from earlier procedures. Under the leadership of Paul Volcker, for example, the Fed never announced changes in its chosen rate; market participants were left to determine for themselves whether or not a change had occurred.¹ The purpose of this paper is to ask whether the recent transparent behaviour of central banks is desirable from the point of view of both central banks and society.

The model is inhabited by a central bank whose welfare is increasing in unexpected inflation (because it increases output) and decreasing in actual inflation. An unobserved stochastic shock, realised after the public's expectation is formed but before monetary policy is made, provides a stabilisation role for central banks. Central banks differ by the weight that they put on the benefit of increased output relative to the cost of increased inflation. This parameter is assumed to be the policy maker's private information and is referred to as the central bank's *type*.

The public has rational expectations; hence, if there were no private information, it would on average correctly predict inflation. However, if there is private information and the public were to believe that the central bank were more inflation averse than it actually is, then on average inflation would be higher than the public expected. Thus, policy makers have an incentive to use their current policy to increase the public's perception of their inflation aversion, and hence, to lower the public's future expectation of inflation.

Central banks choose planned inflation, which they attempt to implement by some action. An inflation-control error causes actual inflation to diverge from planned inflation.

¹See Andrews [1].

The central bank is said to be *transparent* if its action and, equivalently here, its planned inflation are observable. With transparency, the framework is a classical signalling model, similar to those of Vickers [30] and Sibert [25]. In equilibrium, all central banks but the least inflation averse choose inflation that is lower than that that would prevail with no private information and they respond sub-optimally to the shock. In equilibrium, their action perfectly reveals their type.

The central bank is said to be *non-transparent* if its action and planned inflation are unobservable. If the central bank is non-transparent, the public's ability to infer the central bank's action and its planned inflation from observed inflation is decreasing in the relative size of the component of the inflation-control error that is unobservable. Thus, I will refer to this relative size as the degree of non-transparency. This is a standard view of central bank non-transparency; it has been employed by Cukierman and Meltzer [9], who call it *ambiguity*, Faust and Svensson [13], [14], Jensen [19] and Atkeson and Kehoe [2].²

The model of non-transparent central banks that is presented here is quite similar to the one in the seminal work of Cukierman and Meltzer [9], which was extended by Faust and Svensson [13], [14] and Jensen [19]. These previous papers employed infinite-horizon models wherein market participants attempted to infer the time-varying, serially correlated preferences of central banks using the entire history of realisations of observable variables. The sophisticated structure of these models allowed their authors to consider dynamic issues but their complexity made analysis difficult. Cukierman and Meltzer's [9] elegant solution required the assumption that the weight that the central bank puts on output relative to inflation follows an AR(1) process with normally distributed errors, and hence can be negative. I will argue that this assumption helps drive their result that central banks may benefit from ambiguity. Faust and Svensson's [13], [14] extension

²There are other notions of central bank transparency and non-transparency may matter in other ways than the one it does here. See, for example, Gersbach [18] and Geraats [15], Cukierman [8] and Atkeson and Kehoe [2]. See Geraats and Eijffinger [17] for a detailed classification of different notions of transparency and a ranking of some central banks using this scale. See Geraats [16] for a survey of the literature on transparency.

required numerical solution, making the intuition somewhat opaque.

The model presented here has only two periods and it is meant to complement the previous literature by focusing attention on the precise role of private information and non-transparency. For a simple stochastic structure, the model can be solved analytically, enhancing intuition. For a different stochastic structure the model is solved numerically. I am able to obtain a number of results; some support the results of previous work, but some are in stark contrast and others are novel.

I find, as do previous authors, that planned inflation is lower with non-transparency and private information than it would be with full information. I also demonstrate that planned inflation is higher than would prevail in a full-information command optimum. I show the novel result, that policy makers respond optimally to shocks. The lower inflation that results from non-transparency and private information is *not* at the expense of the central bank's stabilisation role. This is in contrast to the scenario where there is private information and transparency.

I find, in line with previous work, that a marginal increase in transparency increases the incentive to signal and lowers equilibrium planned inflation. I also find that increased transparency lowers the variance of planned inflation. I further demonstrate the surprising result, not found in Cukierman and Meltzer [9] and in contrast to what is suggested by Faust and Svensson [13], that increased transparency can *worsen* the public's ability to infer the central bank's private information. Finally, I show that – no matter what their preferences – both the central bank and society are always better off with increased transparency. This is in contrast to Cukierman and Meltzer's [9] and Faust and Svensson's [13] results.

In addition to looking at marginal changes in transparency, I compare transparent with non-transparent regimes. This is novel; in previous work, Cukierman and Meltzer [9], Faust and Svensson [13] and Jensen [19] all consider the limit of the non-transparent regime as the variable representing non-transparency goes to zero. However, the simpler structure employed here makes it clear that the outcome in this case is not the outcome

of the transparent regime. I find that planned inflation can be higher or lower in a transparent regime than in a non-transparent regime, but both central bank and social expected welfare is higher in the transparent regime than in the non-transparent regime.

In Section 2, a model of transparent central banks is briefly reviewed. In Section 3, I describe the model of non-transparent central banks. In Section 4, I analyse the effects of marginal changes in transparency in the non-transparent regime and compare transparent and non-transparent regimes. In Section 5, I briefly discuss an alternative notion of central bank transparency and conclude.

2 A Transparent Central Bank

The two-period model is inhabited by a monetary policy maker, referred to as the central bank, and a private sector.³ In each period, the central bank's welfare is increasing in output and decreasing in squared deviations of inflation from its optimal level, normalised to zero.⁴ Either nominal wage contracting and rational expectations, as in the Barro-Gordon [4] framework, or a Lucas [20] expectations view of aggregate supply ensure that output is increasing in unexpected inflation.

The central bank has private information about the weight, χ , that it puts on the benefit of increased output relative to the cost of higher inflation. I assume that this weight is drawn from a known distribution with a continuous density function f and support $[\chi_L, \chi_U] \subset \mathbb{R}_+$. This preference parameter might reflect a desire to obtain a future job, pressure from the government, the press, the electorate, or a special interest group, as well as fundamental preferences or beliefs about the economy.

³Many modern central banks have monetary policy committees made up of policy makers with overlapping finite terms rather than either infinite-lived or finite-lived single policy makers. Sibert [26] models reputation building in a monetary policy committee.

⁴This linear-quadratic specification is used by Backus and Driffill [3], Cukierman and Meltzer [9] and Sibert [25]. Faust and Svensson [13], [14] assume that central bank welfare is decreasing in both squared inflation and squared deviations from a long-run employment target. This target is the private information of the central bank. See Sibert [25] for a discussion of the loss function assumed here relative to one which is quadratic in output (or employment), as well as inflation. Faust and Svensson [13] object to the linear-quadratic specification, saying that in an infinite-horizon model it causes high- and low-credibility central banks to behave in the same way. This is not relevant in this two-period model. Jensen [19] modifies Faust and Svensson's framework by using a New Keynesian Phillips curve.

The welfare of a central bank with preference parameter χ is

$$\chi\mu_0(\pi_0 - \pi_0^e) - \pi_0^2/2 + \beta[\chi\mu_1(\pi_1 - \pi_1^e) - \pi_1^2/2], \quad 0 < \beta < 1, \quad (1)$$

where π_t is period- t inflation, π_t^e is the public's beginning-of-period- t expectation of period- t inflation, $t = 0, 1$, and β is a commonly known discount factor. The random variable μ_t , $t = 0, 1$, has mean one, support $[\mu_L, \mu_U] \subset \mathbb{R}_+$ and can be viewed as a shock to the expectations-augmented Phillips curve; a higher value of the shock creating a more favourable tradeoff between output and unexpected inflation. I will refer to this variable as a *Phillips curve shock*. In the Barro-Gordon [4] framework with a Cobb-Douglas production function, it can be interpreted as a shock to labour's share of output; in the Lucas [20] framework it is a shock to the variance of aggregate demand.

Period- t inflation is the sum of the time- t action of the central bank, a_t , and an unobservable mean-zero inflation control error δ_t :

$$\pi_t = a_t + \delta_t, \quad t = 0, 1. \quad (2)$$

The variables $\chi, \mu_0, \mu_1, \delta_0$ and δ_1 are assumed to be mutually independent.

The within-period timing of events is as follows. First, the public forms its expectation of the period's inflation. Second, the Phillip's curve shock is realised and observed by the public and the central bank. Third, the central bank chooses its action and this is observed by the public. Fourth, inflation is realised and is observed by the public and the central bank.

The model is a *classic signalling* model, of the type originally described by Spence [28]. This monetary policy version is similar to the model in Sibert [25] and to the model in Vickers [30] (where χ can take on two possible values and there is no Phillip's curve shock). The solution is as follows. In period one, as there is no future to consider, the central bank chooses the action that maximises its within-period expected welfare.

As the public's expectation of period-one inflation is formed before the central bank chooses its period-one action this expectation does not affect the central bank's optimisation problem in period one. Using equations (1) and (2), the central bank maximises $E[\chi(a_1 + \delta_1)\mu_1 - (a_1 + \delta_1)^2/2]$, where the expectation is with respect δ_1 . The solution is $a_1 = \chi\mu_1$. Given that none of the random variables are correlated and that μ_1 has mean one, the public's expectation of period-one inflation is its expectation of χ .

I consider outcomes where the central bank's period-zero action reveals χ . The public conjectures that the central bank follows the policy rule $a_0 = a_0^*(\chi, \mu_0)$. I conjecture that $a_0^*(\cdot, \mu_0)$ is differentiable and invertible.⁵ Thus, upon observing a_0 and μ_0 the public infers that $\chi = a_0^{*-1}(a_0, \mu_0)$, where $a_0^{*-1}(\cdot, \mu_0)$ is the inverse of a_0^* with respect to χ . Thus, $\pi_1^e = a_0^{*-1}(a_0, \mu_0)$.

Using this, employing equations (1) and (2) and ignoring terms that do not affect period-zero optimisation, the central bank's period-zero objective function can be written as

$$\chi a_0 \mu_0 - a_0^2/2 - \beta \chi a_0^{*-1}(a_0, \mu_0). \quad (3)$$

The first- and second order conditions are

$$\chi \mu_0 - a_0 - \beta \chi \partial a_0^{*-1}(a_0, \mu_0) / \partial a_0 = 0 \quad (4)$$

$$1 + \beta \chi \partial^2 a_0^{*-1}(a_0, \mu_0) / \partial a_0^2 > 0, \quad (5)$$

respectively.

Rational expectations requires that the public's conjecture is correct so that $a_0 =$

⁵This is seen to be correct in equilibrium. Using Mailleth's [21] results, no non-differentiable rule can be a separating equilibrium.

$a_0^*(\chi, \mu_0) = a_0(\chi, \mu_0)$. Substituting this into (4) and (5) yields⁶

$$a_0(\chi, \mu_0) = \chi\mu_0 - \frac{\beta\chi}{\partial a_0(\chi, \mu_0)/\partial\chi} \quad (6)$$

$$1 - \frac{\beta\chi\partial^2 a_0(\chi, \mu_0)/\partial\chi^2}{(\partial a_0(\chi, \mu_0)/\partial\chi)^3} > 0. \quad (7)$$

A solution to equations (6) and (7) is a perfect Bayesian equilibrium. The central bank is maximising its own welfare while taking into account the effect of its actions on the public's beliefs. The public's beliefs are consistent with Bayes rule and are formed using the correct conjecture about the central bank's equilibrium strategy and the public's observation of the central bank's action.

Equation (6) is a first-order differential equation with no boundary condition. It is typical in signalling models to generate a boundary condition by assuming that the sender of the poorest quality signal (that is, the least inflation-averse central bank) chooses its within-period optimum:

$$a(\chi_U, \mu_0) = \chi_U\mu_0. \quad (8)$$

The reasoning is that the central bank's type is revealed in equilibrium. For the type with the $\chi = \chi_U$, expectations could be no worse. Thus, this central bank might as well choose its within-period optimum.⁷

Sibert [25] provides sufficient conditions for a unique solution to equations (6) - (8) to exist and demonstrates four striking features of the equilibrium. First, $\partial a_0(\chi, \mu_0)/\partial\chi > 0$: the equilibrium is separating and it is *revealing*; the central bank's type is perfectly inferred from its action. Second, $0 < a_0(\chi, \mu_0) < \chi\mu_0$, $\chi \in [\chi_L, \chi_U]$: all central bankers but the least inflation averse inflate less than they would without private information. Third, a central bank's action depends solely on its own type and the type of the least inflation-averse central banker; it is otherwise invariant to the distribution of χ . Fourth, $\partial a_0(\chi, \mu_0)/\partial\mu_0 > \chi$, $\chi \in [\chi_L, \chi_U]$: central bankers with private information stabilise too

⁶I employ the rules: $f^{-1'}(f(x)) = 1/f'(x)$ and $f^{-1''}(f(x)) = -f''(x)/f'(x)^3$.

⁷Only this outcome satisfies the D1 criterion. See Ramey [23].

much relative to the full-information command optimum.⁸

Despite the suboptimal stabilisation, it appears possible that the private information may improve welfare; this is certainly true if the variance of the Philip's curve shock is sufficiently small. With perfect information, the type- χ , $\chi < \chi_U$, policy maker would like to be able to commit to inflating less than $\chi\mu_0$, but cannot do this; once period-zero expectations are in place, it is optimal to choose inflation of $\chi\mu_0$. With private information, the policy maker has an incentive to signal that he is not less inflation averse than he is by lowering his inflation below $\chi\mu_0$. The threat that the public might otherwise update its beliefs in a harmful way serves as a commitment device.

In addition to this separating equilibrium, where the action of the central bank is a one-to-one function of its type, there exist pooling equilibria where central banks of different types select the same action. These equilibria are supported by out-of-equilibrium beliefs that are not ruled out by the Bayesian equilibrium concept, but which are typically viewed as unappealing.⁹

The model is in sharp contrast to Backus and Driffill's [3] model of uncertainty about central bank types and inflation. In that framework there are two types of central bank: one is strategic and puts some weight on output and one is mechanistic and always votes for zero inflation. Here, all central banks are strategic, but differ in their degree of inflation aversion. In Backus and Driffill's model there may be pooling in period zero as strategic types vote for zero inflation in an attempt to masquerade as non-strategic types. Here there is separation in period zero as inflation-averse types inflate less than they would if there were no private information to signal that they are not a less inflation-averse type. In Backus and Driffill's T -period model, once a strategic policy maker chooses positive inflation he will choose it for ever after. In a more general T -period version of the model presented here, even without time-varying preferences, signalling can occur in all periods

⁸The full-information command optimum has $a_0(\chi, \mu_0) = \chi(\mu_0 - 1)$.

⁹This is formalised with the result that these equilibria violate the D1 criterion. See Ramey [23]. the D1 criterion is the continuum-of-types analogue of Cho and Kreps's [6] Intuitive Criterion.

except the last.¹⁰

In this next section, I show that adding noise to the public's observation of the central bank's action dramatically changes the qualitative features of the equilibrium.

3 A Non-transparent Central Bank

In this section I present a model of a non-transparent central bank. In modelling non-transparency, one might view the central bank as choosing planned inflation. Then, depending on its forecasts and its current beliefs about how the economy works, it chooses an action. The outcome of the action is inflation. Non-transparency might enter in two ways. First, the public might not know the central bank's forecast or its current view of how the economy functions. Thus, the action of the central bank might depend not only on its type, but on other random variables observed by the central bank but not the public. Second, the central bank's action might be unobservable and actual inflation does not perfectly reveal the action because the inflation-control error is unobservable. Here, I focus solely on the second type of transparency, returning briefly to the first type in the conclusion.

This type of non-transparency assumed here has appeared previously in the monetary policy literature in papers by Cukierman and Meltzer [9] and Faust and Svensson [13]. Both of these pairs of authors view the central bank's choice variable as planned inflation, rather than as an action. However, because they also focus on non-transparency arising from unobservable noise (the inflation-control error) rather than from noise which is the central bank's private information (for example, its forecast), this distinction is unimportant in their papers as it is unimportant in this one.¹¹

¹⁰The reason that signalling continues even after the central bank's type is revealed is because the public expects this and updates its beliefs if it observes an out-of-equilibrium event. See Vincent [31] and Sibert [25]. In Backus and Driffill [3] it is technologically impossible for a mechanistic type to chose positive inflation; hence an observation of positive inflation implies that the central bank must be strategic. If preferences here were to vary stochastically over time and were serially correlated, then information revealed in period $t - 1$ would have no effect on the outcome in period t , $t < T$. This is because the outcome is invariant to the distribution of χ .

¹¹Neither Cukierman and Meltzer [9] nor Faust and Svensson [13] assume that the action of the central bank is uobservable; indeed Faust and Svensson state that it is observable. However, in neither paper

The *noisy signalling* model here is also similar in structure to Matthew and Mirman's [22] two-action (for the receiver of the signal) signalling model of a firm facing a potential entrant. There, the incumbent firm has private information about its own costs of production. It chooses an unobservable intent (its planned price) and carries out an unobservable action (the choice of a quantity). An unobservable control error (a demand shock) then causes the actual price to deviate from the planned price.

I continue to use equation (2) from the previous section: $\tilde{\pi}_t = a_t + \delta_t$, except that I now denote time- t inflation by $\tilde{\pi}_t$ and I assume that the time- t action a_t is unobservable by the public. Alternatively, I could assume that the public sees some noisy intermediate consequence of the policy maker's action, such as money growth, and assume that inflation equals money growth plus an error term. The linear-quadratic preference specification ensures that this would be equivalent.

To focus on transparency, rather than on the central bank's ability to control inflation, I follow Faust and Svensson [13] in defining

$$\epsilon_t = (1 - \eta) \delta_t, \quad \xi_t = \eta \delta_t, \quad 0 \leq \eta < 1, \quad t = 0, 1, \quad (9)$$

where ξ_t is observable by the public by the end of period t while ϵ_t is not and these two variables are uncorrelated. I then view the variance of δ_t as fixed and define an increase in transparency to be a decrease in the variance of ϵ_t , denoted by σ_ϵ^2 , resulting from an increase in η . Increases in transparency might arise from changes in the quality and frequency of documents such as inflation reports that educate the public about the inflation-transmission mechanism.

I assume that ϵ_0 and ϵ_1 have the known density function g . Cukierman and Meltzer [9] and Faust and Svensson [13] assume that the support of ϵ_0 and ϵ_1 is the real line, and I will temporarily assume this as well. Faust and Svensson [14] view σ_ϵ^2 as the choice of the central bank; I model it as exogenous. This distinction is irrelevant if all central

does the public attempt to use an observation of the action to infer the central bank's type.

bank types would choose the same σ_ϵ^2 . However, if different types were to prefer different values of σ_ϵ^2 and could choose this variable then their choice would convey information about their type and this would be used by the public in its estimation of χ .

I further define $\pi_t \equiv \tilde{\pi}_t - \xi_t$. I will refer to π_t from now on as inflation, but it should be understood as inflation, less the observable component of the inflation-control error. Thus,

$$\pi_t = a_t + \epsilon_t. \tag{10}$$

The action a_t can be interpreted as, and will be referred to as, *planned inflation*. The distinction between π_t and $\tilde{\pi}_t$ has no consequence for any of the results.

Before proceeding with the analysis, it is useful to ask when the model of the previous section is a better description of central bank behaviour and when the model of this section is more appropriate. Currently, many central banks appear to be transparent; they choose a short-term interest rate and publicly announce their decision. In the United Kingdom, for example, the Monetary Policy Committee of the Bank of England chooses a rate at its monthly meeting and instructs its operational staff to trade at this rate. This action is widely reported in the media and it is impossible to argue that it is unobservable. In the United States, Australia and Brazil the central bank's monetary policy committee chooses a rate at its monthly meeting and instructs its staff to conduct open-market operations to attempt to attain this rate. Atkeson and Kehoe [2] argue that the realised interest rate in this case is a noisy signal of the operational staff's action and this is true. However, unless one believes that the central bank is covertly directing its staff to deviate from its wishes, the central bank's action is the widely publicised choice of an announced interest rate target and this is perfectly observable. These types of central banks may be best modelled with the framework of the previous section.

A situation where the central bank's action may not be observable is where the central bank has target ranges for various measures of the money supply and then conducts open-market operations with the intent of attaining some specific outcome. In this case the

action of the central bank is difficult or impossible to observe. This is the scenario that Cukierman and Meltzer [9] appear to have in mind. The Bank of Japan's monetary policy committee, for example, publicly announces only guidelines for its money market operations. Another scenario where the central bank's action is unobservable would be where the central bank tells its operational staff to attempt to attain some interest rate, but does not announce the targeted rate. This was how the Federal Reserve conducted policy prior to 1994. A further example would be where the central bank intervenes in the foreign exchange market with the intent of achieving a particular exchange rate within a band, but does not announce its target exchange rate and does not publish its intervention.

Returning to the analysis, I now specify the optimisation problem of the central bank, given the the public's beliefs and information. As in the previous section, in period one, when there is no future to consider, the central bank sets planned inflation equal to its within-period optimum of $\chi\mu_1$.

I assume that the Philip's curve shocks, the inflation-control shocks and the preference parameter are contemporaneously and serially uncorrelated; hence, the public's expectation of period-one inflation is equal to its expectation of χ , given its conjecture about central bank behaviour and its observations of π_0 and μ_0 . As in the previous section, the public conjectures that period-zero planned inflation is a function of the central bank's type and is given by $a_0 = a_0^*(\chi, \mu_0)$.

Given the public's conjecture, equation (10) implies that there is a stochastic relationship between π_0 and χ . The joint density function of π_0 and χ , conditional on the conjectured policy rule and μ_0 is then $g(\pi_0 - a_0^*(\chi, \mu_0))f(\chi)$, where it is recalled that f is the density function of χ . The marginal density function of π_0 is then $\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0^*(s, \mu_0))f(s)ds$. Thus, the density function of χ conditional on the conjectured policy rule, μ_0 and π_0 is

$g(\pi_0 - a_0^*(\chi, \mu_0))f(\chi) / \int_{\chi_L}^{\chi_U} g(\pi_0 - a_0^*(s, \mu_0))f(s)ds$ and

$$\pi_1^e = E(\chi|\pi_0, \mu_0, a_0^*(\cdot, \mu_0)) = \int_{\chi_L}^{\chi_U} \frac{xg(\pi_0 - a_0^*(x, \mu_0))f(x)}{\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0^*(s, \mu_0))f(s)ds} dx. \quad (11)$$

The central bank's expectation of this, given its choice of planned inflation, μ_0 and the public's conjecture is

$$E(\pi_1^e|a_0, \mu_0, a_0^*(\cdot, \mu_0)) = \int_{\mathbb{R}} \int_{\chi_L}^{\chi_U} \frac{xg(\pi_0 - a_0^*(x, \mu_0))f(x)}{\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0^*(s, \mu_0))f(s)ds} dx g(\pi_0 - a_0) d\pi_0. \quad (12)$$

Using equations (1) and (12) and ignoring terms that do not affect the optimisation problem, the central bank's objective function can be written as

$$\chi a_0 \mu_0 - a_0^2/2 - \chi\beta E(\pi_1^e|a_0, \mu_0, a_0^*(\cdot, \mu_0)). \quad (13)$$

Using equation (12), the first-order and second-order conditions are

$$a_0 = \chi\mu_0 + \chi\beta \int_{\mathbb{R}} \int_{\chi_L}^{\chi_U} \frac{xg(\pi_0 - a_0^*(x, \mu_0))f(x)}{\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0^*(s, \mu_0))f(s)ds} dx g'(\pi_0 - a_0) d\pi_0 \quad (14)$$

and

$$1 + \chi\beta \int_{\mathbb{R}} \int_{\chi_L}^{\chi_U} \frac{xg(\pi_0 - a_0^*(x, \mu_0))f(x)}{\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0^*(s, \mu_0))f(s)ds} dx g''(\pi_0 - a_0) d\pi_0 > 0, \quad (15)$$

respectively.

In equilibrium, the public's conjecture about the action rule is correct and $a_0 = a_0(\chi, \mu_0) = a_0^*(\chi, \mu_0)$. Substituting this into equations (14) and (15) yields the following definition.

Definition 1 An *equilibrium* is a function $a_0(\chi, \mu_0)$ such that for every $\chi \in [\chi_L, \chi_U]$ and $\mu_0 \in [\mu_L, \mu_U]$,

$$a_0(\chi, \mu_0) = \chi\mu_0 + \chi\beta \int_{\mathbb{R}} \int_{\chi_L}^{\chi_U} \frac{xg(\pi_0 - a_0(x, \mu_0))f(x)}{\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0(s, \mu_0))f(s)ds} dx g'(\pi_0 - a_0(\chi, \mu_0)) d\pi_0. \quad (16)$$

$$1 + \chi\beta \int_{\mathbb{R}} \int_{\chi_L}^{\chi_U} \frac{xg(\pi_0 - a_0(x, \mu_0))f(x)}{\int_{\chi_L}^{\chi_U} g(\pi_0 - a_0(s, \mu_0))f(s)ds} dx g''(\pi_0 - a_0(\chi, \mu_0)) d\pi_0 > 0. \quad (17)$$

The set of possible equilibria here appears to be starkly different than the one in the previous section. In the last section, a continuum of perfect Bayesian separating equilibria were given by a differential equation without a boundary condition and co-existed with a continuum of pooling equilibria. Additional refinements ruled out all but one separating equilibrium. Here, it appears possible for the perfect Bayesian equilibrium to be unique – at least there is not a continuum of equilibria – and it is separating.¹² In the previous section, the equilibrium was revealing, here it is not.¹³

In the next section, I consider how marginal changes in transparency affect inflation and welfare and I compare welfare in regimes with and without transparency.

4 The Effects of Imperfect Inflation Control and Transparency

I now consider the effect of transparency on the equilibrium outcome. I first consider an analytical model and then a numerical one.

4.1 Analytical results

Assume that

$$\chi \sim U(\chi_L, \chi_U), \epsilon_t \sim U(-w/2, w/2). \quad (18)$$

This departs from the assumptions made so far in that the distribution of ϵ_t has finite support. A consequence of this departure is that if the planned inflation rule $a_0^*(\cdot, \cdot)$ is bounded, then the distribution of π_0 has a finite support; hence, out-of-equilibrium beliefs matter. Unappealing out-of-equilibrium beliefs can support odd pooling equilibria.

An example of such an equilibrium is provided in the Appendix immediately following

¹²This local uniqueness is also found by Matthews and Mirman [22]. The result here provides some justification for Cukierman and Meltzer's [9] and Faust and Svensson [13]'s strategy of conjecturing that their equilibrium in linear and then solving for the coefficients.

¹³The equilibrium here is structurally similar to the one in Dewatripont, Jewitt and Tirole [10]. There a worker has unobserved (even by himself) ability and exerts effort (which is not observed by his employer), resulting in observed output. The employer uses his observation of output to infer the worker's ability.

the Proof of Proposition 1.¹⁴ These pooling equilibria have the property that if the public observes period-zero inflation that is too low to be consistent with the pooling equilibrium's planned inflation, then the public believes that the central bank is the *least* inflation-averse type. I rule these pooling equilibria out by assuming for the rest of section 4.1 that a (strict) increase in observed period-zero inflation cannot lead to a (strict) decrease in π_1^e .

Assume further that for every $\mu_0 \in [\mu_L, \mu_U]$

$$w \geq \frac{\beta(2\chi_U - \chi_L)}{2\mu_0}, \left(\frac{w}{\chi_U - \chi_L} \right)^2 - \frac{\mu_0 w}{\chi_U - \chi_L} + \frac{\beta}{2} \geq 0. \quad (19)$$

These assumptions are satisfied if the noise w is sufficiently large.

Proposition 1 *There exists an equilibrium where*

$$a_0(\chi, \mu_0) = (\mu_0 - \alpha)\chi \quad (20)$$

$$E(\pi_1^e | \chi, \mu_0) = \hat{\chi} + \alpha(\mu_0 - \alpha)(\chi - \hat{\chi}) / \beta \quad (21)$$

$$\text{where } \alpha \equiv \beta(\chi_U - \chi_L) / (2w)$$

and where $\hat{\chi}$ is the mean value of χ .

Proof. See the Appendix.

The variable α can be interpreted as the central bank's expected discounted period-one marginal benefit from lowering planned inflation in period zero. It is increasing in both the variance of χ (equal to $(\chi_U - \chi_L)^2 / 12$) and transparency (as $w = (12\sigma_\epsilon^2)^{1/2}$). ■

In the following Proposition, I consider the effect of imperfect inflation control and non-transparency on planned inflation in period zero.

Proposition 2 *For every type- χ central bank:*

(i) *Period-zero planned inflation is higher than the full-commitment optimum planned inflation when the central bank's type is public information, but lower than the planned inflation that would prevail when the central bank's type is public information and there is discretionary policy making.*

(ii) *A marginal increase in transparency lowers period-zero planned inflation.*

¹⁴See Matthews and Mirman [22] for a discussion of this point.

Proof. The proof is obvious from equation (20) and $\alpha \in (0, 1)$, which follows from assumption (19). ■

Part (i) is novel and says that with non-transparency, all central bank types set planned inflation above the full-information command optimum ($a_0 = \chi(\mu_0 - 1)$) and below the full-information discretionary outcome ($a_0 = \chi\mu_0$). This differs from the transparent regime, where planned inflation need not be greater than full-information command optimum planned inflation and where the least inflation-averse type chooses the full-information discretionary outcome.

Part (ii) was also obtained by Cukierman and Meltzer [9] and Faust and Svensson [13]. The intuition is that when transparency increases, for a given planned-inflation rule, actual inflation becomes more informative. This increases the incentive of the central bank to lower inflation.

Proposition 3 *The central bank responds to a shock in the same way that it would with full information or in a full-information command optimum; that is, $\partial a_0(\chi, \mu_0) / \partial \mu_0 = \chi$.*

Proof. Obvious from equation (20). ■

This is a surprising result. Proposition 2 tells us that private information and non-transparency reduces the inflation bias associated with full-information policy making. This proposition says that it does so without affecting the central bank's stabilisation role. Since Rogoff's [24] seminal paper, the prevailing belief among macroeconomists has been that decreasing the inflation bias must be at the expense of suboptimal stabilisation.

The above result is in sharp contrast to the model of the previous section, where transparency and private information leads to suboptimal stabilisation. The intuition behind the outcome of the previous section is that the type- χ_U central banker responds to an increase in μ_0 by increasing planned inflation at rate χ_U . Hence, a marginally more inflation averse type- χ central banker can respond to the shock at a rate greater than χ and still separate himself from the type- χ_U central banker. Thus, by a backwards recursion argument, all central bankers with $\chi < \chi_U$ respond to a shock by increasing

inflation at rate greater than their type χ . This does not happen here because the type- χ_U central banker does not choose his within-period optimum.

In the next proposition, I consider how imperfect inflation control and transparency affect the public's ability to infer the central bank's type, given their observation of π_0 and μ_0 . The public's squared forecast error is $E\{[\chi - E(\chi|\pi_0, \chi)]^2 | \mu_0\}$. The public's forecasting ability is said to be *improved* if this error falls.

Proposition 4 (i) *The public believes that the type- χ central bank is more (less) inflation averse than it actually is if $\chi < (>) \hat{\chi}$.*

(ii) *A marginal increase in transparency lowers the variance of planned inflation (conditional solely on μ_0).*

(iii) *A marginal increase in transparency improves (worsens) the public's forecasting ability if $\mu_0 > (<) 2a$.*

Proof. Obvious from equations (20) and (21). ■

Part (i) is a straightforward consequence of non-transparency. To see this, suppose that the public observes higher than average inflation. Then it infers that it is more likely than not that actual inflation is higher than intended. Likewise, if the public observes lower than average inflation it infers that inflation is lower than intended. The further is actual inflation from mean inflation, the larger the public's estimation of the magnitude of the unobservable control error. This tends to bias the public's estimate of the central bank's type toward the mean.

Result (ii) is more surprising. Inflation-averse central banks set their inflation below less inflation-averse central banks to signal that they are more inflation averse. However, as transparency increases the central banks "separate" less. The reason is as follows. As transparency increases and inflation becomes more informative, the incentive to lower inflation increases. But, this incentive increases more for less inflation-averse types than for more inflation-averse types. This is because the less inflation-averse the central bank is, the more weight it puts on output in period one and the greater the benefit of being thought less inflation averse. Thus, increased transparency lowers planned inflation more

for less inflation-averse types than for more inflation-averse types. This flattens planned inflation as a function of the central bank's type.

Quite surprisingly, part (iii) says that increased transparency need *not* improve the public's forecasting ability. This is the opposite of what Faust and Svensson [13] suggest and is a consequence of parts (i) and (ii). For a given inflation rule, increased transparency improves the public's ability to forecast. But, it also changes the planned-inflation rule, making planned inflation less informative.

I define a central bank's expected welfare to be its welfare conditional on its type, but not on μ_0 . I define a society of type- χ^* 's welfare to be

$$\chi^* (\pi_0 - \pi_0^e) \mu_0 - \pi_0^2/2 + \beta [\chi^* (\pi_1 - \pi_1^e) \mu_1 - \pi_1^2/2], 0 < \beta < 1, \chi^* \in [\chi_L, \chi_U] \quad (22)$$

and its expected welfare to the expected value of this conditional on its own preference parameter, χ^* , but not on μ_0 or the central bank's preference parameter, χ . The next proposition shows how private information and non-transparency affects a central bank's welfare and society's welfare.

Proposition 5 *(i) A central bank with $\chi \geq \hat{\chi}$ is better off with private information and non-transparency than with full information. If χ_L is sufficiently close to zero, then a central bank with χ sufficiently close to χ_L may be made worse off.*

(i) A marginal increase in transparency raises the expected welfare of the type- χ central bank for every $\chi \in [\chi_U, \chi_L]$.

(ii) A marginal increase in transparency raises the expected welfare of the type- χ^ society for every $\chi^* \in [\chi_U, \chi_L]$*

Proof. See the Appendix. ■

In the game theory literature, it is usual for private information and costly signalling to make the senders of the signal worse off. Here, the presence of the time-inconsistency problem causes the private information to improve matters for some central banks. The incentive to separate themselves from less inflation-averse types makes it credible that central banks will choose lower inflation than they would if there were no private information. The benefits of the private information are not spread evenly, however, across

central banks. Proposition 4 suggests that less inflation-averse types benefit more as they are thought to be more inflation averse than they are and more inflation-averse types are thought to be less inflation averse than they are. Thus, less inflation-averse central banks are made better off by the presence of private information. But if a central bank is sufficiently inflation averse, it appears possible that it could be made worse off. This turns out not to be the case.

Part (ii) says that increased transparency benefits all central banks and all societies. It lessens the inflation bias associated with the time-inconsistency problem without affecting stabilisation. This is sufficiently important to less inflation-averse types that it offsets the negative impact of the higher period-one inflationary expectations.

Part (ii) is in contrast to the results in Cukierman and Meltzer [9] and Faust and Svensson [13] where an increase in transparency can *lower* average (over χ) central bank expected welfare. The reason for the difference may be due to a different specification of the distributions of shocks and the specification of preferences. Rather than assuming a time-varying output shock, μ_t , Cukierman and Meltzer assume that the private-information component of preferences is time-varying, following an AR(1) process with a mean-zero normally distributed error. This means that their analogue of χ can be negative; indeed, as the fraction of preferences accounted for by the unknown component goes to one, it becomes as likely to be negative as positive. As a result, policy makers may dislike output and may want to produce less output than is expected. This means that, not only do high- χ types benefit at times from being thought less inflation averse than they are, but low- χ types may benefit at times from being thought more inflation-averse than they are.

4.2 Numerical results

In this subsection I assume that the inflation control errors are normally distributed as in Cukierman and Meltzer [9] and I solve the functional equation (16) for $a_0(\cdot, \cdot)$. The functional equation is solved using a variant of Nyström's method due to Tauchen and

Hussey [29].¹⁵

I continue to assume that χ is uniformly distributed. To ensure a positive weight is always put on output, I assume a mean-one gamma distribution for the Phillips curve shock. I assume that $\beta = 0.8$ and consider both a case where the central bank population is relatively inflation averse ($\hat{\chi} = 2$) and a case where it is less inflation averse ($\hat{\chi} = 4$). For each of these cases I consider different degrees of initial uncertainty about the policy maker population ($\chi_U = \hat{\chi} + 0.5, 1.0, 1.5$) and different variances for the Phillip's curve shock ($\sigma_\mu^2 = 0.5, 1.0, 1.5$). For each scenario I let $\sigma_\epsilon^2 = 0.5, 1.0, \dots, 5.0$. I continue to define the central bank's expected welfare and society's expected welfare as in the previous section. The result is stark.

Numerical Result. *In all of the numerical experiments considered, increased transparency increased the expected welfare of all central bank types and all society types.*

While these experiments hardly exhaust the parameter space, this result does suggest that it cannot be usual for transparency to be undesirable. This is again in contrast to Cukierman and Meltzer's analytical results and is in agreement with Faust and Svensson [13]'s more extensive numerical results for their different preference specification.

4.3 Comparison across regimes

In this section I compare the non-transparent regimes of Sections 3 and 4 with the transparent regime of Section 2. At first glance it might seem that this is unnecessary. One might simply use the non-transparent model and allow σ_ϵ^2 to go to zero. This is the strategy of Cukierman and Meltzer [9] and Faust and Svensson [13].¹⁶ The solution to the analytical model in subsection 4.1 is not valid for small values of σ_ϵ^2 , but one might use the numerical model of subsection 4.2, which is valid for arbitrarily small values of σ_ϵ^2 . This tactic seems questionable however. With perfect transparency there are an uncountably infinite number of Bayesian equilibria; what would ensure that the limit

¹⁵The program is written in Lahey-Fujitsu Fortran 90/95. Details are available on request.

¹⁶Faust and Svensson [13] view the case where $\sigma_\epsilon^2 \rightarrow 0$ as equivalent to the case where planned inflation is observable; Cukierman and Meltzer [9] analyse the case where $\sigma_\epsilon^2 \rightarrow 0$ but do not assert that it is equivalent to the transparent case.

would be the separating equilibrium that is the unique equilibrium possessing sensible out-of-equilibrium beliefs?

Equations (6) and (8) imply that in the model of Section 2, the least-inflation averse type chooses his within-period optimum and $\partial a_0(\chi, \mu_0)/\partial \chi \rightarrow \infty$ as $\chi \rightarrow \chi_U$. The intuition, bolstered by Propositions 2 and 4 (for not-too-small values of σ_ϵ^2), is that with non-transparency *all* types of central banks set planned inflation below their within-period optimum and that, moreover, increased transparency lowers planned inflation and flattens the planned-inflation rule. This suggests that the equilibrium that prevails when σ_ϵ^2 goes to zero may be qualitatively different than the one where σ_ϵ^2 equals zero.

This is also the case in the numerical experiments of Section 4.2. Figure 1 depicts the case where $\beta = 1.0$, $[\chi_L, \chi_U] = [3.5, 4.5]$, $\sigma_\mu^2 = 0$: a typical example. It is apparent that as σ_ϵ^2 becomes small, the equilibrium does not converge to the equilibrium where $\sigma_\epsilon^2 = 0$. Instead, the planned inflation rule flattens, suggesting that, in the limit, the equilibrium is one of the pooling equilibria, rather than a separating equilibrium. This is consistent with Carlsson and Dasgupta's [5] theoretical result for a two-action signalling model. Their intuition is that as the equilibrium in the noisy signalling model is never revealing, in the limit as the variance of the noise goes to zero, the senders of the signal must pool.

As pooling equilibria require implausible out-of-equilibrium beliefs, I continue to view the separating outcome of Section 2 as the sensible outcome for the case of perfect transparency. Thus, to compare transparent and non-transparent regimes I compare the outcomes in the two different models of Section 2 and Sections 3 and 4. An immediate result follows from equation (8) and Proposition 3:

Proposition 6 *Transparent central banks do not necessarily choose lower planned inflation than non-transparent central banks.*

In the transparent regime, the least inflation-averse type has no incentive to signal as his type is revealed. This is not the case in the non-transparent regime.

A further difficulty with the comparison between regimes is that when μ_0 is sufficiently small and the central bank is transparent, a separating equilibrium may fail to exist and it is not possible to say what the outcome is in this case. I side-step this problem by assuming in this subsection that $\sigma_\mu^2 = 0$. Otherwise, I consider the same set of numerical experiments as in the previous section. The result is as follows.

Numerical Result. *In all of the numerical experiments considered, the expected welfare of all central bank types and all society types was higher with transparency than non-transparency.*

This appears to be a result of the lower inflation bias in the case of perfect transparency.

5 Conclusion

This paper demonstrates that increased central bank transparency leads to lower inflation without affecting the central bank's stabilisation role. Surprisingly, increased transparency need not improve the public's ability to infer the central bank's private information. The results suggest that increased transparency increases the welfare of all central bank types and all society types. Completely transparent central banks are shown to produce qualitatively different inflation outcomes than would occur if there were a small amount non-transparency. Numerical results suggest that *all* central banks and societies are better off with complete transparency than with any amount of non-transparency.

The above results were obtained using a particular and popular notion of transparency. The central bank is assumed to have private information about its preferences. It then chooses planned inflation, which it attempts to implement by carrying out an action. An inflation-control error causes planned and actual inflation to diverge. The choice of planned inflation is made with the intent of directly affecting the central bank's welfare and also with the intent of affecting the public's inference about the central bank's type. With complete transparency, the public observes the central bank's action and in equilibrium this action reveals the central bank's type.

With non-transparency, the central bank's action, and hence its intent, are not observed. An unobservable component of the inflation-control error keeps the public from inferring the central bank's action and its planned inflation from its observation of inflation. Thus, the public is only able to form an estimate of the central bank's private information. Here, the degree of non-transparency is measured by the fraction of the inflation-control error that is unobservable.

Other notions of non-transparency are possible. An alternative would be a scenario where a shock causes the central bank's action to be a random function of its planned inflation. If this shock is unobservable, the scenario is similar to the one analysed here; the public solves a statistical inference problem to estimate the central bank's type. However, a plausible alternative is that the shock is the private information of the central bank, perhaps its inflation forecast or its current view of the inflation-transmission mechanism. While this situation is realistic it is complicated to model. The outcome would depend upon the number and types of shocks that are the private information of the central bank and the number of ways that the central bank could signal its private information. Analysis is difficult because some central banks might face high costs in signalling some types of private information and low costs in signalling other types. It is possible that the outcome is a separating equilibrium as in Section 2 or there may be multiple partially revealing equilibria.¹⁷

¹⁷See Engers [12] for a general proof of existence in an oligopoly model with multiple signals and many types of private information. See Stamland [27] for a model with partially revealing equilibria.

6 Appendix

Proof of Proposition 1. Suppose that $0 < a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0) \leq w$. Then $\pi_1^e = E[\chi|\pi_0, \mu_0, a_0^*(\cdot, \mu_0)] =$

$$\begin{cases} \pi_{1L}^e(\pi_0, \mu_0) = \frac{\chi_L + a_0^{*-1}(\pi_0 + w/2, \mu_0)}{2} & \text{if } \pi_0 \in [a_0^*(\chi_L, \mu_0) - w/2, a_0^*(\chi_U, \mu_0) - w/2] \\ \hat{\chi} & \text{if } \pi_0 \in [a_0^*(\chi_U, \mu_0) - w/2, a_0^*(\chi_L, \mu_0) + w/2] \\ \pi_{1U}^e(\pi_0, \mu_0) = \frac{a_0^{*-1}(\pi_0 - w/2, \mu_0) + \chi_U}{2} & \text{if } \pi_0 \in [a_0^*(\chi_L, \mu_0) + w/2, a_0^*(\chi_U, \mu_0) + w/2]. \end{cases} \quad (23)$$

We have $\pi_{1L}^e(a_0^*(\chi_L, \mu_0) - w/2, \mu_0) = \chi_L$ and $\pi_{1U}^e(a_0^*(\chi_U, \mu_0) + w/2, \mu_0) = \chi_U$; hence the assumption that beliefs are monotonic implies that out-of-equilibrium beliefs are given by

$$\pi_1^e = E[\chi|\pi_0, \mu_0, a_0^*(\cdot, \mu_0)] = \begin{cases} \chi_L & \text{if } \pi_0 < a_0^*(\chi_L, \mu_0) - w/2 \\ \chi_U & \text{if } \pi_0 > a_0^*(\chi_U, \mu_0) + w/2. \end{cases} \quad (24)$$

By equation (23), if $a_0 \in [a_0^*(\chi_L, \mu_0), a_0^*(\chi_U, \mu_0)]$, then $E[\pi_1^e|a_0, \mu_0, a_0^*(\cdot, \mu_0)] =$

$$\frac{1}{w} \left[\int_{a_0 - w/2}^{a_0^*(\chi_U, \mu_0) - w/2} \pi_{1L}^e(\pi_0, \mu_0) d\pi_0 + \hat{\chi} \int_{a_0^*(\chi_U, \mu_0) - w/2}^{a_0^*(\chi_L, \mu_0) + w/2} d\pi_0 + \int_{a_0^*(\chi_L, \mu_0) + w/2}^{a_0 + w/2} \pi_{1U}^e(\pi_0, \mu_0) d\pi_0 \right] \quad (25)$$

and $\partial E[\pi_1^e|a_0, \mu_0, a_0^*(\cdot, \mu_0)]/\partial a_0 = [\pi_{1U}^e(a_0 + w/2, \mu_0) - \pi_{1L}^e(a_0 - w/2, \mu_0)]/w = (\chi_U - \chi_L)/(2w) = \alpha/\beta$. Thus, by equation (13), the solution to the first-order condition associated with the central bank's optimisation problem is given by equation (20). The first inequality in assumption (19) implies $a_0(\chi, \mu_0) > 0$ for every $\chi \in [\chi_L, \chi_U]$ and the second inequality in assumption (19) is necessary and sufficient for $0 \leq a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0) \leq w$.

By (25) and (20), $E[\pi_1^e|a_0, \mu_0, a_0^*(\cdot, \mu_0)] =$

$$\begin{aligned} & \int_{a_0 - w/2}^{a_0(\chi_U, \mu_0) - w/2} \frac{\chi_L + \frac{\pi_0 + w/2}{\mu_0 - \alpha}}{2w} d\pi_0 + \frac{\hat{\chi}}{w} \int_{a_0(\chi_U, \mu_0) - w/2}^{a_0(\chi_L, \mu_0) + w/2} d\pi_0 + \int_{a_0(\chi_L, \mu_0) + w/2}^{a_0 + w/2} \frac{\frac{\pi_0 - w/2}{\mu_0 - \alpha} + \chi_U}{2w} d\pi_0 \\ & = \hat{\chi} + (\alpha/\beta)[a_0 - \hat{\chi}(\mu_0 - \alpha)]. \end{aligned} \quad (26)$$

Equations (20) and (26) yield equation (21).

I now show that no central bank has an incentive to deviate from the equilibrium by setting a_0 strictly greater than $a_0(\chi_U, \mu_0)$ or strictly less than $a_0(\chi_U, \mu_0)$. To show the former, it is sufficient to show that it is true for type χ_U . If type χ_U were to deviate and set $a_0 > a_0(\chi_U, \mu_0)$, then equations (23) and (24) imply

$$E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) = \begin{cases} \frac{1}{w} \left[\hat{\chi} \int_{a_0-w/2}^{a_0(\chi_L, \mu_0)+w/2} d\pi_0 + \int_{a_0(\chi_L, \mu_0)+w/2}^{a_0(\chi_U, \mu_0)+w/2} \pi_{1U}^e(\pi_0, \mu_0) d\pi_0 + \right. \\ \left. \chi_U \int_{a_0(\chi_U, \mu_0)+w/2}^{a_0+w/2} d\pi_0 \right] & \text{if } a_0 \in [a_0(\chi_U, \mu_0), a_0(\chi_L, \mu_0) + w] \\ \frac{1}{w} \left[\int_{a_0-w/2}^{a_0(\chi_U, \mu_0)+w/2} \pi_{1U}^e(\pi_0, \mu_0) d\pi_0 + \chi_U \int_{a_0(\chi_U, \mu_0)+w/2}^{a_0+w/2} d\pi_0 \right] \\ & \text{if } a_0 \in [a_0(\chi_L, \mu_0) + w, a_0(\chi_U, \mu_0) + w] \\ \chi_U & \text{if } a_0 > a_0(\chi_U, \mu_0) + w. \end{cases} \quad (27)$$

Using equations (20), (21) and (23), equation (27) can be rewritten as

$$E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) = \begin{cases} \hat{\chi} + \frac{\alpha}{\beta} [a_0 - a_0(\hat{\chi}, \mu_0)] \\ & \text{if } a_0 \in [a_0(\chi_U, \mu_0), a_0(\chi_L, \mu_0) + w] \\ \frac{\chi_U}{4w} [2a_0 - a_0(\chi_U, \mu_0) + 2w] - \frac{(a_0-w)^2}{4w(\mu_0-\alpha)} \\ & \text{if } a_0 \in [a_0(\chi_L, \mu_0) + w, a_0(\chi_U, \mu_0) + w] \\ \chi_U & \text{if } a_0 > a_0(\chi_U, \mu_0) + w. \end{cases} \quad (28)$$

By equation (13), the type- χ_U central bank's welfare is decreasing in a_0 if $a_0 > \chi_U \{ \mu_0 - \beta \partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 \}$. For $a_0 \in [a_0(\chi_U, \mu_0), a_0(\chi_L, \mu_0) + w]$, equation (28) implies that $\partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 = \alpha / \beta$; hence, by equation (20) this is true if $a_0 > a_0(\chi_U, \mu_0)$, which is true. For $a_0 \in [a_0(\chi_L, \mu_0) + w, a_0(\chi_U, \mu_0) + w]$, $\partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 = \chi_U / (2w) - (a_0 - w) / [2w(\mu_0 - \alpha)]$; hence, welfare is decreasing in a_0 if $a_0 \{ 1 - \alpha \chi_U / [a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0)] \} > \chi_U \{ \mu_0 - \alpha [a_0(\chi_U, \mu_0) + w] / [a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0)] \}$. That this is true follows from $w \geq a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0) \geq \alpha \chi_U$ (the latter inequality is true if and only if the first inequality in assumption (19) holds) and $a_0 > a_0(\chi_L, \mu_0) + w$. For $a_0 > a_0(\chi_U, \mu_0) + w$, $\partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 = 0$, thus, welfare is decreasing in a_0

if $a_0 > \chi_U \mu_0$. This is true if $\chi_U \mu_0 < a_0(\chi_U, \mu_0) + w$, which is true if the first inequality in assumption (19) holds. Thus, the type- χ_U central bank will not deviate. (If $\chi_U \mu_0$ were strictly greater than $a_0(\chi_U, \mu_0) + w$, it is easy to show that type χ_U would deviate.)

I now show that no central bank has an incentive to deviate from the equilibrium by setting $a_0 < a_0(\chi_L, \mu_0)$. It is sufficient to show that it is true for type χ_L . If type χ_L were to deviate and set $a_0 < a_0(\chi_L, \mu_0)$, then equation (??) implies that

$$E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) = \begin{cases} \frac{1}{w} \left[\chi_L \int_{a_0-w/2}^{a_0(\chi_L, \mu_0)-w/2} d\pi_0 + \int_{a_0(\chi_L, \mu_0)-w/2}^{a_0(\chi_U, \mu_0)-w/2} \pi_{1L}^e(\pi_0, \mu_0) d\pi_0 + \right. \\ \left. \hat{\chi} \int_{a_0(\chi_U, \mu_0)-w/2}^{a_0+w/2} d\pi_0 \right] & \text{if } a_0 \in [a_0(\chi_U, \mu_0) - w, a_0(\chi_L, \mu_0)] \\ \frac{1}{w} \left[\chi_L \int_{a_0-w/2}^{a_0(\chi_L, \mu_0)-w/2} d\pi_0 + \int_{a_0(\chi_L, \mu_0)-w/2}^{a_0+w/2} \pi_{1L}^e(\pi_0, \mu_0) d\pi_0 \right] \\ & \text{if } a_0 \in [a_0(\chi_L, \mu_0) - w, a_0(\chi_U, \mu_0) - w] \\ \chi_L & \text{if } a_0 < a_0(\chi_L, \mu_0) - w. \end{cases} \quad (29)$$

Using equations (20), (21) and (23), equation (29) can be rewritten as

$$E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) = \begin{cases} \hat{\chi} + \frac{\alpha}{\beta} [a_0 - a_0(\hat{\chi}, \mu_0)] \\ & \text{if } a_0 \in [a_0(\chi_U, \mu_0) - w, a_0(\chi_L, \mu_0)] \\ \frac{\chi_L}{4w} [a_0(\chi_L, \mu_0) - 2a_0 + 2w] + \frac{(a_0+w)^2}{4w(\mu_0-\alpha)} \\ & \text{if } a_0 \in [a_0(\chi_L, \mu_0) - w, a_0(\chi_U, \mu_0) - w] \\ \chi_L & \text{if } a_0 < a_0(\chi_L, \mu_0) - w. \end{cases} \quad (30)$$

By equation (13), the type- χ_L central bank's welfare is increasing in a_0 if $a_0 < \chi_L \{ \mu_0 - \beta \partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 \}$. For $a_0 \in [a_0(\chi_U, \mu_0) - w, a_0(\chi_L, \mu_0)]$, equation (28) implies that $\partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 = \alpha / \beta$; hence, by equation (20) this is true if $a_0 < a_0(\chi_L, \mu_0)$, which is true. For $a_0 \in [a_0(\chi_L, \mu_0) - w, a_0(\chi_U, \mu_0) - w]$, $\partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 = -\chi_L / (2w) + (a_0 + w) / [2w(\mu_0 - \alpha)]$; hence, welfare is increasing in a_0 if $a_0 \{ 1 + \alpha \chi_L / [a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0)] \} < \chi_L \{ \mu_0 + \alpha [a_0(\chi_L, \mu_0) - w] / [a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0)] \}$. That this is true follows from $w \geq a_0(\chi_U, \mu_0) - a_0(\chi_L, \mu_0)$ and $a_0 < a_0(\chi_U, \mu_0) - w$. For $a_0 < a_0(\chi_L, \mu_0) - w$, $\partial E(\pi_1^e | \mu_0, a_0(\cdot, \mu_0)) / \partial a_0 = 0$, thus, welfare is increasing in a_0 if

$a_0 < \chi_L \mu_0$, which is true. Thus type χ_L will not deviate.

Proof of Proposition 3. $E(\chi|\pi_0, \chi) = E(\pi_1^e|\chi, \mu_0)$ because μ_1 has mean one and is uncorrelated with the other variables in the model. Thus, by equation (20), $\chi > (<)$ $E(\chi|\pi_0, \chi)$ iff $1 > \alpha(\mu_0 - \alpha)/\beta$. This follows by the second inequality in (19). This proves part (i). By equation (20), $Var(a_0|\mu_0) = [(\mu_0 - \alpha)(\chi_U - \chi_L)/2]^2/3$. Differentiating yields $\partial Var(a_0|\mu_0)/\partial \epsilon_t^2 = (\chi_U - \chi_L)^2 \alpha(\mu_0 - \alpha)/36 > 0$. This yields part (ii). Part (iii) follows from the derivate of the expected squared forecasting error equaling $[(\chi_U - \chi_L)/6] \alpha [1 - \alpha(\mu_0 - \alpha)/\beta] (\mu_0 - 2\alpha) > (<)$ iff $\mu_0 > (<)2\alpha$.

Proof of Proposition 5. By equations (20) and (21), with private information and nontransparency

$$E[\chi(\pi_0 - \pi_0^e)\mu_0 - \pi_0^2/2 - \beta\chi\pi_1^e\mu_1|\chi] = \{\sigma_\mu^2\chi + (1 - \alpha)^2(\chi - 2\hat{\chi}) - 2\beta\hat{\chi}\}\chi/2 \quad (31)$$

With full information

$$E[\chi(\pi_0 - \pi_0^e)\mu_0 - \pi_0^2/2 - \beta\chi\pi_1^e\mu_1|\chi] = (\sigma_\mu^2 - 1 - 2\beta)\chi^2/2 \quad (32)$$

As the other terms in the objective function are the same across regimes, equations (31) and (32) imply that the type- χ central bank prefers private information and non-transparency to full information iff

$$(1 - \alpha)^2(\chi - 2\hat{\chi}) + 2\beta(\chi - \hat{\chi}) + \chi > 0 \quad (33)$$

The left-hand side of the above inequality is increasing in χ and true for $\chi = \hat{\chi}$; hence it is true for all $\chi > \hat{\chi}$. This establishes the first statement in part (i). It is clearly not true at $\chi = \chi_L = 0$; hence a continuity argument establishes the second statement in part (i).

By equations (26) and (21), with private information and nontransparency

$$E [\chi^* (\pi_0 - \pi_0^e) \mu_0 - \pi_0^2/2 - \beta \chi^* \pi_1^e \mu_1] = \hat{\chi} \chi^* \sigma_\mu^2 - E(\chi^2) [\sigma_\mu^2 + (1 - \alpha)^2] / 2 - \beta \chi^* \hat{\chi} \quad (34)$$

With full information

$$E [\chi^* (\pi_0 - \pi_0^e) \mu_0 - \pi_0^2/2 - \beta \chi^* \pi_1^e \mu_1] = \chi^* \hat{\chi} \sigma_\mu^2 - E(\chi^2) (\sigma_\mu^2 + 1) / 2 - \beta \chi^* \hat{\chi} \quad (35)$$

The right-hand side of equation (34) is larger than the right-hand side of equation (35). As the other terms in the objective function are the same across regimes, this establishes part (ii)

As the other terms in the objective function do not depend on σ_ϵ^2 , to prove part (iii) it is sufficient to show that the derivative of the right-hand side of equation (31) with respect to σ_ϵ^2 is negative. Differentiating yields $\alpha(1 - \alpha)(\chi - 2\hat{\chi})\chi/6 < 0$. Similarly, differentiating the right-hand side of equation (35) with respect to σ_ϵ^2 yields $-\alpha(1 - \alpha)E(\chi^2)/6 < 0$, proving part (iv).

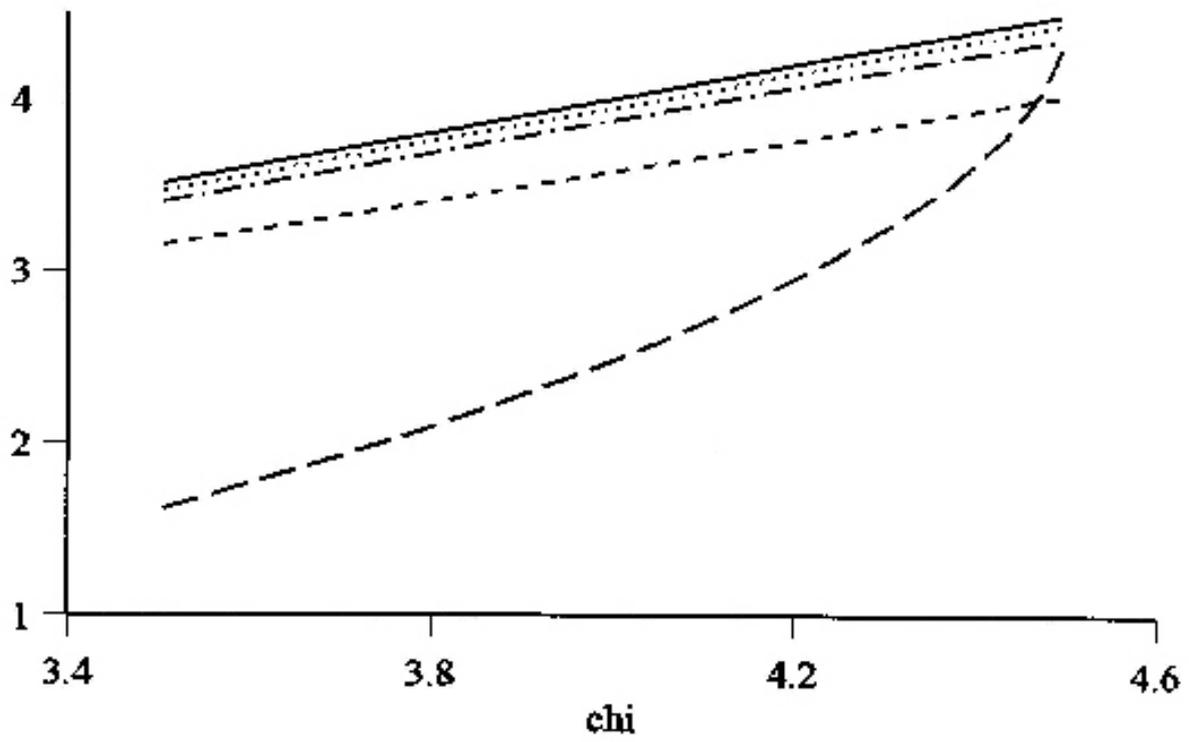
References

- [1] Andrews, Edmund L., 2004, "The Successor to Greenspan Has a Very Tough Act to Follow." *New York Times* 6 Dec.
- [2] Atkeson, A. and P. Kehoe, 2003. "The Advantage of Transparency in Monetary Policy Instruments," Federal Reserve Bank of Minneapolis Research Department Staff Report 297.
- [3] Backus, D. and J. Driffill, 1985. "Inflation and Reputation." *American Economic Review* 75, 530-538.
- [4] Barro, R. and D. Gordon, 1983. "A Positive Theory of Monetary Policy in a Natural rate Model," *Journal of Political Economy* 91, 589-610.
- [5] Carlsson, H. and S. Dasgupta, 1997. "Noise-Proof Equilibria in Two-Action Signalling Games." *Journal of Economic Theory* 77, 432-460.
- [6] Cho, I. and D. Kreps, 1987. "Signalling Games and Stable Equilibria." *Quarterly Journal of Economics* 102, 179-221.

- [7] Cukierman, A., 1992. *Central Bank Strategy, Credibility and Independence: Theory and Evidence*. MIT Press, London.
- [8] Cukierman, A., 2001, "Accountability, Credibility, Transparency and Stabilization Policy in the Eurosystem," in ...
- [9] Cukierman, A. and A. Meltzer, 1986. "A Theory of Ambiguity, Credibility and Inflation under Discretion and Asymmetric Information." *Econometrica* 54, 1099-1128.
- [10] Dewatripont, M., I. Jewitt and J. Tirole, 1999. "The Economics of Career Concerns, Part I: Comparing Information Structures" 66, 183-198.
- [11] Eijffinger, S. and P. Geraats, 2003, "How Transparent are Central Banks?" unpublished paper.
- [12] Engers, M., 1987. "Signalling with Many Signals." *Econometrica* 55, 663-674.
- [13] Faust, J. and L. Svensson, 2001. "Transparency and Credibility: Monetary policy with Unobservable Goals." *International Economic Review* 42, 369-397.
- [14] Faust, J. and L. Svensson, 2002. "The Equilibrium Degree of Transparency and Control in Monetary Policy," *Journal of Money, Credit and Banking* 34, 520-539.
- [15] Geraats, P., 2001. "Precommitment, Transparency and Monetary Policy," Bundesbank Discussion Paper 12/01.
- [16] Geraats, P., 2002. "Central Bank Transparency," *Economic Journal* 112, F532-F565.
- [17] Geraats, P. and S. Eijffinger, 2002. "How Transparent are Central Banks?" CEPR Working Paper 3188.
- [18] Gersbach, H., 1998. "On the Negative Social Value of Central Banks' Knowledge Transparency," unpublished paper.
- [19] Jensen, H., 2002. "Optimal Degrees of Transparency in Monetary Policy Making," *Scandinavian Journal of Economics* 104, 399-422.
- [20] Lucas, R., 1973. "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review* 63, 326-334.
- [21] Mailleth, G., 1987. "Incentive Compatibility in Signalling Games with a Continuum of Types." *Econometrica* 55, 1349-1365.
- [22] Matthews, S. and L. Mirman, 1983. "Equilibrium Limit Pricing: the Effects of Private Information and Stochastic Demand." *Econometrica* 50, 981-996.
- [23] Ramey, G., 1996. "D1 Signalling Equilibria with Multiple Signals and a Continuum of Types." *Journal of Economic Theory* 69, 508-531.
- [24] Rogoff, K., 1985. "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics* 100, 1169-1189.

- [25] Sibert, A., 2002. "Monetary Policy with Uncertain Central Bank Preferences." *European Economic Review* 46, 1093-1109.
- [26] Sibert, A., 2003. "Monetary Policy Committees: Individual and Collective Reputations," *Review of Economic Studies*, 649-666.
- [27] Stamland, T., 1999. "Partially Informative Signalling." *Journal of Economic Theory* 89, 148-161
- [28] Spence, M., 1973. "Job Market Signalling." *Quarterly Journal of Economics* 87, 355-374.
- [29] Tauchen, G. and R. Hussey, 1991. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 371-396.
- [30] Vickers, J., 1986. "Signalling in a Model of Monetary Policy with Incomplete Information." *Oxford Economic Papers* 38, 433-455.
- [31] Vincent, D. R., 1998. "Repeated Signalling Games and Dynamic Trading Relationships." *International Economic Review* 39, 275-293.

Figure 1. Equilibrium Inflation



- no private information
- - - no noise
- variance = 5.0
- · - · variance = 2.0
- - - variance = 0.5